α-MATHEMATICS

Alpha Mathematics
FINAL EXAM PAPER 2025

20 October 2025 **Time: 3 hours**

Grade 12 **Total: 200 marks**

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the examination paper:

1. Answer ALL 10 questions on this paper.
2. Write your name and ID number on the front page of the examination paper.
3. Non-programmable calculators may be used, unless otherwise indicated at a specific question.
4. Unless indicated otherwise, all answers, where applicable, must be given correct to two decimal places.
5. The diagrams in the examination paper are not necessarily drawn to scale.
6. All angles are given in radians. Answers must be given in radians where applicable.
7. This examination paper consists of a front page, 22 pages and a formula sheet.
8. Question 1 consists or **15** multiple choice questions. Answer it on the answer sheet. This answer sheet is at the front of the paper.
   Do not remove the answer sheet.
9. For all other questions, all necessary calculations must be shown clearly. The correct answer on its own will not necessarily lead to full marks.
10. Additional writing space is provided at the end of this examination paper. Clearly indicate if you made use of this to complete a question.
11. Write neatly and legibly.

QUESTION 1 **[30 MARKS]**

- Answer this question **on the answer sheet,** which is attached to the front, by making an X (cross) on A, B, C or D. These questions count 2 marks each.
- Please **DO NOT** detach this page from the paper.

1.1 $\log_5 125 - \log_5 25 =$

(A) 3

(B) 1

(C) $\log_5 100$

(D) $\log_5 5^{\frac{3}{2}}$

1.2 Which one of the following vectors is a unit vector?

(A) (1; 1; 1)

(B) $(\frac{1}{2}; \frac{1}{4}; \frac{3}{4})$

(C) $\frac{1}{3}i + \frac{2}{3}j - \frac{2}{3}k$

(D) $i - j + k$

1.3 If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$, then the function f at the point $x = a$, will **always** be:

(A) continuous

(B) differentiable

(C) integrable

(D) not one of the above

1.4 The graph of $y = -|x - 2| + 3$ is decreasing for $x \in$

(A) $(-\infty; 3]$

(B) $(-\infty; 2]$

(C) $[2; \infty)$

(D) $[-2; \infty)$

1.5 The following statement is always true:

(A) Polynomials of an even grade will always have at least one real zero of the form $a + b\sqrt{c}$ with $c \neq 0$.

(B) If a polynomial with real coefficients can factorise in real factors, the polynomial will also factorise in integer factors.

(C) If a polynomial with real coefficients can factorise in rational factors, the polynomial will also factorise in integer factors.

(D) Polynomials of an even grade, always have at least 2 complex zeros of the form $a + ib$ with $b \neq 0$.

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1.6 The third term in the power series $\sqrt[3]{1+2x}$ is

(A) $-\frac{4}{9}x^2$

(B) $4x^2$

(C) $-4x^2$

(D) $\frac{4}{9}x^2$

1.7 Determine the constant term in the expansion of $\left(-\frac{x}{3}+\frac{2}{x^2}\right)^9$.

(A) $\frac{-1}{3^3}\times 2^6$

(B) $\binom{9}{3}\times\frac{1}{3^6}\times 2^3$

(C) $-\binom{9}{6}\times\frac{1}{3^3}\times 2^6$

(D) $\frac{\mathbf{9!}}{\mathbf{3!}}\times\frac{1}{3^6}\times 2^3$

1.8 If $y=2x-8$, for which values of x will the product xy have a minimum value?

(A) 4

(B) −8

(C) −2

(D) 2

1.9 A polynomial $P(x)$ has a relative maximum at $(-5;5)$, a relative minimum at $(1;2)$ and a relative maximum at $(5;8)$. The polynomial has no other stationary points. What is the maximum number of real zeros that $P(x)$ can have?

(A) One

(B) Four

(C) Three

(D) Two

1.10 Which of the following expressions is the result when the graph of $f(x)=\ln(x+2)$ translates three units left and four units up?

(A) $\ln(x-1)+4$

(B) $\ln(x+2)+4$

(C) $\ln(x+5)+4$

(D) $\ln(x+5)-4$

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1.11 Determine the domain of the function $f(x) = \sqrt{2-x} \times \ln 2x$.

(A) $[2;\ \infty)$

(B) $(-\infty; 2]$

(C) $(0; \infty)$

(D) $(0; 2]$

1.12 Given $f(x) = \frac{5}{2}x^2 - e^x$, then the point $x = \ln 5$ is

(A) a stationary point of f.

(B) a point of inflection of f.

(C) a zero of f.

(D) an undefined point of f.

1.13 If $\frac{dy}{dx} = y.\sec^2 x$, then $y =$

(A) $e^{\tan x}$

(B) $e^{\tan x}.\tan x$

(C) $\tan x$

(D) $\tan x + e^x$

1.14 Which of the following are anti-derivatives (integrals) of $f(x) = \sin x \cos x$?

I $\frac{\sin^2 x}{2}$ II $\frac{\cos^2 x}{2}$ III $-\frac{\cos(2x)}{4}$

(A) Only I.

(B) All three.

(C) Only I and II.

(D) Only I and III.

1.15 The graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1; -6)$. What is the value of b?

(A) 0

(B) 1

(C) 3

(D) It cannot be determined with the given information.

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Answer the following questions **on the exam paper** on the lines provided below each question. Clearly indicate if you use the additional writing space at the end of the paper to complete a question.

QUESTION 2 **[17 MARKS]**

2.1 The half-life of radium-226 is 1590 years. The formula for the mass, m (in milligram) of radium-226 that is left after t years is given by:

$$m(t) = 100e^{-\frac{t(\ln 2)}{1590}}$$

(a) Determine the mass of the radium-226 after 100 years to the closest milligram (mg). (2)

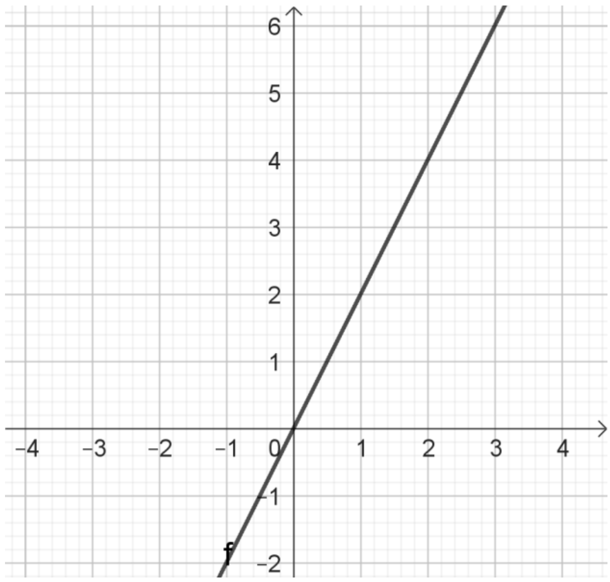
(b) After how many years will the mass decrease to 30 mg? (5)

2.2 Given $f(x) = (x+1)\ln(x+1)$ and $g(x) = e^x - 1$.

(a) Show that $(f \circ g)(x) = xe^x$. (2)

(b) Hence solve for x if $(f \circ g)(x) = 2x$. (4)

(c) The graph of the function of $(f \circ g)(x) = xe^x$ has a minimum turning point at A $(-1; -0{,}4)$ and a horizontal asymptote at $y = 0$. It is increasing for $x > -1$. Draw the graph of this function on the given set of axes below, on which the graph of $y = 2x$ is sketched. Show the turning point A and the point(s) calculated in QUESTION 2.2 (b) with a B (and C) on your sketch. (4)



QUESTION 3 **[16 MARKS]**

3.1 Given $z = \dfrac{4(-1+\sqrt{3}i)^n}{2^n(-\sqrt{3}-i)^2}$. Simplify this expression using de Moivre's theorem and give the final answer in its **simplest polar form** $(r\text{cis}\theta)$ in terms of π and n. (6)

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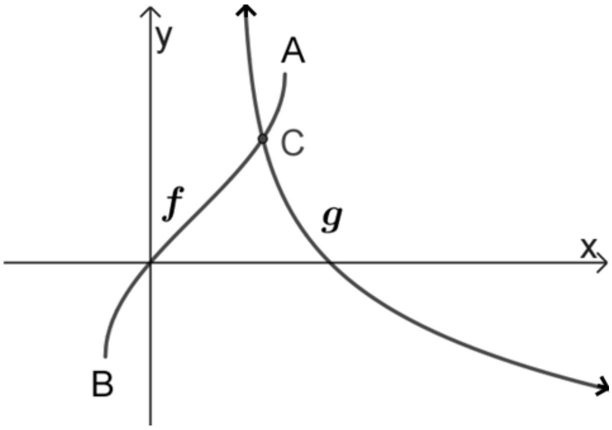
3.2 Given $z^3 = -4 + 4i$. Determine all the possible values of z using de Moivre's theorem. Leave the final answers in **polar form**. (5)

3.3 In the equation $2x^3 + ax^2 + 32x - 17 = 0$, $x = -1 + 4i$ is a root. Determine the value of a. (5)

QUESTION 4 **[19 MARKS]**

4.1 The graph below shows the functions of $f(x) = \arcsin\left(x - \frac{1}{2}\right) + \frac{\pi}{6}$ and $g(x) = -\ln(x - 1)$.



(a) Give the coordinates of A and B, the end points of f. (4)

(b) Give the domain of f^{-1}, the inverse function of f. (1)

(c) The graph of f and g intersect at C. Use Newton's method and determine the x-value of C correct to 4 decimal digits. (Start at $x = 1{,}2$ and show all your steps.) (5)

4.2 Determine the value(s) of x if $|x + 2| = -\frac{x}{4} + 4$. (5)

4.3 The following set of equations is given:

$$2x + y + z = 0$$
$$x - 2y - z = t$$
$$4x + 3y + 2z = 1$$

It is also given that: $|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 4 & 3 & 2 \end{vmatrix} = 3$ and $\boldsymbol{z} = -5$.

Use Cramer's rule and determine the value of $\boldsymbol{t}$. (Clearly show the matrixes that you use.) (4)

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QUESTION 5 **[20 MARKS]**

5.1 Given: vectors $A = (3; -1; 5)$ and $B = 2i - j - k$.

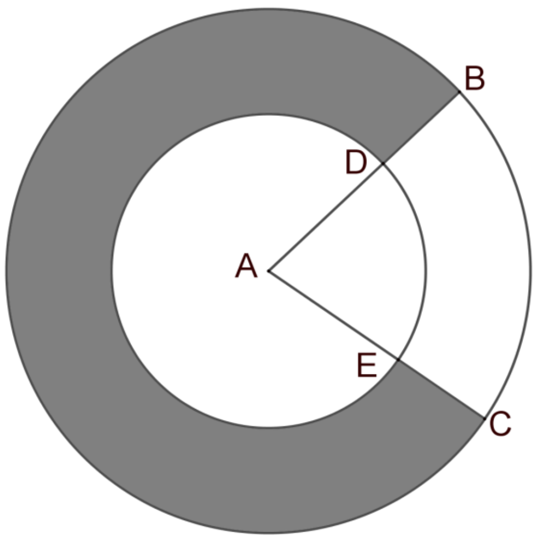
(a) Determine the distance between A and B. Leave the answer in surd form if possible. (2)

(b) Determine the angle between vector A and the y-axis. (3)

(c) Determine the angle between the two vectors. (5)

(d) Determine a vector that is perpendicular to A and B. (4)

5.2 The sketch shows two concentric circles with centre A. AD = 4 is a radius of the smaller circle and AB = 6 is a radius of the bigger circle. ADB and AEC are straight lines. The big area DBCE is shaded.



(a) If the area of the shaded area equals 15π, determine $\mathrm{B\widehat{A}C}$. (4)

(b) Determine the circumference of the shaded area in terms of π. (2)

QUESTION 6 **[20 MARKS]**

6.1 Given:

$$f(x) = \begin{cases} -2\cos x & if \quad x \leq \frac{5\pi}{3} \\ \frac{-\sqrt{3}}{2\pi}\left(x - \frac{2\pi}{3}\right)^2 & if \quad x > \frac{5\pi}{3} \end{cases}$$

(a) Determine algebraically whether f is continuous at $x = \frac{5\pi}{3}$.
If f is not continuous, give the type of discontinuity. (5)

(b) Determine $\lim_{x \to \frac{5\pi}{3}^-} f'(x)$ and $\lim_{x \to \frac{5\pi}{3}^+} f'(x)$. (5)

(c) Is f differentiable at $x = \frac{5\pi}{3}$? Give a reason for your answer. (2)

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6.2 Use mathematical induction to prove that:

$$\sum_{i=1}^{n} i^3 = \frac{n^2}{4}(n+1)^2$$

(8)

QUESTION 7 **[23 MARKS]**

7.1 Determine the derivative of the following as asked. (Answers need not be simplified.)

(a) Determine $\frac{d}{dx}[2^{\operatorname{cosec} 3x}]$ (3)

(b) Determine $g'(x)$ if $g(x) = e^{\tan x}\left(\sqrt[4]{5x^3}\right)$ (5)

7.2 Given: $\cot 2y - y\ln x + x = 2$.

The gradient of the tangent to the graph of the given equation is $\boldsymbol{a} + \boldsymbol{b\pi}$ at the point $(1; \frac{\pi}{8})$. Use implicit differentiation and determine $\boldsymbol{a}$ and $\boldsymbol{b}$. (6)

7.3 Use a Riemann sum and determine the value of $\int_2^4 (3x^2 - 6x)\,dx$. (9)

QUESTION 8 **[17 MARKS]**

Given: $\boldsymbol{f(x)} = \frac{2x^2-18}{x^2-5x-6}$

8.1 Calculate the x- and y-intercepts of f. (3)

8.2 Determine all asymptotes of f. (3)

8.3 Use differentiation to show that f has no stationary points. (4)

8.4 The graph f intersects the horizontal asymptote once at B. Calculate the x-value of this point. (2)

8.5 Draw a sketch graph of f. Clearly show the intercepts with the axes, the asymptotes and the point calculated in QUESTION 8.4 with a D on the graph. (5)

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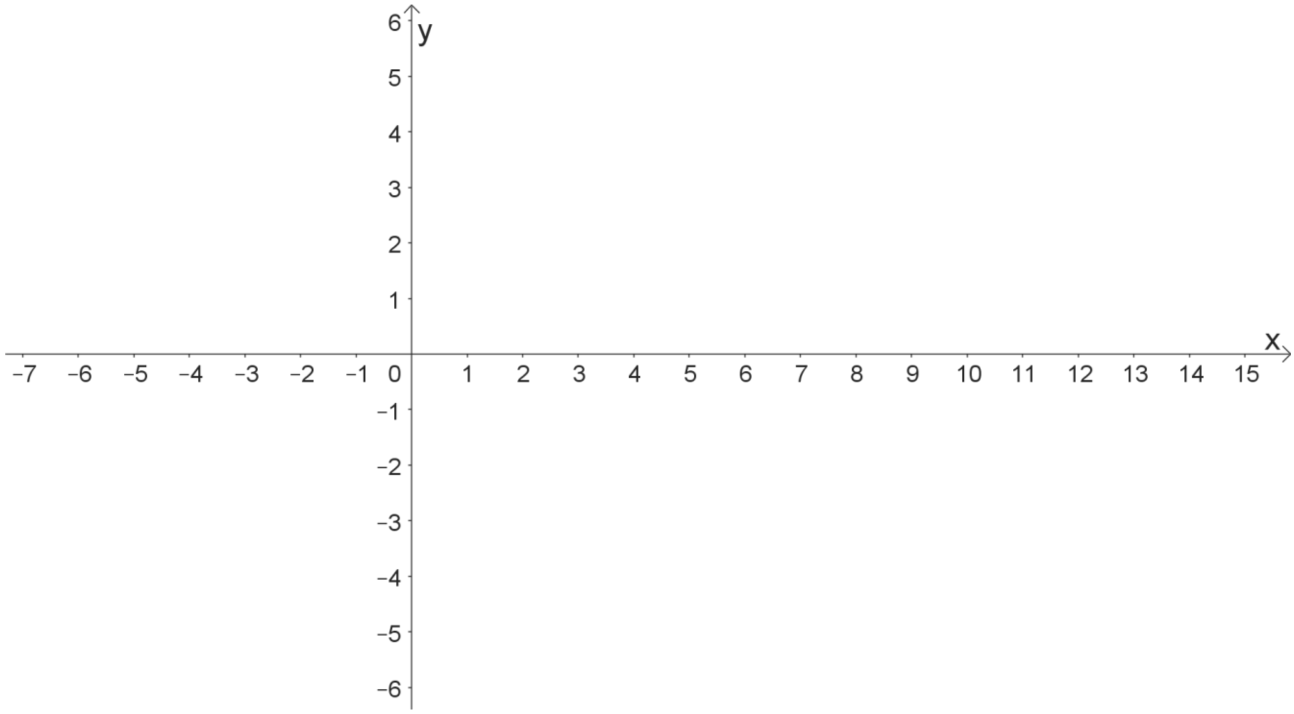

QUESTION 9 **[21 MARKS]**

Determine the following integrals:

9.1 $\int (\left(\frac{8}{(4x-2)\ln 5}\right) + 10)dx.$ (3)

9.2 $\int \frac{3x^2+3x+7}{(x^2+1)(x-2)}dx$ by using partial fractions. (8)

9.3 $\int x^2 . e^x \, dx$ by using factor integration (integration by parts). (6)

9.4 $\int \left(\sin 2x . \frac{1}{2\sqrt{x}} + 2\sqrt{x}\cos 2x\right) dx.$
(Hint: Use the product rule and the fundamental theorem of calculus.) (4)

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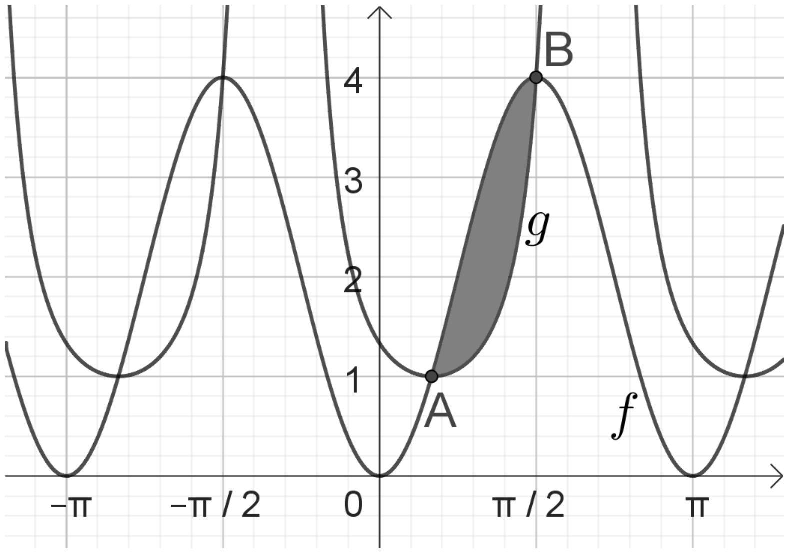
QUESTION 10 **[17 MARKS]**

10.1 The sketch below shows the graphs of $f(x) = (2\sin x)^2$ and $g(x) = \sec^2(x - \frac{\pi}{6})$. The graphs intersect at $\mathrm{A}(\frac{\pi}{6}; 1)$ and $\mathrm{B}(\frac{\pi}{2}; 4)$. The area between the two graphs between A and B is shaded.



(a) Determine a, b and c if the area of this region equals $a\pi + b\sqrt{c}$.
(Show all the steps that you use.) (7)

(b) The shaded area rotates around the x-axis. Write down **only** the equation that can be used to calculate the volume of the body of revolution formed in this way. (3)

10.2 Calculate the value of a if $\int_0^a \frac{1}{\sqrt{3+4x-4x^2}}dx = \frac{\pi}{6}$. (7)

TOTAL: 200 MARKS

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