α-WISKUNDE

Graad 10 Alpha Wiskunde/ Alpha Mathematics
June 2025

MEMORANDUM

Totaal / *Total*: 130 punte / *marks*

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Moderator: Annelize Lippert

Hierdie memorandum bestaan uit 11 bladsye. /
This memorandum consists of 11 pages.

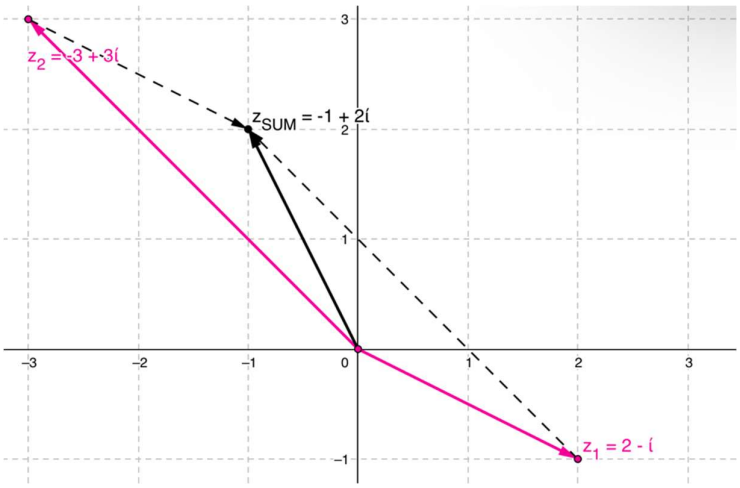
Vraag / *Question* 1 [10 punte / *marks*]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	D	$\sqrt{-2} = \sqrt{-1 \times 2} = \sqrt{-1}\sqrt{2} = \sqrt{2}i$	**2**
1.2	B	1. Ja / Yes. Definisie / Definition 2. Nee / No, Skalaar / Scalar 3. Ja / No, Definisie / Definition 4. Nee / No. Probeer / Try $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$	**2**
1.3	D	Teorie / theory	**2**
1.4	C	$z^* = i - 3$	**2**
1.5	C	AB en B het dieselfde dimensie / has the same dimension	**2**
1.6	C	$g\big(f(1)\big) = g(2) = 1$	**2**
1.7	B	$\boldsymbol{v} = -\frac{1}{2}\boldsymbol{u} \therefore (x; -2) = (\frac{1}{2}; -2) \therefore x = \frac{1}{2}$	**2**
1.8	D	Teorie / theory	**2**
1.9	D	$Im(2 - 6i) = -6$	**2**
1.10	B	$\frac{1}{i} = -i$	**2**

Vraag / *Question* 2 **[27 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1.a)	$x^2 + 3 = 0$ $x^2 - (-3) = 0$ $(x - \sqrt{3}i)(x + \sqrt{3}i) = 0$ $x = \pm\sqrt{3}i$	✓Metode / Method ✓$\pm\sqrt{3}i$ **[2 punte / *marks*]**
2.1.b)	$\frac{4-i}{i} = xi - 1$ $\frac{4-i}{i} = \frac{4-i}{i} \times \frac{i}{i} = \frac{4i - i^2}{i^2} = -4i - 1$ $\therefore x = -4$	✓$4i - i^2$ ✓$-4i - 1$ ✓Finale antwoord/ Final answer **[3 punte / *marks*]**
2.2a)	$a + b$ $= (1 + 2i) + (-2 - 5i)$ $= 1 + 2i - 2 - 5i$ $= -1 - 3i$	✓-1 ✓$-3i$ **[2 punte / *marks*]**
2.2.b)	$a \times b$ $= (1 + 2i)(-2 - 5i)$ $= -2 - 5i - 4i - 10i^2$ $= -9i - 2 - 10(-1)$ $= 8 - 9i$	✓ Maal / multiply ✓ $i^2 = -1$ ✓8 ✓$-9i$ **[4 punte / *marks*]**
2.2 c)	$\frac{ab}{1+i}$ $= \frac{8-9i}{1+i}$ $= \frac{8-9i}{1+i} \times \frac{1-i}{1-i}$ $= \frac{(8-9i)(1-i)}{1-i^2}$ $= \frac{(8-9i)(1-i)}{1-i^2} = \frac{8-9i-8i+9i^2}{1-i^2} = \frac{8-17i-9}{1+1}$ $= -\frac{1}{2} - \frac{17}{2}i$	✓ $1 - i$ ✓ $i^2 = -1$ ✓$-\frac{1}{2}$ ✓$-\frac{17}{2}i$ **[4 punte / *marks*]**

2.2 d)	$i^{209}b^2$ $= i(-2-5i)^2$ $= i(4+25i^2+20i)$ $= -21i+20i^2$ $= -20-21i$	✓ $i^{209}=i$ ✓ Bewerking / Working ✓ -20 ✓ $-21i$ **[4 punte / *marks*]**
2.3	$Re(2z)=4$ $Re(2a+2bi)=4$ $2a=4$ $\therefore a=2$ $\sqrt{a^2+b^2}=\sqrt{5}$ $a^2+b^2=5$ $(2)^2+b^2=5$ $b^2=1$ $\therefore b=\pm 1$	✓✓ $a=2$ ✓ Bewerking / Working ✓ $b=\pm 1$ **Maks 3 indien slegs een antwoord vir *b*** **/ Maximum 3 marks, if only one solution for *b*** **[4 punte / *marks*]**
2.4		✓ z_1 ✓ z_2 ✓ z_1+z_2 ✓ Final antwoord/ Final answer **[4 punte / *marks*]**

Vraag / *Question* 3 **[15 punte / *marks*]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
3.1.a)	$\begin{bmatrix}1 & 1\\ a & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -5\end{bmatrix}$ $detA_y = \lvert A_y\rvert = \begin{vmatrix}1 & \mathbf{5}\\ a & \mathbf{-5}\end{vmatrix} = (1)(-5)-(5)(a) = -5-5a$ $detA = \lvert A\rvert = \begin{vmatrix}1 & 1\\ a & 1\end{vmatrix} = (1)(1)-(1)(a) = 1-a$ $\frac{-5-5a}{1-a} = 10$ $\therefore -5-5a = 10-10a$ $\therefore 5a = 15$ $\therefore a = 3$	✓✓ determinante / determinants ✓ metode/ method ✓$a = 3$ **[4 punte / *marks*]**
3.1 b)	$\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -5\end{bmatrix}$ $detA = \lvert A\rvert = \begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix} = (1)(1)-(1)(-1) = 2$ $detA_y = \lvert A_y\rvert = \begin{vmatrix}1 & \mathbf{5}\\ 1 & \mathbf{-5}\end{vmatrix} = (1)(-5)-(5)(1) = -10$ $y = -\frac{10}{2} = -5$	✓ 2 ✓ -10 ✓ -5 **[3 punte / *marks*]**
3.2 a)	$\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 5 & 0 & -6\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}3\\ 2\\ -1\end{pmatrix}$	✓ 1 1 1 ✓ 1 -1 2 ✓ 5 0 6 **[3 punte / *marks*]**
3.2 b)	$\lvert A\rvert = \begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 5 & 0 & -6\end{vmatrix} = 1\begin{vmatrix}-1 & 2\\ 0 & -6\end{vmatrix} - \begin{vmatrix}1 & 2\\ 5 & -6\end{vmatrix} + \begin{vmatrix}1 & -1\\ 5 & 0\end{vmatrix}$ $= (6-0)-(-6-10)+(0-(-5)) = 27$ $\lvert A_x\rvert = \begin{vmatrix}3 & 1 & 1\\ 2 & -1 & 2\\ -1 & 0 & -6\end{vmatrix} = 3\begin{vmatrix}-1 & 2\\ 0 & -6\end{vmatrix} - \begin{vmatrix}2 & 2\\ -1 & -6\end{vmatrix} + \begin{vmatrix}2 & -1\\ -1 & 0\end{vmatrix}$ $= 3(6-0)-(-12+2)+(0-1) = 27$ $\therefore x = \frac{\lvert A_x\rvert}{\lvert A\rvert} = \frac{27}{27} = 1$	✓✓ $\lvert A\rvert$ ✓✓ $\lvert A_x\rvert$ ✓ $x = 1$ **[5 punte / *marks*]**

Vraag / *Question* 4 **[20 punte / *marks*]**

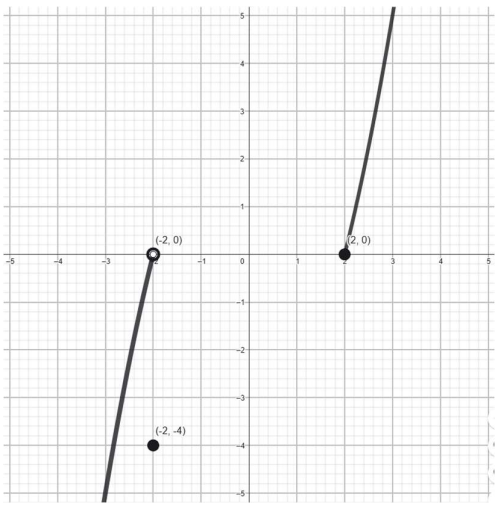
NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
4.1 a)	$A+E=\begin{pmatrix}-1 & 3\\ 2 & 4\end{pmatrix}+\begin{pmatrix}1 & 1\\ -2 & -2\end{pmatrix}=\begin{pmatrix}0 & 4\\ 0 & 2\end{pmatrix}$	✓✓ **[2 punte / *marks*]**
4.1 b)	$A^2=\begin{pmatrix}-1 & 3\\ 2 & 4\end{pmatrix}\begin{pmatrix}-1 & 3\\ 2 & 4\end{pmatrix}=\begin{pmatrix}1+6 & -3+12\\ -2+8 & 6+16\end{pmatrix}=\begin{pmatrix}7 & 9\\ 6 & 22\end{pmatrix}$	✓✓ **[2 punte / *marks*]**
4.1 c)	$\begin{pmatrix}-1 & 3\\ 2 & 4\end{pmatrix}\begin{pmatrix}2\\ -1\end{pmatrix}=\begin{pmatrix}-2-3\\ 4-4\end{pmatrix}=\begin{pmatrix}-5\\ 0\end{pmatrix}$	✓✓ **[2 punte / *marks*]**
4.1 d)	Onmoontlik / Impossible B is 'n 2×1 matriks, en A is 'n 2×2 matriks. Die bewerking is onmoontlik want die aantal kolomme in die eerste matrikse is nie dieselfde as die hoeveelheid rye van die tweede matriks nie / because the number of columns in the first matrix is not the same as the number of rows of the second matrix	✓Onmoontlik / Impossible ✓ Rede / Reason **[2 punte / *marks*]**
4.1 e)	Onmoontlik / Impossible Die dimensies moet dieselfde wees om af te trek / Their dimensions need to be the same to subtract	✓ Onmoontlik / Impossible ✓ Rede / Reason **[2 punte / *marks*]**
4.1 f)	Onmoontlik / Impossible Dit moet 'n vierkantige matriks wees / It needs to be a square matrix. OF / OR Dit is nie 'n 2x2 of 3x3 matriks nie / It is not a 2x2 or 3x3 matrix.	✓Onmoontlik / Impossible ✓ Rede / Reason **[2 punte / *marks*]**
4.1 g)	$dim(C)=1\times 2$	✓✓ **[2 punte / *marks*]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
4.1 h)	$a_{21} = 2$	✓✓ **[2 punte / *marks*]**
4.2 a)	$\|F\| = 0 - 1\begin{vmatrix}1 & 1\\ 0 & 0\end{vmatrix} + 0$ $= 0 - 1(0 - 0) + 0$ $= 0$	✓✓ **[2 punte / *marks*]**
4.2 b)	Nee / No Die hoofmatriks, F, se determinant is nul, en met Cramer se reël deel ons met hierdie waarde. Daar kan nie met 'n nul gedeel word nie. / Die matrix, $F's$ determinant is zero, and when using Cramer's rule we divide with this value. You can not divide by zero.	✓Nee/ No ✓Rede / Reason **[2 punte / *marks*]**

Vraag / *Question* 5 **[10 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.	$\frac{5x-2}{(x-1)(x^2-3x+2)} \equiv \frac{5x-2}{(x-2)(x-1)^2}$ $\equiv \frac{A}{x-2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$ $5x-2 \equiv A(x-1)^2+B(x-1)(x-2)+C(x-2)$ Stel / set $x=1$: $3=-C$ $\therefore C=-3$ Stel / set $x=2$: $8=A$ $\therefore A=8$ Kyk na/ Look at x^2: $0x^2=(A+B)x^2$ $\therefore B=-A$ $\therefore B=-8$ $\therefore \frac{5x-2}{(x-1)(x^2-3x+2)} \equiv \frac{8}{x-2}-\frac{8}{x-1}-\frac{3}{(x-1)^2}$	✓Faktoriseer/Factorise ✓✓ Ontbind / Decompose ✓Vermenigvuldig met KGV / Multiply with LCD ✓$C=-3$ ✓✓$A=8$ ✓✓ $B=-8$ ✓Antwoord/ Answer **[10 punte / *marks*]**

Vraag / *Question* 6 **[22 punte / *marks*]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
6.1 a)	$f(0) = 1$	✓ **[1 punte / *marks*]**
6.1 b)	$f(f(3)) = f(2) = 1.5$ of/or $\frac{3}{2}$	✓✓ **[2 punte / *marks*]**
6.1 c)	$f(2f(1)) = f(2) = 1.5$ of/or $\frac{3}{2}$	✓✓ **[2 punte / *marks*]**
6.2	$g(x) = 2 - x^4$ en / and $f(x) = 2x^{\frac{1}{5}} + \frac{3}{x^{\frac{1}{5}}} + x$ OF / OR $g(x) = x^4$ en / and $f(x) = 2(2-x)^{\frac{1}{5}} + \frac{3}{(2-x)^{\frac{1}{5}}} + 2 - x$ OF / OR $g(x) = \sqrt[5]{2 - x^4}$ en / and $f(x) = 2x + \frac{3}{x} + x^5$	✓ f ✓ g Onthou / Remember: $x^{\frac{1}{5}} = \sqrt[5]{x}$ **[2 punte / *marks*]**
6.3		✓ $(-2; 0)$ ✓ $(-2; -4)$ ✓ Oop kol/ open dot ✓✓ Vorm / Shape **[5 punte / *marks*]**
6.4 a)	$AO = OB = 4\ cm$ (radiusse) $AC = AO - OC = 4 - 3 = 1\ cm$	✓ **[1 punte / *marks*]**
6.4 b)	$AB = r\theta = AO \times \theta = \frac{4\pi}{3}\ cm$	✓✓ **[2 punte / *marks*]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
6.4 c)	$CD = r\theta = OC \times \theta = 3 \times \frac{\pi}{3} = \pi\ cm$ $ACDB = AB + DB + CD + AC = AB + CD + 2AC$ $= AB + CD + 2(1\ cm)$ $= \frac{4\pi}{3} + \pi + 2 = \frac{7\pi}{3} + 2 = 9.33$	✓ CD ✓ Antwoord/ Answer **[2 punte / *marks*]**
6.5 a)	$s = r\theta = 4\pi$ $\therefore \theta = \frac{4\pi}{r}$	✓ **[1 punte / *marks*]**
6.5 b)	$A = \frac{1}{2}r^2\theta = 4\pi$ $\therefore r^2\theta = 8\pi$ $r^2\left(\frac{4\pi}{r}\right) = 8\pi$ $4\pi r = 8\pi$ $\therefore r = 2$ And so / en so: $\therefore \theta = \frac{4\pi}{r} = \frac{4\pi}{2} = 2\pi$	✓ $r = 2$ ✓ $\theta = 2\pi$ **[2 punte / *marks*]**
6.5 c)	$\theta = \frac{4\pi}{r} = \frac{4\pi}{2} = 2\pi \times \frac{180}{\pi} = 360°$	✓✓ **[2 punte / *marks*]**

Vraag / *Question* 7 **[16 punte / *marks*]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
7 a)	$\boldsymbol{a} - 2\boldsymbol{c}$ $= (-2; 3) - 2\left(1; \frac{1}{2}\right)$ $= (-2 - 2; 3 - 1)$ $= (-4; 2)$	✓ -4 ✓ 2 **[2 punte / *marks*]**
7 b)	$\boldsymbol{a}.\boldsymbol{b} = (-2)(2) + (3)(3) = -4 + 9 = 5$	✓✓ **[2 punte / *marks*]**
7 c)	$\lvert\boldsymbol{a}\rvert = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$	✓ **[1 punte / *marks*]**
7 d)	$cos\theta = \frac{\boldsymbol{a}.\boldsymbol{b}}{\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert}$ $cos\theta = \frac{5}{\sqrt{13}\sqrt{13}} = \frac{5}{13}$ $\theta = bgcos\left(\frac{5}{13}\right) = 1.176$	✓ Formule / Formula ✓ In vervang/ Substitution ✓ Answer / Antwoord **[3 punte / *marks*]**
7 e)	$tan\theta = \frac{y}{x} = \frac{1}{2}$ $\theta = 0.464$	✓✓ **[2 punte / *marks*]**
7 f)	$\left(-\frac{2}{\sqrt{13}}; \frac{3}{\sqrt{13}}\right)$	✓ ✓ **[2 punte / *marks*]**
7 g)	$\boldsymbol{a}.\boldsymbol{d} = (-2)(3) + (3)(y) = 0$ $3y = 6$ $\therefore y = 2$	✓ Stel gelyk aan nul / set equal to one ✓$y = 2$ **[2 punte / *marks*]**
7 h)	$2\boldsymbol{c} = 2\left(1; \frac{1}{2}\right) = (2; 1)$	✓✓ **[2 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -