α-MATHEMATICS

Grade 11 **Time: $2\frac{1}{2}$ hours**

Examinator: Pieter van Onselen **Total: 150 marks**

Moderator: Anna Muller

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

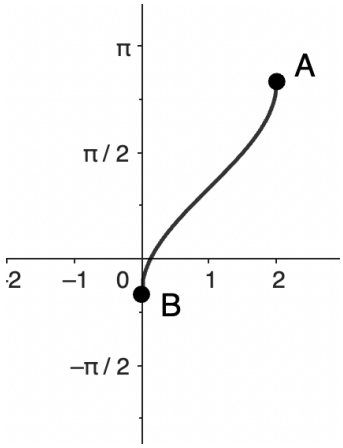
1. This question paper consists of 8 pages and an answer sheet of 2 pages.
2. Answer ALL 6 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, must be given correct to two decimal places, where necessary.
6. Clearly show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neatly and legibly.

Question 1 **[20 marks]**

This question must be answered on the answer sheet.

Every question has **ONLY** one correct answer for TWO marks each. Mark the correct answer with an X on the answer sheet.

1.1 The sketch shows the function $f(x) = bgsin(x + a) + b$ with points $A\left(2; \frac{5\pi}{6}\right)$ and $B\left(0; -\frac{\pi}{6}\right)$.



The values of a and b are:

(A) $a = 1,\ b = -\frac{\pi}{6}$ **(B)** $a = 1,\ b = \frac{\pi}{3}$

(C) $a = -1,\ b = \frac{\pi}{3}$ **(D)** $a = -1,\ b = -\frac{\pi}{6}$

1.2 The function $f(x) = \frac{1}{(3+2x)^3}$ converges if

(A) $|x| > \frac{2}{3}$ **(B)** $|x| < 2$

(C) $|x| < \frac{1}{2}$ **(D)** $|x| < \frac{3}{2}$

1.3 Solve for x in $|x + 2| + 1 = 0$.

(A) $x = -1$

(B) $x \in \mathbb{R}$

(C) $x = 1$ of $x = -1$

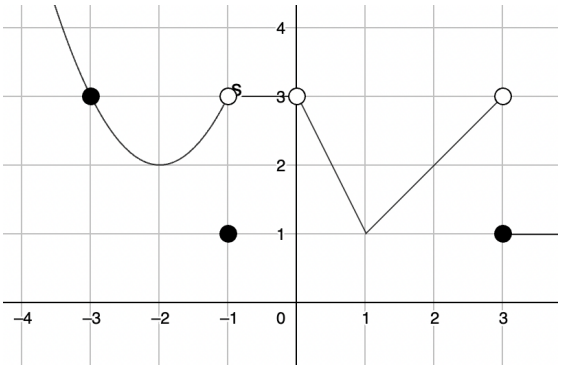
(D) No solution for x

1.4 If $x = 3 - 2i$ is a zero of $x^4 - 6x^3 + 4x^2 + 54x - 117$, then another zero will be:

(A) $x = -3 - 2i$ **(B)** $x = 2i + 3$

(C) $x = -3 + 2i$ **(D)** $x = 2i - 3$

1.5 Given the sketch of $g(x)$.



Which of the following statements are true?

1	$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^+} g(x) \neq g(1)$
2	Jump discontinuity at $x = -1$ and $x = 3$
3	Removable discontinuity at $x = 0$
4	$g(x)$ is differentiable at $x = 1$

(A) $1\ and\ 3$
(B) $3\ and\ 4$
(C) $1{,}3\ and\ 4$
(D) $1{,}2\ and\ 4$

1.6 The two vectors u and w are parallel if:

(A) $u \cdot w = 0$ **(B)** $u \times w = 0$
(C) $|u - w| = 0$ **(D)** $|u + w| = 0$

1.7 How many terms does $x(x - y)^5$ have?

(A) 4 **(B)** 5
(C) 6 **(D)** 7

1.8 Given:

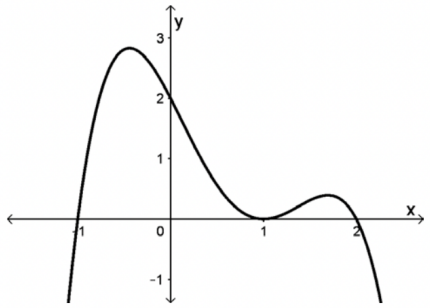
$$f(x) = \begin{cases} 2 & as\ x \leq 1 \\ ax - 1 & as\ x > 1 \end{cases}$$

If f is continuous at $x = 1$, then

(A) $a = 1$ (B) $a = 0$

(C) $a = 2$ (D) $a = 3$

1.9 Given the function $f(x)$ with turning points at $x = -\frac{1}{2}, x = 1$ and $x = 1{,}7$.



If Newton's method is used to find the x-intercept, which starting value can be used to find the x-intercept at $x = 1$.

