α-WISKUNDE/ MATHEMATICS

Junie/ June 2025 **Tyd/ Time: 3 uur/*hours***
Graad/ Grade 12 **Totaal/ Total: 200 PUNTE/ *MARKS***

VRAAG/*QUESTION* 1 **[30 PUNTE/*MARKS*]**

1.1	**A**	**B**	**C**	**D**
1.2	**A**	**B**	**C**	**D**
1.3	**A**	**B**	**C**	**D**
1.4	**A**	**B**	**C**	**D**
1.5	**A**	**B**	**C**	**D**
1.6	**A**	**B**	**C**	**D**
1.7	**A**	**B**	**C**	**D**
1.8	**A**	**B**	**C**	**D**
1.9	**A**	**B**	**C**	**D**
1.10	**A**	**B**	**C**	**D**
1.11	**A**	**B**	**C**	**D**
1.12	**A**	**B**	**C**	**D**
1.13	**A**	**B**	**C**	**D**
1.14	**A**	**B**	**C**	**D**
1.15	**A**	**B**	**C**	**D**

VRAAG/*QUESTION* 2 [16 PUNTE/*MARKS*]

2.1	$\|A\| = \begin{vmatrix} 3 & 5 \\ -1 & -1 \end{vmatrix} = 2$✓ $\|A_y\| = \begin{vmatrix} 3 & 7 \\ -1 & -1 \end{vmatrix}$✓ $= 4$✓ $\therefore y = \frac{4}{2}$✓ $= 2$✓	1: $\|A\| = 2$ 1: $\|A_y\|$ 1: $= 4$ 1: $\frac{4}{2}$ 1: antwoord/*answer* **[5]**
2.2a	$x \geq \frac{1}{2}$: ✓ $2x - 1 = x^2$✓ $\therefore x = 1$ ✓ $x < \frac{1}{2}$: ✓ $-2x + 1 = x^2$✓ $\therefore x = 0{,}41$✓ of/*or* $x = -2{,}41$✓	2: opskrifte/*headings* 1: $2x - 1 = x^2$ 1: $x = 1$ 1: $-2x + 1 = x^2$ 1: $x = 0{,}41$ 1: $x = -2{,}41$ **[7]**
2.2b	$2x + 3 \geq 7$✓ of/*or* $2x + 3 \leq -7$✓ $\therefore x \geq 2$✓ $x \leq -5$✓	1: $2x + 3 \geq 7$ 1: $2x + 3 \leq -7$ 1: $x \geq 2$ 1: $x \leq -5$ **[4]**

VRAAG/*QUESTION* 3 [15 PUNTE/*MARKS*]

3.1	$x = -\sqrt{3}$✓ is ook 'n nulpunt/*is a zero as well* $\therefore x^2 - 3$✓ is 'n faktor/*is a factor* $\therefore f(x) = (x - \sqrt{3})(x + \sqrt{3})(4x^2 + 1)$✓ $= (x - \sqrt{3})(x + \sqrt{3})(2x + i)$✓$(2x - i)$✓	1: $x = -\sqrt{3}$ 1: $x^2 - 3$ 1: faktore in/*factors in* $\mathbb{R}$ 2: faktore in/*factors in* $\mathbb{C}$ **[5]**
3.2a	$a = 2\sqrt{2}$✓$\text{cis}\left(\frac{3\pi}{4}\right)$✓ $b = 8$✓$\text{cis}\left(\frac{3\pi}{2}\right)$✓ of/*or* $b = 8\text{cis}\left(\frac{-\pi}{2}\right)$	1: $2\sqrt{2}$ 1: $\frac{3\pi}{4}$ 1: 8 1: $\frac{3\pi}{2}$ of/*or* $\frac{-\pi}{2}$ **[4]**
3.2b	$a^2b = \left(8$✓$\text{cis}\left(\frac{3\pi}{2}\right)$✓$\right)\left(8\text{cis}\left(\frac{3\pi}{2}\right)\right)$ $= 64$✓$\text{cis}(3\pi)$✓ $= 64(-1 + 0i)$✓ $= -64$✓	1: 8 1: $\frac{3\pi}{2}$ 1: 64 1: 3π 1: $-1 + 0i$ 1: antwoord/*answer* **[6]**

VRAAG/*QUESTION* 4 [12 PUNTE/*MARKS*]

4.1	$P(0) = \frac{10\,000}{5 + 1245e^{0}}$✓ $= 8$✓	1: vervang $t = 0$/ *substitute* $t = 0$ 1: antwoord/*answer* **[2]**
4.2	1000✓ $= \frac{10\,000}{5 + 1245e^{-0,97t}}$ $\therefore \frac{5}{1245} = e^{-0,97t}$✓ $\therefore -0,97t = \ln\frac{5}{1245}$✓ $\therefore t \approx 6$ dae/*days*✓	1: vervang $P(t) = 1000$/ *substitute* $P(t) = 1000$ 1: vereenvoudig/*simplify* 1: omskakeling/*conversion* 1: antwoord/*answer* **[4]**
4.3	2000✓	antwoord/*answer* **[1]**
4.4	$P(t) = 10\,000(5 + 1245e^{-0,97t})^{-1}$ $\therefore P'(t)$ $= -10\,000(5 + 1245e^{-0,97t})^{-2}$✓$(1245e^{-0,97t})$✓$(-0,97)$✓	1:$-10\,000(5 + 1245e^{-0,97t})^{-2}$ 1: $(1245e^{-0,97t})$ 1: $(-0,97)$ **[3]**
4.5	Die afgeleide funksie gee die tempo waarteen mense besmet word./*The derivative function gives the rate at which people are infected.* ✓✓	2: antwoord/*answer* **[2]**

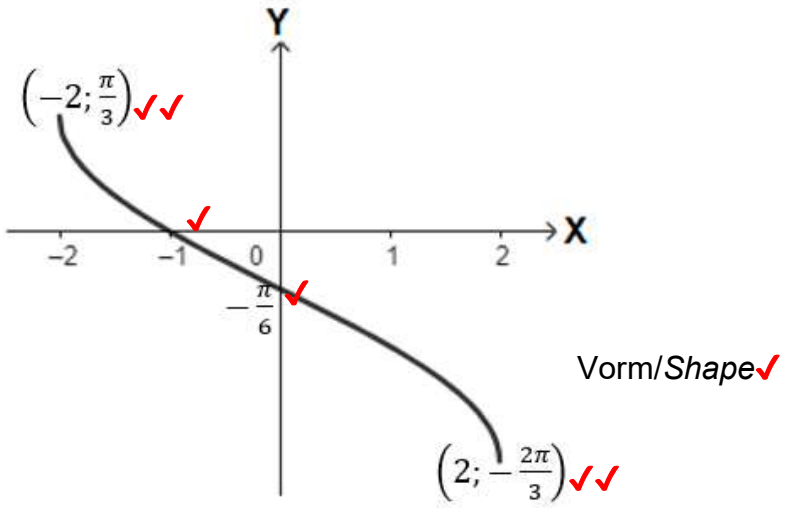
VRAAG/*QUESTION* 5 [18 PUNTE/*MARKS*]

5.1	$n = -3$✓ Derde en vierde terme: $\frac{(-3)(-4)}{2!}(-x)^2$✓ $+ \frac{(-3)(-4)(-5)}{3!}(-x)^3$✓ $= 6x^2$✓ $+ 10x^3$✓	1: $n = -3$ 1: $\frac{(-3)(-4)}{2!}(-x)^2$ 1: $+\frac{(-3)(-4)(-5)}{3!}(-x)^3$ 1: $6x^2$ 1: $+10x^3$ **[5]**
5.2	$\binom{15}{11}$✓$(x^{-2})^4$✓$(x)^{11}$✓ $= 1365x^3$✓	1: $\binom{15}{11}$ 1: $(x^{-2})^4$ 1: $(x)^{11}$ 1: antwoord/*answer* **[4]**
5.3	Stel/*Let* $n = 1$: LK/*LHS* $= 1$ RK/*RHS*$= 1$ ∴ Die bewering is waar as/The statement is true for $n = 1$✓ Aanvaar die bewering is waar vir/ *Accept the statement is true for* $n = k$: $1 - 1 - 3 + \cdots + (3 - 2k) = 2k - k^2$ ✓ Beskou nou/*Consider* $n = k + 1$: LK/*LHS* $= 2k - k^2$✓ $+ (3 - 2(k + 1))$✓ $= -k^2 + 1$ ✓ RK/RHS $= 2(k + 1) - (k + 1)^2$✓ $= -k^2 + 1$ ✓ ∴ LK=RK en die bewering is waar vir $n = k + 1$/ *LHS=RHS and the statement is true for* $n = k + 1$ Volgens die beginsel van wiskundige induksie is die bewering dus waar vir alle $n \in \mathbb{N}$ /*By the principle of mathematical induction the statement is true for all* $n \in \mathbb{N}$ ✓✓	1: Bewys n = 1/ *Prove n = 1* 1: Aanvaar bewering is waar vir $n = k$ / *Accept statement is true for* $n = k$ 1: Vervang/*Substitute* 1: $(k + 1)$de term/ $(k + 1)$*th term* 1: Vereenoudig LK/ *Simplify LHS* 1: RK/*RHS* 1: Vereenoudig RK/ *Simplify RHS* 2: Afleiding/*Deduction* **[9]**

VRAAG/*QUESTION* 6 [12 PUNTE/*MARKS*]

6.1a	$\|\boldsymbol{u}\| = \sqrt{5}$✓ ∴ rigtingskosinusse/*direction cosines*: $\left[\frac{1}{\sqrt{5}}; 0; \frac{2}{\sqrt{5}}\right]$✓	1: $\sqrt{5}$ 1: $\left[\frac{1}{\sqrt{5}}; 0; \frac{2}{\sqrt{5}}\right]$ **N.B. Blokhakies/ Square brackets** **[2]**
6.1b	$\gamma = \text{bgcos}\left(\frac{2}{\sqrt{5}}\right)$✓ of/*or* $\arccos\left(\frac{2}{\sqrt{5}}\right)$ $= 0{,}46$✓	1: $\text{bgcos}\left(\frac{2}{\sqrt{5}}\right)$ / $\arccos\left(\frac{2}{\sqrt{5}}\right)$ 1: antwoord/*answer* **[2]**
6.2a	$PS = (-3; 1; 3)$✓ $ST = (1; 5; -1)$✓	1: PS 1: ST **[2]**
6.2b	$PS \bullet ST = (-3)(1) + (1)(5) + (3)(-1)$✓ $= -1$✓ $\neq 0$✓ ∴ PS en ST is nie loodreg op mekaar nie/ PS *and* ST *is not perpendicular to each other*✓	1: puntproduk/ *point product* 1: -1 1: $\neq 0$ 1: gevolgtrekking/ *deduction* **[4]**
6.2c	$\sqrt{16^2 + 16^2}$✓ $= 16\sqrt{2}$✓	1: $\sqrt{16^2 + 16^2}$ 1: $16\sqrt{2}$ **[2]**

VRAAG/*QUESTION 7* **[15 PUNTE/*MARKS*]**

7.1a	$\widehat{OAB} = \widehat{AOB} = \frac{\pi}{4}$ (binnehoeke van Δ/*sum of angles of* Δ)✓ $\therefore \Delta$ is gelykbenig/*isosceles*✓ $\therefore AB = 2$cm	1: $\widehat{OAB} = \widehat{AOB}$ 1: gelykbenig/*isosceles* **[2]**
7.1b	$PB = 2\left(\frac{\pi}{4}\right)$ ✓ $= \frac{\pi}{2}$ ✓	1: formule/*formula* 1: antwoord/*answer* **[2]**
7.1c	$AO = 2\sqrt{2}$✓ (Pythagoras) $AP = 2\sqrt{2} - 2$ ✓ Omtrek/*Perimeter* $= \frac{\pi}{2} + 2 + 2\sqrt{2} - 2$✓ $= 4{,}4$✓	1: AO 1: AP 1: omtrek/*perimeter* 1: antwoord/*answer* **[4]**
7.2		2: $\left(-2;\frac{\pi}{3}\right)$ 2: $\left(2;\frac{-2\pi}{3}\right)$ 1: $x = -1$ 1: $y = -\frac{\pi}{6}$ 1: vorm/*shape* **[7]**

