α-WISKUNDE/MATHEMATICS

Alpha Wiskunde Graad 11 / ***Alpha Mathematics Grade 11***

Junie eksamen 2025 / ***June examination 2025***

MEMORANDUM

Totaal / ***Total*****: 160 punte /** ***marks***

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Hierdie memorandum bestaan uit 15 bladsye. /
This memorandum consists of 15 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

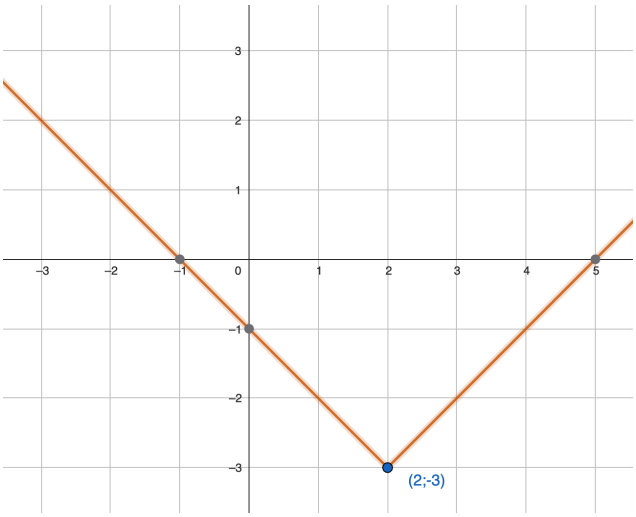
1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
1.1	A $\quad |x-1| = x$: $If/as\ x \geq 1$: $\quad$ $If/as\ x < 1$: $x + 1 = x$ $\quad$ OF $\quad x - 1 = -x$ $No\ solution$ $\quad$ $2x = 1$ $x = \frac{1}{2}$	**(2)**
1.2	D $\quad \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$	**(2)**
1.3	B $\quad f^{-1}(x): x = \cos\left(\frac{y}{2}\right) + 1$ $x - 1 = \cos\left(\frac{y}{2}\right)$ $bgcos(x-1) = \frac{y}{2}$ $y = 2bgcos(x-1)$	**(2)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
1.4	A $x = -1 + i$,$x = -1 - i$ $h(-1 - i)$ $(x + 1 - i)(x + 1 - i)$ $= (x + 1)^2 - i^2$ $= x^2 + 2x + 2$	**(2)**
1.5	B $\left(x + \frac{3}{x}\right)^6 = 5\ terme$ Dus $(x + 1)\left(x + \frac{3}{x}\right)^6 = 6\ terme$	**(2)**
1.6	A By the principle of mathematical induction, if a statement is true for any number m = k, then for its successor m = k + 1, the statement also satisfied, provided the statement is true for m = 1. So, the required answer is $P(k) = m^k + 5$.	
1.7	B $\lvert x - 1\rvert > -1$ $x \in \mathbb{R}$	
1.8	C $\sum_{i=1}^{n}(k)^{2+i} = k^{2+n}$	
1.9	D $y = bgcos(x) - \frac{\pi}{2}$	
1.10	B $A_1 = \frac{1}{2}r^2\theta$ $A_2 = \frac{1}{2}\left(\frac{r}{2}\right)^2(2\theta)$ $= \frac{1}{2}\left(\frac{1}{4}\right)(2)r^2\theta$ $= \frac{1}{2}\left(\frac{1}{2}\right)r^2\theta$ $= \frac{1}{2}A_1$	

Vraag / *Question* 2 **[27 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1(a)	$\|x-5\|=2$ $x-5=2$ ✓ of/or $x-5=-2$ ✓ $x=7$ ✓ $x=3$ ✓	**(4)**
2.1(b)	$\|x+1\|>1$ ✓ $x+1>1$ ✓ $x+1<-1$ ✓ $x>0$ ✓ $x<-2$ ✓	**(5)**
2.2(a)	$y=\|x-2\|-3$ $x-2=0$ $x=2$ ✓ $(2;-3)$ ✓	**(2)**
2.2(b)	x-afsnit: $y=0$: $0=\|x-2\|-3$ $\|x-2\|=3$ $x-2=3$ OF/OR $x-2=-3$ $x=5$ $x=-1$ 	**(5)** $x=-1$ ✓ $x=\mathbf{5}$ ✓ $(2;-3)$ ✓ $(\mathbf{0};-\mathbf{1})$ ✓ $Vorm$ ✓

