α-WISKUNDE/MATHEMATICS

Alpha Wiskunde Graad 11 / ***Alpha Mathematics Grade 11***

Finale eksamen 2025 / ***Final examination 2025***

MEMORANDUM

Totaal / ***Total*****: 150 punte /** ***marks***

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Moderator: Anna Muller

Hierdie memorandum bestaan uit 12 bladsye. /
This memorandum consists of 12 pages.

Vraag / *Question* 1

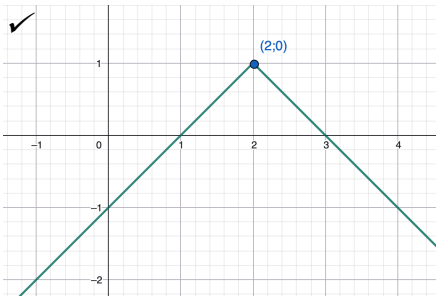
1.1	A	B	**C**	D
1.2	A	B	C	**D**
1.3	A	B	C	**D**
1.4	A	**B**	C	D
1.5	**A**	B	C	D
1.6	A	**B**	C	D
1.7	A	B	**C**	D
1.8	A	B	C	**D**
1.9	A	B	**C**	D
1.10	A	**B**	C	D

Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
1.1	$C \quad bgsin(x-1)+\frac{\pi}{3}$	**(2)**
1.2	$D \quad f(x)=(3-2x)^{-3}$ $=3^{-3}\left(1-\frac{2}{3}x\right)^{-3}$ $\left\|-\frac{2}{3}x\right\|<1$ $\|x\|<\frac{3}{2}$	**(2)**
1.3	$D \quad \|x+2\|=-1$ $Geen\ oplossing/No\ solution$	
1.4	B by $x=3+2i$	**(2)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
1.5	A 1 waar/*true* 2 vals/*false* $x = 3$ Sprong/*Jump* 3 waar/*true* 4 vals/*false* $\lim_{x\to 1^-} f'(x) \neq \lim_{x\to 1^+} f'(x)$	**(2)**
1.6	B $u \times w = 0$	**(2)**
1.7	C 6 terme/*terms* $x^6 - 5x^5\,y + 10x^4y^2 - 10x^3y^3 + 5x^2y^4 - xy^5$	**(2)**
1.8	D $\lim_{x\to 1^-} 2 = \lim_{x\to 1^+} ax - 1$ $2 = a - 1$ $a = 3$	**(2)**
1.9	$Cx_0 = -0{,}4$	**(2)**
1.10	B $\frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} + \frac{\mathrm{E}}{(x-1)}$	**(2)**

Vraag / *Question* 2 **[17 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1.1	$h(x) = -\|2-x\| + 1$ $2 - x = 0 \quad \Rightarrow \quad x = 2$ ✓ $(2;1)$ ✓	**(2)**
2.1.2	$-\|2-x\| + 1 < -2$ $-\|2-x\| < -3$ ✓ $\|2-x\| > 3$ ✓ $2 - x > 3$ *OR*/OF $2 - x < -3$ $x < -1$ ✓ $x > 5$ ✓	**(4)**
2.1.3	$-\|2-x\| + 1 = 2x - 2$ $-\|2-x\| = 2x - 3$ ✓ $\|2-x\| = -2x + 3$ ✓ As/*If* $x \leq 2$: As/*If* $x > 2$: $2 - x = -2x + 3$ $2 - x = 2x - 3$ $x = 1$ ✓ $3x = 5$ $x = \frac{5}{3}$ ✓ Nie van toepassing/*Not applicable* ✓	**(5)**
2.2	✓  	**(4)** $(\mathbf{2;0})$ ✓ $\boldsymbol{x = 3}$ ✓ $\boldsymbol{x = 1}$ ✓ $\boldsymbol{y = -1}$ ✓
2.3	$h(x) = \|x^2 - 1\|$ ✓ $+ 1$ ✓	**(2)**