(A) $x_0 = -0{,}8$ **(B)** $x_0 = 1{,}7$

(C) $x_0 = -0{,}4$ **(D)** $x_0 = 2{,}1$

1.10 If we decompose $\frac{f(x)}{(x^2+1)^2(x-1)}$ into partial fractions, we get:

(A) $\frac{Ax+B}{(x^2+1)^2} + \frac{\mathrm{C}}{(x-1)}$

(B) $\frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} + \frac{\mathrm{E}}{(x-1)}$

(C) $\frac{A}{(x^2+1)} + \frac{B}{(x^2+1)^2} + \frac{\mathrm{C}}{(x-1)}$

(D) $\frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)} + \frac{\mathrm{E}}{(x-1)}$

Question 2 **[17 marks]**

2.1 Given $h(x) = -|2 - x| + 1$ and $g(x) = 2x - 2$.

2.1.1 Determine the salient point of $h(x)$. (2)

2.1.2 Find the value(s) of x for which $h(x) < -2$. (4)

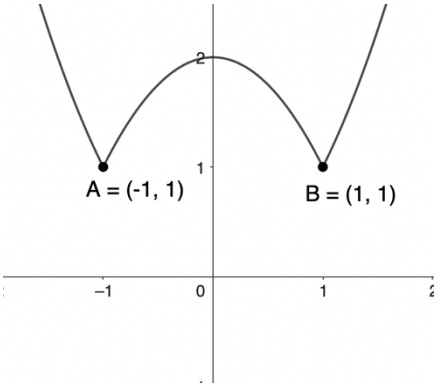
2.1.3 Find the value(s) of x for which $g(x) = h(x)$. (5)

2.2 Sketch the graph h on DIAGRAM SHEET 1. (4)

2.3 The function $f(x) = x^2 - 1$ has x-intercepts at $x = 1$ and $x = -1$ and a y-intercept $y = -1$.

$f(x)$ is transformed to form a new function $h(x)$. (2)

The sketch shows $h(x)$. If $A(-1; 1)$ and $B(1; 1)$, determine the equation of $h(x)$.



Question 3 **[21 marks]**

3.1 If $x - 1 - 3i$ is a zero of $f(x) = 2x^3 - 5x^2 + 22x - 10$, factorise $f(x)$ fully in $R[x]$. (5)

3.2 Decompose $\frac{2x^2-1}{x^4+x^2}$ fully into partial fractions. (8)

3.3 Use mathematical induction to prove that: (8)

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

Question 4 **[8 marks]**

4.1 Given that the fourth term in the power series of $\sqrt[3]{1-ax}$ is $-\frac{5}{3}x^3$, (4)

determine the value of a.

4.2 Find the coefficient of x^4 in $\left(x^4+\frac{2}{x}\right)^6$. (4)

Question 5 **[18 marks]**

5.1 Given three points $A(-1;0;-2)$, $B(1;x;1)$ and $C(2;1;1)$ of triangle ABC.

5.1.1 Determine vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in terms of x. (2)

5.1.2 If ΔABC is a right-angled triangle with $A\widehat{B}C=\frac{\pi}{2}$, use the dot product to find x, where $x>0$ and $x\in\mathbb{R}$. (4)

5.2 Given the following vectors:

$\boldsymbol{a}=-i+2j+k$

$\boldsymbol{b}=(-2;1;5)$

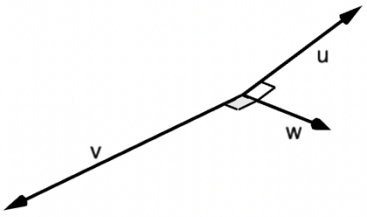
5.2.1 Find $|\boldsymbol{a}|$ en $|\boldsymbol{b}|$, the magnitudes of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. (2)

5.2.2 Using direction cosines, determine the angle between vector $\boldsymbol{a}$ and the y-axis. (2)

5.2.3 Determine the angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. (4)

5.3 The sketch shows two vectors $\boldsymbol{u}=(1.;2;1)$ and $\boldsymbol{v}=(-2;-4;-1)$.

Determine the vector $\boldsymbol{w}$ that is perpendicular to both $\boldsymbol{u}$ and $\boldsymbol{v}$. (4)



Question 6 **[27 marks]**

6.1 Given:

$$f(x)=\begin{cases} -x+\frac{\pi}{2} & if \quad x\leq 0 \\ arccos(x) & if \quad 0\leq x<1 \\ (x-1)^2-1 & if \quad x\geq 1 \end{cases}$$

6.1.1 Discuss, with reasons, the continuity of $f(x)$ at $x=1$. (4)

6.1.2 Given that $f(x)$ is continuous at $x=0$, determine if $f(x)$ is differentiable at $x=0$. (5)

6.2 Differentiate the following:

6.2.1 $D_x\left[(\sin^2 3x)\left(\frac{1}{x^2}\right)\right]$ (4)

6.2.2 $f'(x)$ as $f(x)=\frac{arcsin(2x)}{\cot(5x)}$ (4)

6.3 Given $f(x)=3arctan(x+1)$

6.3.1 Sketch the graph of f. Use DIAGRAM SHEET 2. (4)

6.3.2 Find the inverse of f in die form $y=\cdots$ (3)

6.3.3 Determine the gradient of f at $x=1$. (3)

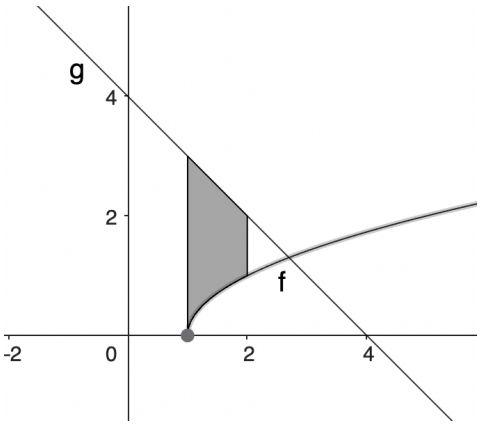
Question 7 **[23 marks]**

7.1 Determine the following integrals.

7.1.1 $\int\left(\frac{3}{\sqrt{x}}+\sec^2(3x)-\frac{\pi}{2}\right)dx$ (4)

7.1.2 $\int\frac{dx}{\sqrt{4-9x^2}}$ (4)

7.2 The sketch shows $f(x)=\sqrt{x-1}$ and $g(x)=4-x$.



7.2.1 Use Newton-Raphson's method to find the x-coordinate of the point of intersection of $f(x)$ and $g(x)$, correct to **five** decimal places. Start with $x = 2$ as the first approximation.

Use $h(x) = 4 - x - \sqrt{x-1}$. (5)

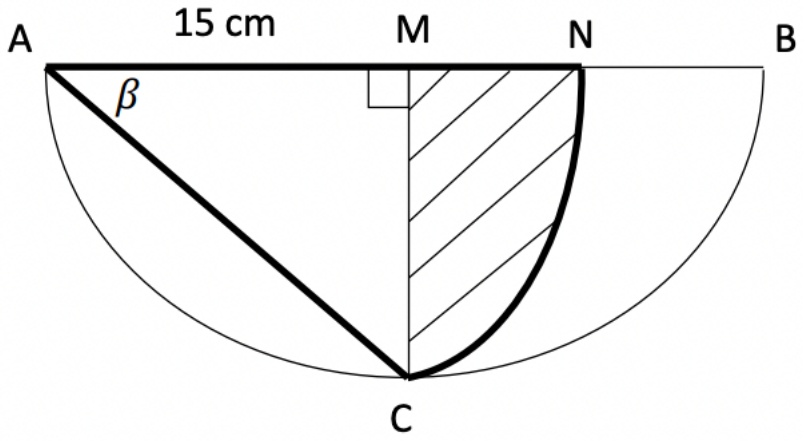
7.2.2 Find the area of the shaded region between $x = 1$ and $x = 2$. (6)

7.2.3 Find the volume of the solid of revolution formed when $f(x)$ is rotated about the x-axis between $x = 2$ en $x = 1$.
Give your answer in terms of π. (4)

Question 8 **[14 marks]**

8.1 ABC is a semicircle with centre M. ACN is a sector with centre A.

$AM = 15\ cm$ and $A\widehat{M}C = \frac{\pi}{2}$.



8.1.1 Determine AC, the radius of sector ACN. Leave the answer in root form. (2)

8.1.2 If $AC = 21$, calculate the perimeter of CMN. (3)

8.1.3 Calculate the area of the shaded region CMN. (5)

8.2 The area of a sector is $20\ cm^2$.

8.2.1 Give the formula for the area of a sector in terms of the arc length(s) and θ. (2)

8.2.2 If the arc length is **doubled**, calculate the **new** area. (2)

- END OF QUESTION PAPER -

Alpha Wiskunde Formuleblad

Alpha Mathematics Formula Sheet

ALGEBRA:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\lvert x\rvert = \begin{cases} x & \text{as / } \textit{if} \quad x \geq 0 \\ -x & \text{as / } \textit{if} \quad x < 0 \end{cases}$	Cramer se reël / *Cramer's rule* $x_i = \frac{\lvert A_i\rvert}{\lvert A\rvert}$
$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$	$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$; mits / *if* $\lvert x\rvert < 1$	

VEKTORE/ *VECTORS*:

$\lvert\boldsymbol{AB}\rvert = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	$\boldsymbol{u} \times \boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$		
$\lvert\boldsymbol{OP}\rvert = \sqrt{a^2 + b^2 + c^2}$	$\boldsymbol{p} \bullet \boldsymbol{q} = \lvert\boldsymbol{p}\rvert\lvert\boldsymbol{q}\rvert \cos\theta$ $\boldsymbol{p} \bullet \boldsymbol{q} = x_1.x_2 + y_1.y_2 + z_1.z_2$	$\boldsymbol{a} \times \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \sin\theta \,.\boldsymbol{n}$	$\alpha = \underset{arccos}{\text{bgcos}}\left(\frac{u_n}{\lvert\boldsymbol{u}\rvert}\right)$

CALCULUS:

$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$	$\int f'\big(g(x)\big).g'(x)\,dx = f\big(g(x)\big) + k$
$V = \pi \int_a^b [f(x)]^2\,dx$	$\int f(x).g'(x)\,dx = f(x).g(x) - \int f'(x).g(x)\,dx + k$

TRIGONOMETRIE / *TRIGONOMETRY:*

In 'n sektor / *In a sector:* $s = r\theta$ en / *and* $A = \frac{1}{2}r^2\theta$		
Identiteite / *Identities*:		
$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$\cot^2 x + 1 = \text{cosec}^2 x$
$\text{cosec}\, x = \frac{1}{\sin x}$	$\sec x = \frac{1}{\cos x}$	$\cot x = \frac{1}{\tan x}$

TABEL MET AFGELEIDES / *TABLE WITH DERIVATIVES*

$F(x)$	$F'(x)$
ax^n	nax^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \,.\, \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \,.\, \cot x$
$\text{bgsin}\, x$ $\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\text{bgcos}\, x$ $\arccos x$	$\dfrac{-1}{\sqrt{1-x^2}}$
$\text{bgtan}\, x$ $\arctan x$	$\dfrac{1}{x^2+1}$
a^x	$a^x . \ln a$
$\log_a x$	$\dfrac{1}{x . \ln a}$
$f(x).g(x)$	$f'(x).g(x) + f(x).g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x).g(x) - f(x).g'(x)}{[g(x)]^2}$
$f[g(x)]$	$f'[g(x)].g'(x)$

Alpha Mathematics Grade 11 - Final examination 2025
ANSWER SHEET

Name en Surname:

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Question Total	1 [20]	2 [17]	3 [21]	4 [8]	5 [20]	6 [25]	7 [23]	8 [14]	TOTAL 150
Learner mark									

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1 Question 2.2

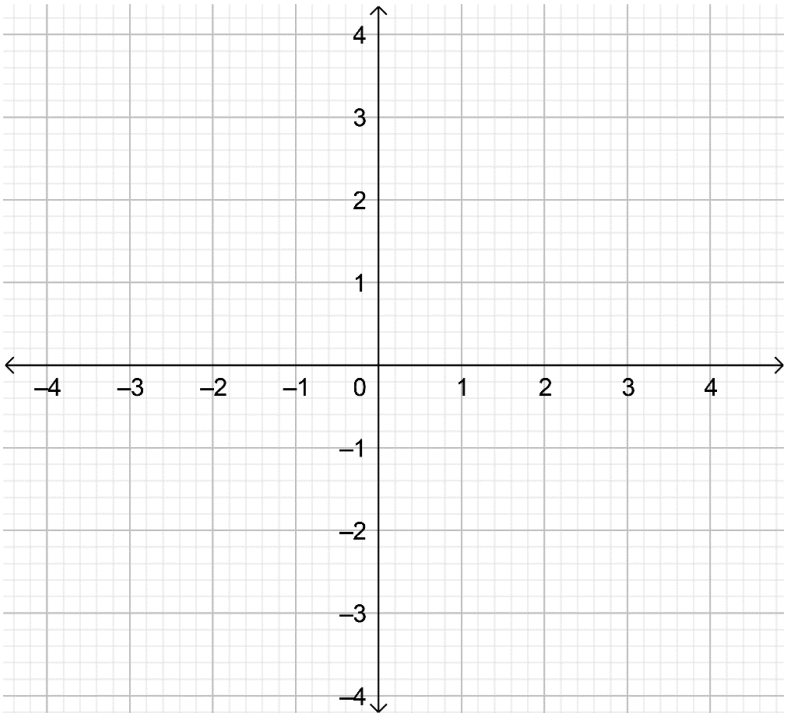

DIAGRAM SHEET 2 Question 6.3.1