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.2(c)	$\lfloor x-2 \rfloor - 3 = 3x+1$ $\lfloor x-2 \rfloor = 3x+4$ ✓ As $x \geq 2$: $x-2 = 3x+4$ ✓ $2x = -6$ $x = -3$ N.v.t ✓ As $x < 2$: $x-2 = -3x-4$ ✓ $4x = -2$ $x = -\frac{1}{2}$ ✓	**(5)** $x = -\frac{1}{2}$
2.3	$\lvert x^2-3 \rvert = x+1$ As $-2 < x < 2$: $-(x^2-3) = x+1$ ✓ $-x^2+3 = x+1$ $x^2+x-2 = 0$ ✓ $(x+2)(x-1) = 0$ $x = -2$ of $x = 1$ ✓ n.v.t As $x \geq 2$: $(x^2-3) = x+1$ ✓ $x^2-x-4 = 0$ ✓ $x = \frac{1}{2} \pm \frac{\sqrt{1-4(4)}}{2}$ $x = \frac{1}{2} \pm \frac{\sqrt{17}}{2}$ $x = \frac{1}{2} + \frac{\sqrt{17}}{2}$ ✓	**(6)** $\boldsymbol{x = 1}$ $x = \frac{1}{2} + \frac{\sqrt{17}}{2}$

Vraag / *Question* 3 **[20 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1	$\frac{x^3+3x^2+2}{(x^2+x)(x^2+1)}$ $\frac{x^3+3x^2+2}{x(x+1)(x^2+1)} \checkmark \equiv \frac{A}{x}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1} \checkmark$ $x^3+3x^2+2 \equiv A(x+1)(x^2+1)+B(x)(x^2+1)$ $+(Cx+D)(x)(x+1) \checkmark\checkmark$ $x^3+3x^2+2 \equiv Ax^3+Ax^2+Ax+A+Bx^3+Bx+Cx^3+Cx^2+Dx^2$ $+Dx \checkmark$ Stel/Set $x=-1$: $-1+3+2=B(-1)(1+1) \Rightarrow 4=-2B$ $B=-2 \checkmark$ Stel/Set $x=0$: $A=2 \checkmark$ x^3: $1=A+B+C$ $C=1 \checkmark$ x^2: $3=A+C+D$ OR/OF x: $0=A+B+D$ $3=2+1+D$ $D=0 \checkmark$ $D=0$ $\frac{x^3+3x^2+2}{x(x+1)(x^2+1)} \equiv \frac{2}{x}-\frac{2}{x+1}+\frac{x}{x^2+1} \checkmark$	**(10)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.2(a)	$\frac{2x^3+x^2-5x+10}{x^2-4} = ax + b + \frac{cx+d}{x^2-4}$ $2 + 1$ $1 + 0 - 4 \;\big|\; 2 + 1 - 5 + 10$ $2 + 0 - 8$ $1 + 3 + 10$ $1 + 0 - 4$ $3 + 14$ $\frac{2x^3 + x^2 - 5x + 10}{x^2 - 4} = 2x + 1 + \frac{3x + 14}{x^2 - 4}$ $a = 2$ ✓ $b = 1$ ✓ $c = 3$ ✓ $d = 14$ ✓	
3.2(b)	$\frac{3x + 14}{(x-2)(x+2)}$ ✓ $\equiv \frac{A}{x-2} + \frac{B}{x+2}$ ✓ $3x + 14 \equiv A(x + 2) + B(x - 2)$ ✓ Stel $x = -2$: $3(-2) + 14 = B(-2 - 2)$ $8 = -4B$ $B = -2$ ✓ Stel $x = 2$: $3(2) + 14 = A(2 + 2)$ $20 = 4A$ $A = 5$ ✓ $\frac{2x^3 + x^2 - 5x + 10}{x^2 - 4} = 2x + 1 + \frac{5}{x-2} - \frac{2}{x+2}$ ✓ OF NET $\frac{5}{x-2} + \frac{-2}{x+2}$	

Vraag / *Question* 4 **[26 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1(a)	$x^2 + 7 = 0$ $x^2 = -7$ ✓ $x = \pm\sqrt{-7} = \pm\sqrt{7}\sqrt{-1} = \pm\sqrt{7}i$ ✓	**(2)**
4.1(b)	$x = \sqrt{-44}$ $x = \sqrt{44}\sqrt{-1} = 12i$ ✓	**(1)**
4.2(a)	$-1 \quad 4 - 4 - 13 - 5$ $\quad\quad -4 + 8 \;\; + 5$ $\quad\quad 4 - 8 - 5 \quad\; 0$ ✓ ✓ ✓	**(3)**
4.2(b)	$4x^2 - 8x - 5$ is 'n faktor $(2x + 1)$ ✓$(2x - 5)$ ✓ $f(x) = (x + 1)$ ✓$(2x + 1)(2x - 5)$ OF/OR $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(4)}$ $x = \frac{8}{8} \pm \frac{\sqrt{64 - 4(4)(-5)}}{2}$ $= 1 \pm \frac{\sqrt{144}}{8}$ $= 1 \pm \frac{12}{8}$ $x = 1 + 1\frac{1}{2} = \frac{5}{2}$ of $x = 1 - 1\frac{1}{2} = -\frac{1}{2}$ $f(x) = (x + 1)$ ✓$\left(x + \frac{1}{2}\right)$ ✓$\left(x - \frac{5}{2}\right)$ ✓	**(4)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.3	As $x-5-\sqrt{2}$ 'n nulpunt is dan is $x-5+\sqrt{2}$ ook 'n faktor. *If $x-5-\sqrt{2}$ is a factor then $x-5+\sqrt{2}$ is also a factor.* ✓ $(x-5-\sqrt{2})(x-5+\sqrt{2})$ OF/*OR* $x-5=\pm\sqrt{2}$ ✓ $=(x-5)^2-2$ ✓ $(x-5)^2=2$ $=x^2-10x+25-2$ $x^2-10x+25-2=0$ ✓ $=x^2-10x+23$ ✓ $8x^4-82x^3+201x^2-16x-69 \div x^2-10x+23$ 8 ✓ -2 ✓ -3 ✓ $1-10+23 \mid 8-82+201-16-69$ $8-80+184$ $-2+17-16$ $-2+20-46$ $-3+30-69$ $-3+30-69$ 0 Ander faktor/*Other factor:* $8x^2-2x-3$ $8x^2-2x-3=(2x+1)(4x-3)$ ✓ $x=5\pm\sqrt{2}$ ✓of/or $x=-\frac{1}{2}, x=\frac{3}{4}$ ✓	**(9)**
4.4(a)	As $x+2+2i$ 'n faktor is dan is $x+2-2i$ ook 'n faktor. *If $x+2+2i$ is a factor then $x+2-2i$ is also a factor.* ✓	**(1)**
4.4(b)	$(x+2-2i)(x+2-2i)$ OF/*OR* $x=2\pm 2i$ $=(x+2)^2-(2i)^2$ ✓ $x+2=\pm 2i$ $=x^2+4x+4+4$ $(x+2)^2=(2i)^2$ ✓ $=x^2+4x+8$ ✓ $x^2+4x+4=4i^2$ $x^2+4x+8=0$ ✓ $m=8$ ✓	**(3)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.4(a)	$3\checkmark + 5\checkmark$ $1 + 4 + 8 \;\big|\; 3 + 17 + 44 + 40$ $3 + 12 + 24$ $5 + 20 + 40$ $5 + 20 + 40$ 0 Reële faktor/*Real factor:* $3x + 5\checkmark$	**(3)**

Vraag / *Question* 5 **[27 punte / *marks*]**

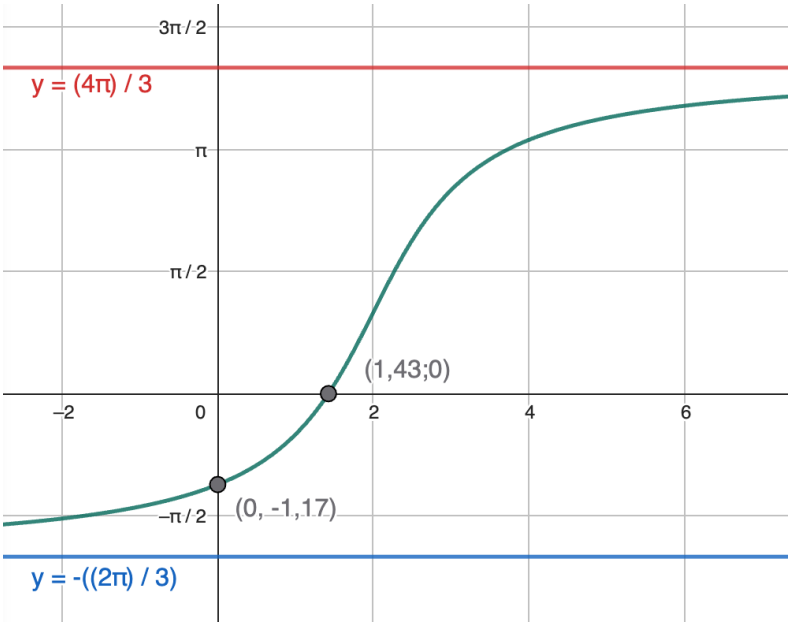
NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1(a)	$\sum_{i=1}^{n}(-1)^{i-1}\,(i)^2$ $= (-1)^0(1^2) + (-1)^1(2^2) + (-1)^2(2^3) + (-1)^3(2^4)$ $= 1\checkmark - 4\checkmark + 8\checkmark - 16\checkmark$	**(4)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1(b)	**Bewys as $i = 1$:** ✓ LHS = 1 RHS = $\frac{(-1)^{1-1}1(1+1)}{2} = 1$ ∴ LK=RK vir / *LHS=RHS* ✓ Aanvaar dat bewering is waar vir $n = i$ / *Assume that statement is true for* $n = i$ ✓ $\sum_{i=1}^{n}(-1)^{i-1}(i)^2 = (-1)^{i-1}\frac{i(i+1)}{2}$ ✓ Beskou nou /*Now look at* $n = i + 1$: ✓ LK/*LHS*= $\sum_{i=1}^{n}(-1)^{i-1}(i)^2 + (-1)^i(i+1)^2$ ✓ $= (-1)^{i-1}\frac{i(i+1)}{2} + (-1)^i(i+1)^2$ $= \frac{(-1)^i(-1)^{-1}i(i+1)}{2} + (-1)^i(i+1)^2$ $= \frac{(-1)^i(i+1)\big(-1(i)+2(i+1)\big)}{2}$ $= \frac{(-1)^i(i+1)(i+2)}{2}$ ✓ RK/*RHS*: $(-1)^{i+1-1}\frac{(i+1)(i+1+1)}{2}$ ✓ $= \frac{(-1)^i(i+1)(i+2)}{2}$ ✓ ∴LK=RK/*LHS=RHS* ✓ Volgens die beginsel van wiskundige induskie is die bewering waar vir alle $n \in \mathbb{N}$. / The equation is true for all $n \in \mathbb{N}$ according to the principal of mathematical. ✓	**(11)**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.2(a)	$\left(\frac{1}{3x}-2x^3\right)^8$ $\left(\frac{1}{3x}-2x^3\right)^8=\sum_{r=0}^{8}\binom{8}{r}\left(\frac{1}{3x}\right)^{8-r}(-2x^3)^r$ $x^0=x^{-(8-r)}(x^3)^r$ ✓ $x^0=x^{-8+r+3r}$ $0=-8+r+3r$ ✓ $8=4r$ $r=2$ ✓	**(3)**
5.2(b)	$\left(\frac{1}{3x}-2x^3\right)^8=\sum_{r=0}^{8}\binom{8}{r}\left(\frac{1}{3x}\right)^{8-r}(-2x^3)^r$ $\binom{8}{2}\left(\frac{1}{3}\right)^{8-2}(-2)^2$ ✓ $=28\left(\frac{1}{3^6}\right)(4)$ $=\frac{112}{729}$ $=0.15$ ✓	**(2)**
5.3(a)	$\sqrt[3]{1-\frac{x}{2}}=\left(1-\frac{x}{2}\right)^{\frac{1}{3}}$ ✓ $\left(1+\frac{x}{2}\right)^{\frac{1}{3}}$ $=1+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{2}x\right)}{1}+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{1}{2}x\right)^2}{2}$ ✓ $=1$ ✓$+\frac{x}{6}$ ✓$-\frac{x^2}{36}$ ✓	**(5)**
5.3(b)	$\left\|-\frac{x}{2}\right\|<1$ ✓ $\|x\|<2$ ✓	**(2)**

Vraag / *Question* 6 **[22 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1(a)	$y = bgcos(ax)$ waardeversameling/*range* $y \in [0;\ \pi]$ $y = bgcos(ax) + b$ waardeversameling/*range: :* $y \in \left[-\frac{\pi}{4}; \frac{3\pi}{4}\right]$ $y = 0 + b = -\frac{\pi}{4} \implies b = -\frac{\pi}{4}$ *OF* $y = \pi + b = \frac{3\pi}{4} \implies b = \frac{3\pi}{4} - \pi \implies b = -\frac{\pi}{4}$ *OF* met inspeksie/*with inspection* $b = -\frac{\pi}{4}$ ✓ $bgcos(x)$ definisie versameling/*domain x* $\in [-1; 1]$ $bgcos(ax)$ definisie versameling/*domain x* $\in [-2; 2]$ $a(-1) = -2 \implies a = 2$ OF $a(1) = 2 \implies a = 2$ OF met inspeksie/*with inspection* $a = 2$ ✓	**(2)**
6.1(b)	$f(x) = bgcos(2x) - \frac{\pi}{4} = 0$ ✓ $bgcos(2x) = \frac{\pi}{4}$ $2x = \cos\left(\frac{\pi}{4}\right) \implies 2x = \frac{1}{\sqrt{2}}$ $x = \frac{2}{\sqrt{2}} = 1{,}41$ ✓	**(2)**
6.1(c)	$g(x) = bgcos(2x - 1$ ✓$) - \frac{\pi}{4}$	**(1)**
6.2(a)	$f^{-1}(x)$: $x = 2bgtan(y - 2) + \frac{\pi}{3}$ $x - \frac{\pi}{3}$ ✓$= 2bgtan(y - 2)$ $\frac{x}{2} - \frac{\pi}{6} = bgtan(y - 2)$ OF $\quad \frac{x-\frac{\pi}{3}}{2} = bgtan(y - 2)$ $\tan\left(\frac{x}{2} - \frac{\pi}{6}\right) = y - 2$ $\quad \tan\left(\frac{x-\frac{\pi}{3}}{2}\right) = y - 2$ $y = \tan\left(\frac{x}{2} - \frac{\pi}{6}\right) + 2$ $\quad y = \tan\left(\frac{x-\frac{\pi}{3}}{2}\right) + 2$	(5) Vervang/*substitute* x met/*with* y ✓ $-\frac{\pi}{3}$ ✓ $\div 2$ ✓ tan ✓ *Answer* ✓

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.2(b)	$f(x) = 2bgtan(x-2) + \frac{\pi}{3} = 0$ $2bgtan(x-2) = -\frac{\pi}{3}$ ✓ $\Rightarrow$ $(x-2) = tan(-\pi/6)$ ✓ $x = \tan\left(-\frac{\pi}{6}\right) + 2$ $x \approx 1{,}43$ OF $x = -\frac{1}{\sqrt{3}} + 2$ ✓	(3)
6.2(c)	$f(x) = 2bgtan(-2) + \frac{\pi}{3}$ ✓ $y = -1{,}17$ ✓	(2)
6.2(d)	$2bgtan(x-2) + \frac{\pi}{3} = 2\left(\frac{\pi}{2}\right) + \frac{\pi}{3} = \frac{4\pi}{3}$ $2bgtan(x-2) + \frac{\pi}{3} = 2\left(-\frac{\pi}{2}\right) + \frac{\pi}{3} = -\frac{2\pi}{3}$	(2)
6.2(e)		(5) $y = -\frac{2\pi}{3}$ $y = \frac{4\pi}{3}$ $(1{,}43; 0)$ $(0; -1{,}17)$ Vorm/*form*

Vraag / *Question* 7 **[18 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1(a)	$\sin(\theta) = \frac{CD}{AC}$ $\theta = bgsin\left(\frac{5\sqrt{3}}{10}\right)$ ✓ $\theta = \frac{\pi}{3}$ ✓	**(2)**
7.1(b)	$CB = r\theta = 10\left(\frac{\pi}{3}\right)$ ✓ $= \frac{10\pi}{3} = 10{,}472\ cm$ ✓ $\frac{CD}{AD} = \tan(\theta)$ ✓ $AD = \frac{10}{\tan\left(\frac{\pi}{3}\right)} = \frac{10}{\sqrt{3}}$ ✓ $BD = AB - AD = 10 - \frac{10}{\sqrt{3}} = 4{,}226\ cm$ ✓ Omtrek $BCD = CB + CD + BD$ $= \frac{10\pi}{3} + 5\sqrt{3} + 10 - \frac{10}{\sqrt{3}}$ $= 23{,}36\ cm$ ✓	**(6)**
7.1(c)	$A_{ABC} = \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2\left(\frac{\pi}{3}\right)$ ✓ $= \frac{100\pi}{2(3)} = \frac{50\pi}{3}cm^2$ ✓ $A_{ACD} = \frac{1}{2}bh = \frac{1}{2}\left(\frac{10}{\sqrt{3}}\right)\left(5\sqrt{3}\right)$ ✓ $= 25\ cm^2$ ✓ $A_{BCD} = A_{ABC} - A_{ACD}$ ✓ $= \frac{50\pi}{3} - 25 = 27{,}36\ cm^2$ ✓	**(6)**
7.2	$\theta = \frac{s}{r}$ $r = \frac{s}{\theta}$ ✓ $r < \frac{43}{\frac{\pi}{6}}$ ✓ $r < \frac{43(6)}{\pi}$ $r < 82{,}12\ cm$ ✓ Maksimum lengte van pendulum is $82{,}12\ cm$ ✓	**(4)**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -