α-WISKUNDE/ MATHEMATICS

Rekordeksamen/*Preliminary Exam* 2025 **Tyd/ Time: 3 uur/*hours***
Graad/*Grade* 12 **Totaal/ Total: 200 PUNTE/*MARKS***

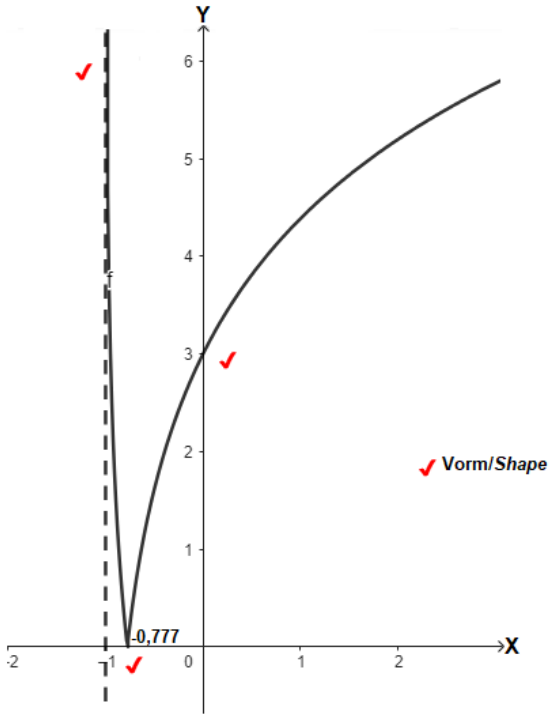
VRAAG/*QUESTION* 1 [30 PUNTE/*MARKS*]

1.1	**A**	**B**	**C**	**D**
1.2	**A**	**B**	**C**	**D**
1.3	**A**	**B**	**C**	**D**
1.4	**A**	**B**	**C**	**D**
1.5	**A**	**B**	**C**	**D**
1.6	**A**	**B**	**C**	**D**
1.7	**A**	**B**	**C**	**D**
1.8	**A**	**B**	**C**	**D**
1.9	**A**	**B**	**C**	**D**
1.10	**A**	**B**	**C**	**D**
1.11	**A**	**B**	**C**	**D**
1.12	**A**	**B**	**C**	**D**
1.13	**A**	**B**	**C**	**D**
1.14	**A**	**B**	**C**	**D**
1.15	**A**	**B**	**C**	**D**

VRAAG/*QUESTION* 2 [21 PUNTE/*MARKS*]

2.1	$x = 1 - \sqrt{2}$ is ook 'n nulpunt/*is a zero as well*✓ $\therefore (x - 1 - \sqrt{2})(x - 1 + \sqrt{2})$ is 'n faktor/*is a factor*✓ $= x^2 - 2x - 1$✓ $\therefore f(x) = (x - 1 - \sqrt{2})(x - 1 + \sqrt{2})(x + 3)$✓	1: $x = 1 - \sqrt{2}$ 1: $(x - 1 - \sqrt{2})(x - 1 + \sqrt{2})$ 1: $x^2 - 2x - 1$ 1: $x + 3$ **[4]**
2.2a	$p = 6\text{cis}\pi$✓ $t = 2$✓$\text{cis}\left(\frac{7\pi}{6}\right)$✓ of/*or* $t = 2\text{cis}\left(\frac{-5\pi}{6}\right)$	1: $6\text{cis}\pi$ 1: 2 1: $\frac{7\pi}{6}$ of/*or* $-\frac{5\pi}{6}$ **[3]**
b	$\left(3$✓$\text{cis}\left(-\frac{\pi}{6}\right)$✓$\right)^3$ of/*or* $\left(3\text{cis}\left(\frac{11\pi}{6}\right)\right)^3$ $= 27$✓$\text{cis}\left(-\frac{\pi}{2}\right)$✓ of/*or* $27\text{cis}\left(\frac{11\pi}{2}\right)$ $= 27(0 + i(-1))$✓ $= -27i$✓	1: 3 1: $-\frac{\pi}{6}$ of/*or* $\frac{11\pi}{6}$ 1: 27 1: $-\frac{\pi}{2}$ of/*or* $\frac{11\pi}{2}$ 1: $27(0 + i(-1))$ 1: $-27i$ **[6]**
c	$\sqrt{t} = \sqrt{2}$✓$\text{cis}\left(\frac{7\pi}{12}\right)$✓ en/*and* $\sqrt{t} = \sqrt{2}\text{cis}\left(\frac{19\pi}{12}\right)$✓ of/*or* $\sqrt{t} = \sqrt{2}\text{cis}\left(\frac{-5\pi}{12}\right)$ en/*and* $\sqrt{t} = \sqrt{2}\text{cis}\left(\frac{7\pi}{12}\right)$	1: 2 1: $\frac{7\pi}{12}$ of/*or* $\frac{-5\pi}{12}$ 1: $\frac{19\pi}{12}$ of/*or* $\frac{7\pi}{12}$ **[3]**
2.3	$b = 2$✓ $2(2) + c = 0$✓ $\therefore c = -4$✓ $(0; -2): -2 = a\lvert 0 - 4\rvert + 2$✓ $\therefore a = -1$✓	1: $b = 2$ 1: $2(2) + c = 0$ 1: $c = -4$ 1: Vervang/*Substitute* 1: $a = -1$ **[5]**

VRAAG/*QUESTION* 3 [22 PUNTE/*MARKS*]

3.1a	$2{,}7\checkmark = 5{,}9\left(1 - e^{-2(3)k}\right)\checkmark$ $\frac{32}{59} = e^{-6k}\checkmark$ $\therefore -6k = \ln\left(\frac{32}{59}\right)\checkmark$ $\therefore k = 0{,}102\checkmark$	2: Vervang/*Substitute* 1: Vereenvoudig/*Simplify* 1: Skakel om na ln / Convert to ln 1: Antwoord/*Answer* **[5]**
b	$V'(t)\checkmark = -5{,}9e^{-0{,}2t}\checkmark(-0{,}2)\checkmark$ $V'(2) = -5{,}9e^{-0{,}2(2)}(-0{,}2)\checkmark$ $= 0{,}79$ volt/sek$\checkmark$	1: Differensieer/ *Differentiate* 1: $-5{,}9e^{-0{,}2t}$ 1: $-0{,}2$ 1: Vervang/*Substitute* 1: Antwoord/*Answer* **[5]**
3.2a	$f(0) = 2\ln(1) + 3\checkmark$ $y = 3\checkmark$	1: Vervang/*Substitute* 1: Antwoord/*Answer* **[2]**
b	$f'(x) = \frac{2}{x+1}\checkmark$ $x_{n+1} = x_n - \frac{2\ln(x_n+1)+3}{\frac{2}{x_n+1}}\checkmark\checkmark$ $x_1 = -0{,}77811\checkmark$ $x \approx -0{,}777\checkmark$	1: $f'(x) = \frac{2}{x+1}$ 2: Formule/*Formula* 1: $-0{,}77811$ 1: Antwoord/*Answer* **[5]**
c	$x = -1\checkmark$	Antwoord/*Answer* **[1]**
d		1: $x = -1$ 1: $y = 3$ 1: $x = -0{,}777$ 1: Vorm/*shape* **[4]**

VRAAG/*QUESTION* 4 [15 PUNTE/*MARKS*]

4.1	Stel/*Let* $n = 1$: LK/*LHS* $= 2$ RK/*RHS* $= 2$ ∴ Die bewering is waar as/*The statement is true for* $n = 1$✓ Aanvaar die bewering is waar vir /*Accept the statement is true for* $n = k$: $2^1 + 2^0 + \cdots 2^{2-n} = 4 - 2^{2-k}$✓ Beskou nou/*Consider* $n = k + 1$: LK/*LHS* $= 4 - 2^{2-k}$✓ $+ 2^{2-(k+1)}$✓ $= 4 - 2^{-k}(2^2 - 2^1)$✓ $= 4 - 2^{-k+1}$✓ RK/*RHS* $= 4 - 2^{2-(k+1)}$ ✓ $= 4 - 2^{1-k}$✓ $LK = RK$ en die bewering is dus waar vir $n = k + 1$/ $LHS = RHS$ *and the statement is therefore true for* $n = k + 1$. Volgens die beginsel van wiskundige induksie is die bewering dus waar vir alle $n \in \mathbb{N}$ /*By the principle of mathematical induction the statement is true for all* $n \in \mathbb{N}$ ✓✓	1: Bewys waar vir n = 1/ *Prove true for n = 1* 1: Aanvaar waar vir/ Accept true for $n = k$ 1: Vervang/*Substitute* 1: $(k+1)$de term/$(k+1)$*th term* 2: Vereenoudig LK/ *Simplify LHS* 1: Rk Vervang/*Substitute* 1: Vereenoudig RK/ *Simplify RHS* 2: Afleiding/*Deduction* **[10]**
4.2	$n = \frac{1}{2}$✓ $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(ax)^3$✓ $\therefore \frac{-x^3}{128} = \frac{1}{16}a^3x^3$✓ $\therefore -\frac{1}{8} = a^3$✓ $\therefore a = -\frac{1}{2}$✓	1: $n = \frac{1}{2}$ 1: Formule/*Formula* 2: Vereenoudig/ *Simplify* 1: Antwoord/*Answer* **[5]**

VRAAG/*QUESTION* 5 [16 PUNTE/*MARKS*]

5.1a	$: x = 2\text{bgtan}(y-1) - \frac{\pi}{3}$ ✓ of/*or* $x = 2\arctan(y-1) - \frac{\pi}{3}$ $\frac{1}{2}\left(x + \frac{\pi}{3}\right) = \text{bgtan}(y-1)$ ✓ of/*or* $\frac{1}{2}\left(x + \frac{\pi}{3}\right) = \arctan(y-1)$ $\therefore y = \tan\left(\frac{x}{2} + \frac{\pi}{6}\right) + 1$ ✓	1: Ruil x en y/ Swop x and y 1: Vereenoudig/ *Simplify* 1: Antwoord/*Answer* **[3]**
b	$y \in \left(-\frac{4\pi}{3}\checkmark; \frac{2\pi}{3}\checkmark\right)$	1: $-\frac{4\pi}{3}$ 1: $\frac{2\pi}{3}$ **[2]**
c	$g(x) = \frac{2}{3}\checkmark\text{bgtan}(-x\checkmark - 1) - \frac{\pi}{6}\checkmark$ of/*or* $g(x) = \frac{2}{3}\arctan(-x-1) - \frac{\pi}{6}$	1: $\frac{2}{3}$ 1: $-x$ 1: $-\frac{\pi}{6}$ **[3]**
5.2a	$2\pi r = 20$ ✓ $\therefore r = \frac{10}{\pi}$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* **[2]**
b	$6^2 = \left(\frac{10}{\pi}\right)^2 + \left(\frac{10}{\pi}\right)^2 - 2\left(\frac{10}{\pi}\right)\left(\frac{10}{\pi}\right)\cos \hat{\text{BAC}}$ ✓ $-0{,}776\ldots = \cos \hat{\text{BAC}}$ ✓ $\therefore \hat{\text{BAC}} = 2{,}460$ ✓	1: cos-reël/*cos rule* 1: Vereenoudig/ *Simplify* 1: Antwoord/*Answer* **[3]**
c	Inspringende hoek/*Reflex angle* $\hat{\text{BAC}} = 2\pi - 2{,}5 = 3{,}78\ldots$ ✓ Opp/*Area* $= \frac{1}{2}\left(\frac{10}{\pi}\right)^2 (3{,}78\ldots)$ ✓ $= 19{,}17$ ✓	1: Inspringende hoek/ *Reflec angle* 1: Formule/*Formula* 1: Antwoord/*Answer* **[3]**

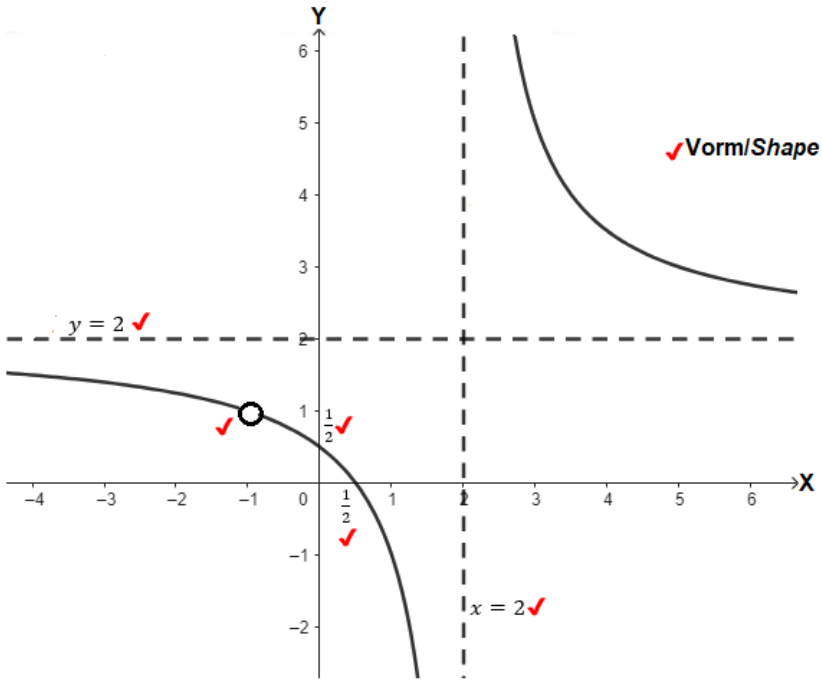
VRAAG/*QUESTION* 6 [11 PUNTE/*MARKS*]

6.1	$\|\text{PT}\| = \sqrt{(-1-1)^2 + \big(2-(-1)\big)^2 + (0-3)^2}$ ✓ $= \sqrt{22}$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* **[2]**
6.2a	$\begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -1 & 1 & 0 \end{vmatrix}$ ✓ $= i[0-(-1)] - j[0-1] + k[1-(-2)]$ ✓ $\boldsymbol{w} = i + j + 3k$ ✓✓✓	1: Matriks/*Matrix* 1: Vereenvoudig/ *Simplify* 3: Antwoord/*Answer* **[5]**
b	Nee/*No* ✓ $\boldsymbol{w} \neq 0$ ✓	1: Afleiding/*Deduction* 1: Motivering/*Motivation* **[2]**
c	$1(-1) + 2(1) - 1(0) = \sqrt{6}\sqrt{2}\cos\theta$ ✓ $\theta = 1{,}28$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* **[2]**

VRAAG/*QUESTION 7* [22 PUNTE/*MARKS*]

7.1a	$\lim\limits_{x\to p^-} f(x) = 2^{3p} + 1$ ✓ $\lim\limits_{x\to p^+} f(x) = 2$ ✓ $\therefore 2^{3p} + 1 = 2$ ✓ $\therefore p = 0$ ✓	1: $\lim\limits_{x\to p^-} f(x)$ 1: $\lim\limits_{x\to p^+} f(x)$ 1: = 1: Antwoord/*Answer* **[4]**
7.1b	$f'(x) = \begin{cases} 2^{3x}.3\ln 2 & \text{as } x \le 0 \text{ ✓} \\ 0 & \text{as } x > 0 \text{ ✓} \end{cases}$ $\lim\limits_{x\to 0^-} f'(x) = 3\ln 2$ ✓ $\lim\limits_{x\to 0^+} f'(x) = 0$ ✓ $\lim\limits_{x\to 0^-} f'(x) \neq \lim\limits_{x\to 0^+} f(x)$ ✓ f is nie differensieerbaar in $x = 0$/*is not differentiable* ✓	2: $f'(x)$ 1: $\lim\limits_{x\to 0^-} f'(x)$ 1: $\lim\limits_{x\to 0^+} f'(x)$ 1: $\neq$ 1: Afleiding/ *Deduction* **[6]**
7.2a	$f'(x) = \sec^2(\text{bgcos}(2x))$ ✓$.\dfrac{-2}{\sqrt{1-4x^2}}$ ✓	1: $\sec^2(\text{bgcos}(2x))$ 1: -2 1: $\frac{1}{\sqrt{1-4x^2}}$ **[3]**
b	$2y^3$ ✓ $+ 2x$ ✓$3y^2$ ✓$.\dfrac{dy}{dx}$ ✓ $-\dfrac{1}{5}y^{-2}$ ✓$.\dfrac{dy}{dx}$ ✓ $= e$ ✓ $\therefore \dfrac{dy}{dx} = \dfrac{e - 2y^3}{2x.3y^2 - \frac{1}{5}y^{-2}}$ ✓	1: $2y^3$ 1: Produkreël/ Product rule 1: $3y^2$ 1: $\frac{dy}{dx}$ 1: $-\frac{1}{5}y^{-2}$ 1: $\frac{dy}{dx}$ 1: e 1: $e - 2y^3$ 1: $2x.3y^2 - \frac{1}{5}y^{-2}$ **[9]**

VRAAG/*QUESTION* 8 [15 PUNTE/*MARKS*]

8.1	Vertikaal/*Vertical:* ✓ $x = 2$✓ Horisontaal/*Horizontal:* ✓ $y = 2$✓	1: Vertikaal/*Vertical* 1: $x = 2$ 1: Horisontaal/ *Horizontal* 1: $y = 2$ **[4]**
8.2	$f'(x)$✓ $= \dfrac{2(x-2)-(2x-1)}{(x-2)^2}$✓ $= \dfrac{-3}{(x-2)^2}$✓ < 0✓ $\therefore f$ daal vir alle $x \in \mathbb{R}$/*decreases for all* $x \in \mathbb{R}$✓	1: $f'(x)$ 1: Kwosiëntreël/ Quotient rule 1: Vereenvoudig/ *Simplify* 1: < 0 1: Afleiding/*Deduction* **[5]**
8.3		1: $y = 2$ 1: $x = 2$ 1: Ongedefinieerd by/ *Undefined at* $x = -1$ 1: x –afsnit/*intercept* 1: y-afsnit/*intercept* 1: Vorm/*Shape* **[6]**

VRAAG/*QUESTION* 9 [16 PUNTE/*MARKS*]

9.1	$f'(x)$✓ $= 5(2x-3)^4$✓(2)✓ $= 0$✓ $\therefore x = \frac{3}{2}$✓ $f\left(\frac{3}{2}\right)$✓ $= e$ $\therefore \left(\frac{3}{2}; e\right)$✓	1: $f'(x)$ 1: $5(2x-3)^4$ 1: 2 1: $= 0$ 1: $x = \frac{3}{2}$ 1: $f\left(\frac{3}{2}\right)$ 1: Antwoord/*Answer* **[7]**
9.2	$f''(x) = 40(2x-3)^3(2)$✓ $f''\left(\frac{3}{2}\right)$✓ $= 0$✓ Moontlike buigpunt by/*Possible inflection point at* $x = \frac{3}{2}$✓ Toets/*Test:* $f''(1)$✓ < 0✓ $f''(2)$✓ > 0✓ $\therefore$ buigpunt by/*inflection point at* $\left(\frac{3}{2}; e\right)$✓	1: $40(2x-3)^3(2)$ 1: $f''\left(\frac{3}{2}\right)$ 1: $= 0$ 1: Afleiding: moontlike buigpunt/*Deduction: possible inflection point* 1: $f''(1)$ 1: < 0 1: $f''(2)$ 1: > 0 1: Afleiding/*Deduction* **[9]**

VRAAG/*QUESTION* 10 [16 PUNTE/*MARKS*]

10.1a	$\frac{1}{2}\int(1-\cos(6x))dx$✓ $-\int \operatorname{cosec}^2 x\,dx$ $=\frac{1}{2}x$✓ $-\frac{1}{2}.\frac{\sin(6x$✓$)}{6$✓$}+\cot x$✓ $+k$	1: $\frac{1}{2}\int(1-\cos(6x))dx$ 1: x 1: $\sin(6x)$ 1: $\frac{1}{6}$ 1: $+\cot x$ **[5]**
b	$\frac{5^{4x^3-1}$✓$}{12$✓$\ln 5$✓$}+k$	1: 5^{4x^3-1} 1: $\frac{1}{12}$ 1: $\frac{1}{\ln 5}$ **[3]**
c	$\frac{1}{3}$✓e^{3x-1}✓ $+x\ln 2$✓ $+k$	1: $\frac{1}{3}$ 1: e^{3x-1} 1: $x\ln 2$ **[3]**
10.2a	$-x^2+4x+1=\mathrm{A}(1+x^2)+2(2x+1)$✓ Stel/*Let* $x=-\frac{1}{2}$ $\therefore \mathrm{A}=-1$✓	1: Maal met KGV/ *Multiply with LCM* 1: Antwoord/*Answer* **[2]**
b	$-\frac{\ln(2x+1)$✓$}{2$✓$}+2\mathrm{bgtan}x$✓ $+k$ of/*or* $-\frac{\ln(2x+1)}{2}+2\arctan x+c$	1: $-\ln(2x+1)$ 1: $\frac{1}{2}$ 1: $2\mathrm{bgtan}x/2\arctan x$ **[3]**

VRAAG/*QUESTION* 11 [16 PUNTE/*MARKS*]

11.1	Stel/*Let* $f(x)=\ln x$✓ *en/*and $g'(x)=2x+1$ ✓ $\therefore \quad f'(x)=\frac{1}{x} \qquad g(x)=x^2+x$ $\int_1^e[(2x+1)\ln x]$ $=[(x^2+x)\ln x]\|_1^e-\int_1^e\frac{1}{x}(x^2+x)dx$ ✓ $=(e^2+e)\ln e-(1^2+1)\ln 1-\left[\frac{x^2}{2}+x\right]\Big\|_1^e$ ✓ $=e^2+e-\left(\frac{e^2}{2}+e-\frac{1}{2}-1\right)$ ✓ $=e^2+e-\frac{e^2}{2}-e+\frac{3}{2}$ ✓ $=\frac{1}{2}(e^2+3)$ ✓	1: $f(x)=\ln x$ 1: $g'(x)=2x+1$ 1: Formule/*Formula* 1: Integreer/*Integrate* 1: Vervang/*Substitute* 1: Vereenvoudig/*Simplify* 1: Antwoord/*Answer* **[7]**
11.2	$\Delta x_i=\frac{2}{n}$ ✓ $\quad x_i=1+\frac{2i}{n}$ ✓ $f(x_i)=-\left(1+\frac{2i}{n}\right)^2+1=\frac{-4i}{n}-\frac{4i^2}{n^2}$ ✓ $f(x_i).\Delta x_i=\frac{-8i}{n^2}-\frac{8i^2}{n^3}$ ✓ $\therefore \sum_{i=1}^{n} f(x_i).\Delta x_i=\frac{-8}{n^2}\sum_{i=1}^{n} i-\frac{8}{n^3}\sum_{i=1}^{n} i^2$ ✓ $=\frac{-8}{n^2}\left(\frac{n^2}{2}+\frac{n}{2}\right)$ ✓ $-\frac{8}{n^3}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)$ ✓ $=-4-\frac{4}{n}-\frac{8}{3}-\frac{4}{n}-\frac{4}{3n^2}$ ✓ $\therefore \lim_{n\to\infty}\sum_{i=1}^{n}(f(x_i).\Delta x_i)=-\frac{20}{3}$ ✓ $\int_1^3(-x^2+1)dx=-\frac{20}{3}$ **OF/*OR***	1: Δx_i 1: x_i 1: $f(x_i)$ 1: $f(x_i).\Delta x_i$ 1: Sigma 1: Vervang/ *Substitute* i^2 1: Vervang/ *Substitute* i 1: Vereenvoudig/ *Simplify* 1: Antwoord/ *Answer*

$\Delta x_i = \frac{2}{n}$ ✓ ; $x_i = 1 + \frac{2i}{n}$ ✓

$f(x_i) = -\left(1 + \frac{2i}{n}\right)^2 + 1 = \frac{-4i}{n} - \frac{4i^2}{n^2}$ ✓

$\sum_{i=1}^{n}\left(\frac{-4i}{n} - \frac{4i^2}{n^2}\right) = \frac{-4}{n}\sum_{i=1}^{n} i - \frac{4}{n^2}\sum_{i=1}^{n} i^2$ ✓

$= \frac{-4}{n}\left(\frac{n^2}{2} + \frac{n}{2}\right)$ ✓ $- \frac{4}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$ ✓

$= -2n - 2 - \frac{4n}{3} - 2 - \frac{2}{3n}$

$= -2n - 4 - \frac{4n}{3} - \frac{2}{3n}$ ✓

$\int_1^3 (-x^2 + 1)dx$ = basis x hoogte

$= \frac{2}{n} \times \left(-2n - 4 - \frac{4n}{3} - \frac{2}{3n}\right) = -4 - \frac{8}{n} - \frac{8}{3} - \frac{4}{3n^2}$ ✓

$\int_1^3 (-x + x^2)dx = \lim_{n\to\infty}\left(-\frac{8}{n} - \frac{20}{3} - \frac{4}{3n^2}\right) = -\frac{20}{3}$ ✓

$\therefore \int_1^3 (-x^2 + 1)dx = -\frac{20}{3}$

[9]