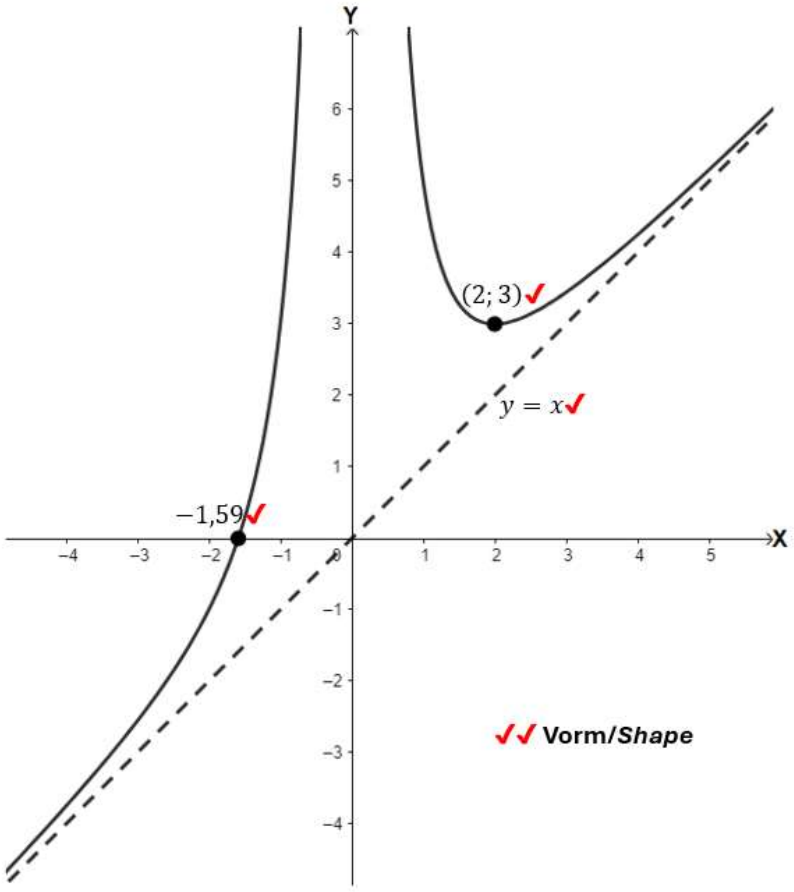
VRAAG/*QUESTION* 8 [20 PUNTE/*MARKS*]

8.1a	$x = -2$✓ $\lim_{x\to -2} f(x) \neq f(-2)$ ✓ $x = 6$✓ $f(6)$ bestaan nie/*does not exist*✓	1: $x = -2$ 1: $\lim_{x\to -2} f(x) \neq f(-2)$ 1: $x = 6$ 1: $f(6)$ bestaan nie/ *does not exist* **[4]**
8.1b	$x = 4$✓ $\lim_{x\to -4^-} f(x) \neq \lim_{x\to 4^+} f(x)$ ✓	1: $x = 4$ 1: $\lim_{x\to -4^-} f(x) \neq \lim_{x\to 4^+} f(x)$ **[2]**
8.2a	onwaar/*false*✓✓	onwaar/*false* **[2]**
8.2b	waar/*true*✓✓	waar/*true* **[2]**
8.2c	waar/*true*✓✓	waar/*true* **[2]**
8.2d	onwaar/*false*✓✓	onwaar/*false* **[2]**
8.2e	waar/*true*✓✓	waar/*true* **[2]**
8.2f	onwaar/*false*✓✓	onwaar/*false* **[2]**
8.2g	onwaar/*false*✓✓	onwaar/*false* **[2]**

VRAAG/*QUESTION* 9 [14 PUNTE/*MARKS*]

9.1	$f'(x) = \frac{3}{5\sqrt[5]{x^2}} - 1$✓ $x_{n+1} = x_n - \frac{\sqrt[5]{x_n{}^3} - x_n + 1}{\frac{3}{5\sqrt[5]{x_n{}^2}} - 1}$✓ $x_1 = 2{,}891063 \ldots$ ✓ $x \approx 2{,}89055$✓✓	1: $f'(x)$ 1: formule/*formula* 1: x_1 2: antwoord/*answer* **[5]**
9.2	2^{7y}✓$.\ln 2$✓$.7\frac{dy}{dx}$✓ $= 2\cos(7x)$✓$.(-\sin(7x))$✓$.7$✓ $+ \pi\frac{dy}{dx}$✓ $\therefore \frac{dy}{dx} = \frac{2\cos(7x).(-\sin(7x)).7}{2^{7y}.\ln(2).7 - \pi}$✓	1: 2^{7y} 1: $\ln(2)$ 1: $7\frac{dy}{dx}$ 1: $2\cos(7x)$ 1: $-\sin(7x)$ 1: 7 1: $\pi\frac{dy}{dx}$ 1:$2\cos(7x).(-\sin(7x)).7$ 1: $2^{7y}.\ln 2.7 - \pi$ **[9]**

VRAAG/*QUESTION* 10 [17 PUNTE/*MARKS*]

10.1	Vertikaal/*Vertical*✓ $x = 0$✓ Skuins/*Oblique*✓ $y = x$✓	1: Vertikaal/*Vertical* 1: $x = 0$ 1: Skuins/*Oblique* 1: $y = x$ **[4]**
10.2	$f'(x) = 1 - 8x^{-3}$✓ $= 0$✓ $\therefore x = 2$✓ $f(2)$✓ $= 3$ Stasionêre punt/*Stationary point*: $(2; 3)$✓	1: $f'(x)$ 1: $= 0$ 1: $x = 2$ 1: $f(2)$ 1: antwoord/*answer* **[5]**
10.3	Vervang $x = 2$ in die tweede afgeleide. ✓ Indien minimum, sal $f''(2) > 0$✓/ *Substitute* $x = 2$ *in the second derivative.* *If minimum, then* $f''(2) > 0$	1: Vervang in tweede afgeleide/*Substitute in second derivative* 1: $f''(2) > 0$ **[2]**
10.4	$x = -1{,}59$✓	1: $x = -1{,}59$ **[1]**
10.5		1: $x = -1{,}59$ 1: $(2; 3)$ 1: $y = x$ 2: vorm/*shape* **[5]**

VRAAG/*QUESTION* 11 [11 PUNTE/*MARKS*]

11.1	$f'(x) = \frac{11}{5}\sqrt[5]{x^6}$✓ $f''(x) = \frac{11}{5}.\frac{6}{5}\sqrt[5]{x}$✓ $= 0$✓ $\therefore x = 0$✓ $f(0)$✓ $= e$ $\therefore (0; e)$✓	1: $f'(x)$ 1: $f''(x)$ 1: $= 0$ 1: $x = 0$ 1: $f(0)$ 1: $(0; e)$ **[6]**
11.2	$f''\left(-\frac{1}{2}\right)$✓ > 0✓ $f''\left(\frac{1}{2}\right)$✓ < 0✓ $\therefore (0; e)$ is 'n buigpunt/*is a point of inflection*✓	1: $f''\left(-\frac{1}{2}\right)$ 1: > 0 1: $f''\left(\frac{1}{2}\right)$ 1: < 0 1: afleiding/*deduction* **[5]**

VRAAG/*QUESTION* 12 [20 PUNTE/*MARKS*]

12.1	$f'(x)$ $= \left[-\frac{1}{\pi x^2}\checkmark + \text{cosec}x\cot x\checkmark\right]\left[\ln(5x) + \text{bgcos}\left(-\frac{x}{2}\right)\right]$ $+ \left[\frac{1}{\pi x} - \text{cosec}x\right]\checkmark\left[\frac{1}{x}\checkmark + \frac{\frac{1}{2}\checkmark}{\sqrt{1-\frac{x^2}{4}}\checkmark}\right]$	1: $-\frac{1}{\pi x^2}$ 1: $+\text{cosec}x\cot x$ 1: produkreël/ *product rule* 1: $\frac{1}{x}$ 1: $+\frac{1}{2}$ 1: $\frac{1}{\sqrt{1-\frac{x^2}{4}}}$ **[6]**
12.2a	$\frac{2^{4x}\checkmark}{4\checkmark\ln 2\checkmark} + \frac{2x^5}{5}\checkmark + k$	1: 2^{4x} 1: 4 1: $\ln 2$ 1: $\frac{2x^5}{5}$ **[4]**
12.2b	$\frac{2\ln(7x-1)\checkmark}{7\checkmark} - \frac{(7x-1)^3\checkmark}{3\checkmark.7\checkmark} + k$	1: $\ln(7x-1)$ 1: $\frac{2}{7}$ 1: $(7x-1)^3$ 1: 3 1: 7 **[5]**
12.3	$\text{Vol} = \pi\int_2^4\left(\sqrt{x^2-p}\right)^2 dx\checkmark$ $= \pi\int_2^4(x^2-p)dx\checkmark$ $= \pi\left[\frac{x^3}{3} - px\right]\Big\|_2^4\checkmark$ $= \pi\left[\frac{4^3}{3} - 4p - \left(\frac{2^3}{3} - 2p\right)\right]\checkmark$ $= \pi\left(\frac{56}{3} - 2p\right)\checkmark$	1: formule/*formula* 1: vereenvoudig/ *simplify* 1: integreer/*integrate* 1: vervang/*substitute* 1: antwoord/*answer* **[5]**