Vraag / *Question* 3 **[21 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1	As $x-1-3i$ 'n faktor is dan is $x-1+3i$ ook 'n faktor. *If $x-1-3i$ is a factor then $x-1+3i$ is also a factor.* ✓ $(x-1-3i)(x-1+3i)$ OF/*OR* $x-1=3i$ $=(x-1)^2-9i^2$ ✓ $(x-1)^2=9i^2$ ✓ $=x^2-2x+1+9$ $x^2-2x+1+9=0$ $=x^2-2x+10$ ✓ $x^2-2x+10=0$ ✓ $2-1$ ✓ $1-2+10 \mid 2-5+22-10$ $2-4+20$ $-1+2-10$ $x^2-2x+10$ is ook $-1+2-10$ 'n faktor 0 $(x^2-2x+10)(2x-1)$ ✓	**(5)**
3.2	$\frac{2x^2-1}{x^2(x^2+1)} \equiv \frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$ ✓ $2x^2-1 \equiv Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2$ ✓ $2x^2-1 \equiv Ax^3+Ax+Bx^2+B+Cx^3+Dx^2$ ✓ Stel/Set $x=0$: $-1=B$ ✓ x: $0=A$ ✓ x^2: $2=B+D$ $D=3$ ✓ x^3: $0=A+C$ $C=0$ ✓ $\frac{2x^2-1}{x^2(x^2+1)} \equiv -\frac{1}{x^2}+\frac{3}{x^2+1}$ ✓	**(8)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.3	**Toets as $n = 1$:**	**(8)**

LHS $= 1$ RHS = $\frac{1(2+1)(2+1)}{6} = 1$

$\therefore$ LK=RK vir / *LHS=RHS* ✓

Aanvaar dat bewering is waar vir $n = k$ /
Assume that statement is true for $n = k$

$$\sum_{r=1}^{k}(r^2) = \frac{k(k+1)(2k+1)}{6} \checkmark$$

Beskou nou /*Now look at* $n = k + 1$: ✓
LK/*LHS*=

$$\sum_{r=1}^{k}(r^2) + (k+1)^2 \checkmark$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6\checkmark(k+1)^2}{6}$$

$$= \frac{(k+1)\left(2k^2 + k + 6k + 6\right)}{6}$$

$$= \frac{(k+1)\left(2k^2+7k+6\right)}{6}$$

$$= \frac{(k+1)(2k+3)(k+2))}{6} \checkmark$$

RK/*RHS*:

$$\frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6} \checkmark$$

$\therefore LK = RK / LHS = RHS$

Volgens die beginsel van wiskundige induskie is die bewering waar vir alle $n \in \mathbb{N}$. /. *The equation is true for all* $n \in \mathbb{N}$ *according to the principal of mathematical.*
✓

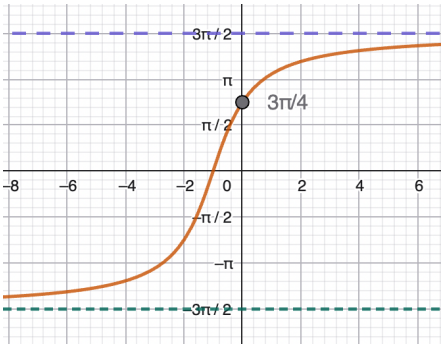
Vraag / *Question* 4 **[8 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1	$(1-ax)^{\frac{1}{3}}$ ✓ $\frac{n(n-1)(n-2)x^3}{6} = \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)(-ax)^3}{6} = -\frac{5}{81}a^3x^3$ ✓ $-\frac{5}{81}a^3x^3 = -\frac{5}{3}x^3$ ✓ $-\frac{5}{81}a^3 = -\frac{4}{3}$ $-\frac{5}{81}a^3 = -\frac{5}{3}\left(-\frac{81}{5}\right)$ $a^3 = 27$ $a = 3$ ✓	**(4)**
4.2	$\left(x^4+\frac{2}{x}\right)^6 = \sum_{r=0}^{6}\binom{6}{r}(x^4)^{6-r}(2x^{-1})^r$ ✓ $x^4 = x^{24-4r-r}$ ✓ $4 = 24-5r$ $r = 4$ ✓ $\binom{6}{4}(2^4) = 240$ ✓	**(4)**

Vraag / *Question* 5 **[20 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1.1	$BA = A - B = (-1-1; 0-x; -2-1) = (-2; -x; -3)$ ✓ $BC = C - B = (2-1; 1-x; 1-1) = (1; 1-x; 0)$ ✓	(2)
5.1.2	$B = \frac{\pi}{2}; \quad BA \cdot BC = 0$ ✓ $-2 - x(1-x) - 3(0) = 0$ $-2 - x + x^2 = 0$ ✓ $(x-2)(x+1) = 0$ ✓ $x = 2$ OF/*OR*. $x = -1$ Maar/*But* $x > 0 \quad \therefore x = 2$ ✓	(4)
5.2.1	$\|a\| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$ ✓ $\|b\| = \sqrt{(-1)^2 + 2^2 + 5^2} = \sqrt{30}$ ✓	(2)
5.2.2	$\beta = bgcos\left(\frac{2}{\sqrt{6}}\right)$ ✓ $= 0{,}62$ ✓	(2)
5.2.3	$a \cdot b = (-1)(-2) + 2(1) + 1(5) = 9$ ✓ $\theta = bgcos\left(\frac{a \cdot b}{\|a\|\|b\|}\right)$ ✓ $= bgcos\left(\frac{9}{\sqrt{6}\sqrt{30}}\right)$ ✓ $= 0{,}84\ radiale$ ✓	(4)
5.3	$w = u \times v$ ✓ $= \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -2 & -4 & -1 \end{vmatrix}$ ✓ $= \begin{vmatrix} 2 & 1 \\ -4 & -1 \end{vmatrix} i$ ✓ $- \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix}$ ✓ $+ \begin{vmatrix} 1 & 1 \\ -2 & -4 \end{vmatrix} k$ ✓ $= (-2+4)i - (-1+2)j + (-4+4)k$ $= 2i - j + 0k$ ✓ OF/*OR* $2i - j$	(6)

Vraag / *Question* 6 **[27 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1.1	$\lim_{x\to 1^-} bgcos(x) = 0$ ✓ $\lim_{x\to 1^+} (x-1)^2 - 1 = -1$ ✓ $f(x)$ het sprongdiskontinuïteit by $x = 1$ ✓want $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$ ✓	(4)
6.1.2	$Dx[bgcos(x)] = -\frac{1}{\sqrt{1-x^2}}$ ✓ $Dx\left[-x+\frac{\pi}{2}\right] = -1$ ✓ $\lim_{x\to 0^+} f'(x) = \frac{1}{\sqrt{1-(0)^2}} = -1$ ✓ $\lim_{x\to 0^-} f'(x) = -1$ ✓ $f(x)$ is differensieerbaar by $x = 0$ want $\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^+} f'(x)$ ✓	(5)
6.2.1	$D_x\left[(\sin^2 3x)\left(\frac{1}{x^2}\right)\right]$ $= 2sin3x$ ✓$(cos3x)(3)$ ✓$x^{-2} + \sin^2 3x(-2)(x^{-3})$ ✓	(4) Twee terme/ *two other terms* ✓
6.2.2	$\frac{bgsin(2x)}{\cot(5x)} = \frac{\frac{1}{\sqrt{1-4x^2}}(2)$ ✓$\cot(5x) - (cosec^2 5x(5)$ ✓$)bgsin2x}{\cot^2 5x$ ✓	(4) Boonste terme/ *Top terms* ✓
6.3.1	$f(x) = 3bgtan(x+1)$ y= $3bgtan(0+1)$ $3bgtan(x+1) = 0$ $y = 3\left(\frac{\pi}{4}\right) = \frac{3\pi}{4}$ ✓ $x + 1 = \tan(0)$ $x = -1$ ✓  	(4) Asimptote/ *Asymtotes* $y = \pm\frac{3\pi}{2}$ ✓ *VORM/FORM* ✓

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.3.2	$x = 3bgtan(y+1)$ $\frac{x}{3}$ ✓$= bgtan(y+1)$ $y = \tan\left(\frac{x}{3}\right)$ ✓-1 ✓	(3)
6.3.3	$f'(x) = \frac{3}{1+(x+1)^2}$ ✓ $f'(1)$ ✓$= \frac{3}{1+4} = \frac{3}{5}$ ✓	(3)

Vraag / *Question* 7 **[23 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1.1	$\int \left(3x^{-\frac{1}{2}} + \sec^2(3x) - \frac{\pi}{2}\right) dx$ $= \frac{3x^{\frac{1}{2}}}{\frac{1}{2}}$ ✓$+ \frac{\tan(3x)}{3}$ ✓$- \frac{\pi}{2}x$ ✓$+ k$	(4)
7.1.2	$\int \frac{dx}{\sqrt{4-9x^2}} = \int \frac{dx}{2\sqrt{1-\left(\frac{3x}{2}\right)^2}}$ ✓ ✓ $= \frac{\frac{1}{2}bgsin\left(\frac{3x}{2}\right)}{\frac{3}{2}}$ ✓ $+ k$	(4)
7.2.1	$h'(x) = -1$ ✓$- \frac{1}{2}(x-1)^{-\frac{1}{2}}$ ✓ $x_{n+1} = x_n - \frac{h(x_n)}{h'(x_n)}$ ✓ $= x_n - \frac{4-x_n-\sqrt{x_n-1}}{-1-\frac{1}{2}(x_n-1)^{-\frac{1}{2}}}$ ✓ $x_0 = 2$ $x_1 = \frac{8}{3}$ $x_2 = 2{,}697185$ ... $x \approx 2{,}69722$ ✓	(5)
7.2.2	$\int_1^2 \left(4 - x - \sqrt{x-1}\right) dx$ $= 4x$ ✓$- \frac{x^2}{2}$ ✓$- \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}}$ ✓$\Big]_1^2$	(6)

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
	$= 4(2) - \frac{2^2}{2} - \frac{(2-1)^{\frac{3}{2}}}{\frac{3}{2}}$ ✓$- \left(4(1) - \frac{1^2}{2} - \frac{(1-1)^{\frac{3}{2}}}{\frac{3}{2}}\right)$ ✓ $= 8 - 2 - \frac{2}{3} - 4 + \frac{1}{2}$ $= \frac{11}{6}\ eenhede^2/units^2$ ✓ OF/*OR* $\int_1^2 (4-x)dx - \int_1^2 \sqrt{x-1}dx$ $= \left[4x \checkmark - \frac{x^2}{2} \checkmark\right]_1^2 - \left[\frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} \checkmark\right]_1^2$ $= 4(2) - \frac{2^2}{2} - 4(1) + \frac{1^2}{2}$ ✓$- \left(\frac{(2-1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(1-1)^{\frac{3}{2}}}{\frac{3}{2}}\right)$ ✓ $= 2\frac{1}{2} - \left(\frac{2}{3}\right)$ $= \frac{11}{6}$ ✓	
7.2.3	$V = \pi \int_1^2 \left(\sqrt{x-1}\right)^2 dx$ ✓ $= \pi \int_1^2 (x-1)$ ✓dx $= \pi \left[\frac{x^2}{2} - x\right]_1^2$ $= \pi \left(\frac{2^2}{2} - 2 - \left(\frac{1^2}{2} - 1\right)\right)$ ✓ $= \frac{\pi}{2}$ ✓	(4)

Vraag / *Question* 8 **[10 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1.1	Driehoek AMC: $AM = MC$ $AC = \sqrt{AM^2 + MC^2}$ ✓ $= \sqrt{15^2 + 15^2}$ $= 15\sqrt{2}$ ✓	(2)
8.1.2	$\beta = \frac{\pi}{4}$ ✓ Sektor $NC = 21\left(\frac{\pi}{4}\right)$ ✓ $= 16{,}49$ ✓	(3)
8.1.3	Oppervlakte $AMC = \frac{1}{2}(15)(15)$ ✓ $= \frac{225}{2} / \ 112{,}5 \ eenhede^2/units^2$ ✓ Oppervlakte sektor $ACN = \frac{1}{2}\left(\frac{\pi}{4}\right)(21)^2$ ✓ $= \frac{441}{6}\pi = 173{,}18 \ eenhede^2/units^2$ ✓ $CNM = 173{,}18 - 112{,}5$ $= 60{,}68 \ eenhede^2/units^2$ ✓	(5)
8.2.1	$Area = \frac{1}{2}r^2\theta$ $r = \frac{s}{\theta}$ ✓ $Area = \frac{1}{2}\left(\frac{s}{\theta}\right)^2 \theta$ ✓	(2)
8.2.2	$Area = \frac{1}{2}\left(\frac{s}{\theta}\right)^2 \theta = 20 \ cm^2$ $Nuwe \ Area = \frac{1}{2}\left(\frac{2s}{\theta}\right)^2 \theta = 4\left(\frac{1}{2}\left(\frac{s}{\theta}\right)^2 \theta\right)$ ✓ $= 4(20 \ cm^2)$ $= 80 \ cm^2$ ✓	(2)

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -