α-MATHEMATICS

Grade 10 **Time: $2\frac{1}{2}$ hours**

Examiner: Pieter van Onselen **Totaal: 150 marks**

Moderator: Lanice Liebenberg

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 9 pages and an answer sheet of 2 pages.
2. Answer ALL 8 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, must be given correct to two decimal places, where necessary.
6. Clearly show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neatly and legibly.

Question 1 [20 marks]

This question must be answered on the **answer sheet**.

Every question has **ONLY** one correct answer for **TWO** marks each. Mark the correct answer with an **X** on the answer sheet.

1.1 If $x^2 + 4 = 0$, then:

(A) $x = \pm 2$ (B) $x = \pm 2i$ (C) $x = \pm 4i$ (D) $x = \pm 4$

1.2 Given: $\frac{x-1}{x^2} \equiv \frac{1}{x} + \frac{B}{x^2}$

(A) $B = -1$ (B) $B = 1$ (C) $B = 2$ (D) $B = -2$

1.3 Given the vectors $w = (2; -1)$ and $v = (x; 1)$ and the dot produk $v \cdot w = 1$.
Determine the value of x.

(A) -1 **(B)** $\frac{-(2-x)^{\frac{1}{2}}}{2}$ **(C)** $-\frac{1}{2\sqrt{2-x}}$ **(D)** $\frac{1}{2}(x-2)^{\frac{1}{2}}$

1.4 Which of the following statements are NOT true:

(A) A radian is the ratio between the arc length and the radius.

(B) $1\ radian \approx 57{,}3^0$

(C) Arc length is the radius divided by the angle of the sector.

(D) $30^0 = \frac{30^0}{180^0} \times \pi\ radians$

1.5 Determine $\begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix}$

(A) 0 (B) 4 (C) 3 (D) 5

1.6 Given $f'(x) = 3x^2$ and $f(x) = \int 3x^2 dx$. The function $f(x)$ goes through the point $(1; 0)$.
Determine the equation of $f(x)$.

(A) $f(x) = 3x^2 + 1$ (B) $f(x) = 3x^2 - 1$

(C) $f(x) = x^3 + 1$ (D) $f(x) = x^3 - 1$

1.7 If $\frac{f(x)}{x^3+x^2}$ is decomposed using partial fractions, then $\frac{f(x)}{x^3+x^2} \equiv$

(A) $\frac{A}{x}+\frac{B}{x}+\frac{C}{x+1}$

(B) $\frac{Ax+B}{x^2}+\frac{C}{x+1}$

(C) $\frac{A}{x^2}+\frac{B}{x+1}$

(D) $\frac{A}{x+1}+\frac{B}{x^2}+\frac{C}{X}$

1.8 Determine $\frac{1}{i^{47}}$

(A) $\frac{1}{i}$ (B) $-\frac{1}{i^3}$ (C) i (D) $-i$

1.9 Determine $arccos\left(\frac{\sqrt{3}}{2}\right)$

(A) $\frac{\pi}{3} \times 180°$

(B) $30°$

(C) $\frac{180^0}{\pi} \times 3$

(D) $60°$

1.10 Determine the value of a if $\int_0^1 ax\ dx = 1$.

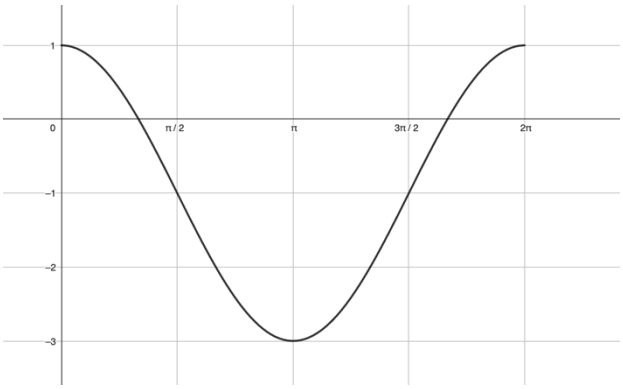
(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{2}{x^2}$

Question 2 [19 marks]

2.1 Decompose $\frac{2x^2-3x-1}{x^3-x}$ into partial fractions. (12)

2.2 Determine the quotient of $\frac{4x^3+4x^2-5x+1}{2x^2+3x-1}$. Use long division with detached coefficients. (4)

2.3 The sketch shows the function $f(x) = acosx + b$.



(a) Determine the value of a and b. (2)

(b) Write down the function formed when $f(x)$ is moved 2 units up. (1)

Question 3 [18 marks]

3.1 Determine the following:

(a) $\sqrt{-4}(\sqrt{2}-2i)$ (3)

(b) $\left(\frac{1}{2}-i\right)\left(1+\frac{i}{4}\right)$ (3)

(c) $\frac{2+i}{i}$ (3)

3.2 Given $a = 1 - i$ and $b = -2 + i$:

(a) Determine a^* (1)

(b) Determine $Im(b)$ the imaginary part of b. (1)

3.3 Given $(3a-2i)-(1+ai)$.

(a) Determine the value of a such that the expression is only real. (2)

(b) Find the answer of $(3-2i)+(1+i)$ graphically.

Use **DIAGRAM SHEET 1** on the answer sheet. (5)

Question 4 [21 marks]

4.1 The compound function $F(x) = \sqrt[3]{1-x} + (x-1)$ is given.

Determine $f(x)$ and $g(x)$. (3)

4.2 Given $f(x) = \sqrt{x+2}$ and $g(x) = x^3$.

Determine:

(a) $(g \circ f)(x)$ (2)

(b) $(f \circ g)(-1)$ (4)

4.3 Sketch the following piecewise-defined function on **DIAGRAM SHEET 2** provided. (5)

$$f(x) = \begin{cases} 2x+3 & if \ \ x < -1 \\ 3 & if \ -1 \leq x < 1 \\ x & if \ \ x > 1 \end{cases}$$

4.4 Given $f(x) = x^3 - 2x^2 - 5x + 6$.

(a) Show that $x = 3$ is a zero of $f(x)$. (2)

(b) Factorise $f(x)$ completely. (5)

Question 5 [21 marks]

5.1 Given the following system of equations. (8)

$$x + 2y = -3$$
$$3x + y = 1$$

Write the system of equations in matrix form and then use Cramer's method to determine x and y.

5.2 Given: (6)

$$A = \begin{bmatrix} 2 & -5 & 3 \\ 0 & 7 & -2 \\ -1 & 4 & 1 \end{bmatrix}$$

Determine $|A|$ the determinant of A.

5.3 Given the matrices $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 6 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 \\ a & 1 \end{bmatrix}$.

Determine the following, if possible. If not, give a reason why.

(a) B^T (1)

(b) $A \times B$ (3)

(c) $2B + C = \begin{bmatrix} -1 & 15 \\ 3 & 3 \end{bmatrix}$, determine the value of a. (3)

Question 6 [19 marks]

6.1 Differentiate the functions, with regards to the variable:

(a) $f(x) = \frac{3}{\sqrt[3]{x}} + \frac{\pi}{x^2} + 5x^7$ (3)

(b) $f(x) = \frac{9}{x-1}$ (2)

(c) $f(z) = \sqrt{z^2 - z}$ (3)

6.2 Determine the of the following.

(a) $\int \left(x \left(\frac{1}{x} + 2x \right) \right) dx$ (3)

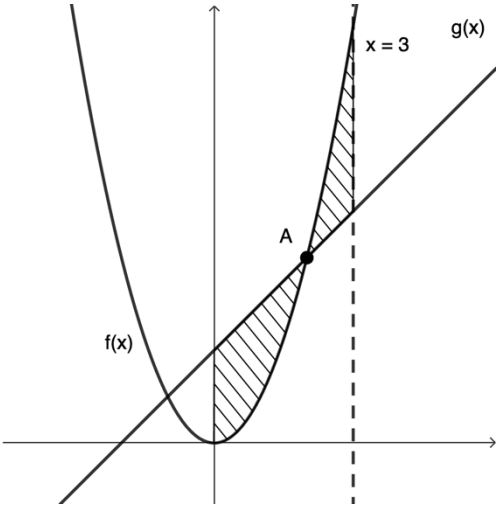
(b) $\int_0^1 (\sqrt{x} + \frac{1}{x^3})\, dx$ (5)

(c) $\int \sqrt{1 - 2x}\, dx$ (3)

Question 7 [14 marks]

7.1 Given the functions $f(x) = x^2$ and $g(x) = x + 2$ and the line $x = 3$.

The area between the functions $f(x)$ and $g(x)$ between $x = 3$ and A is $\frac{11}{6} unit^2$.



(a) Determine x-value of A the intercept of the functions $f(x)$ and $g(x)$. (3)

(b) Determine the area between the functions $f(x)$ and $g(x)$ between A and $x = 0$. (5)

7.2 The volume of the rotating body which is formed when $f(x) = \sqrt{3x^2 + a}$ is rotated around the $x - axis$, between $x = 0$ and $x = 3$ is 30π. Determine the value of a. (6)

Question 8 [18 marks]

8.1 Given the vectors $a = (2; 5)$ and $b = (-1; 2)$.

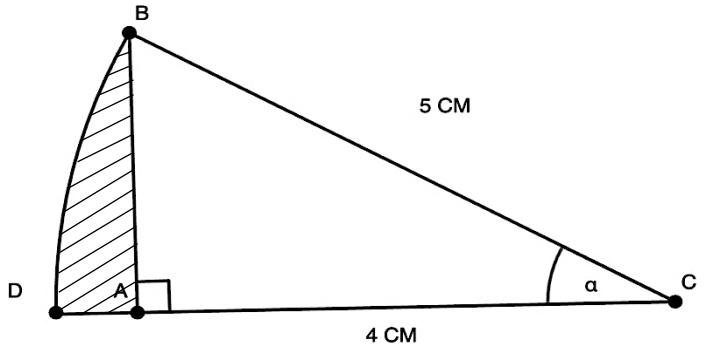
(a) Determine the magnitute a. (1)

(b) Determine the angle between the vector b and the x-axis. (2)

(c) Determine the unit vector of a. (1)

(c) Determine the angle between the vectors a and b. (4)

8.2 In the sketch BD is the arc of the sector BCD with the angle α at the center C. The radius BC of the sector BCD is $5\ cm$. The length of AC is $4\ cm$. Triangle ABC is a right angled triangle. The angle $A\hat{B}C$ is $\frac{\pi}{3}$ radians.



(a) If $\alpha = \frac{\pi}{6}$. Determine the circumference of the area ABD if $\alpha = \frac{\pi}{6}$. (5)

(c) Determine the area of ABD if $\alpha = \frac{\pi}{6}$. (5)

- END OF EXAM PAPER -

ALPHA MATHEMATICS FORMULE SHEET

Matrices and vectors:

Cramer's rule: $x_i = \frac{|A_i|}{|A|}$

$\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$

$\boldsymbol{a} \bullet \boldsymbol{b} = a_x b_x + a_y b_y$

CALCULUS:

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

$$\int_a^b x^n \, dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

TRIGONOMETRY:

In a sector: $s = r\theta$ en $A = \frac{1}{2} r^2 \theta$

TABLE OF DERIVATIVES:

$\boldsymbol{F(x)}$	$\boldsymbol{F'(x)}$
ax^n	nax^{n-1}
$f[g(x)]$	$f'[g(x)].g'(x)$

Alpha Mathematics Grade 10 – Final examination 2024

ANSWER SHEET

Name and Surname:

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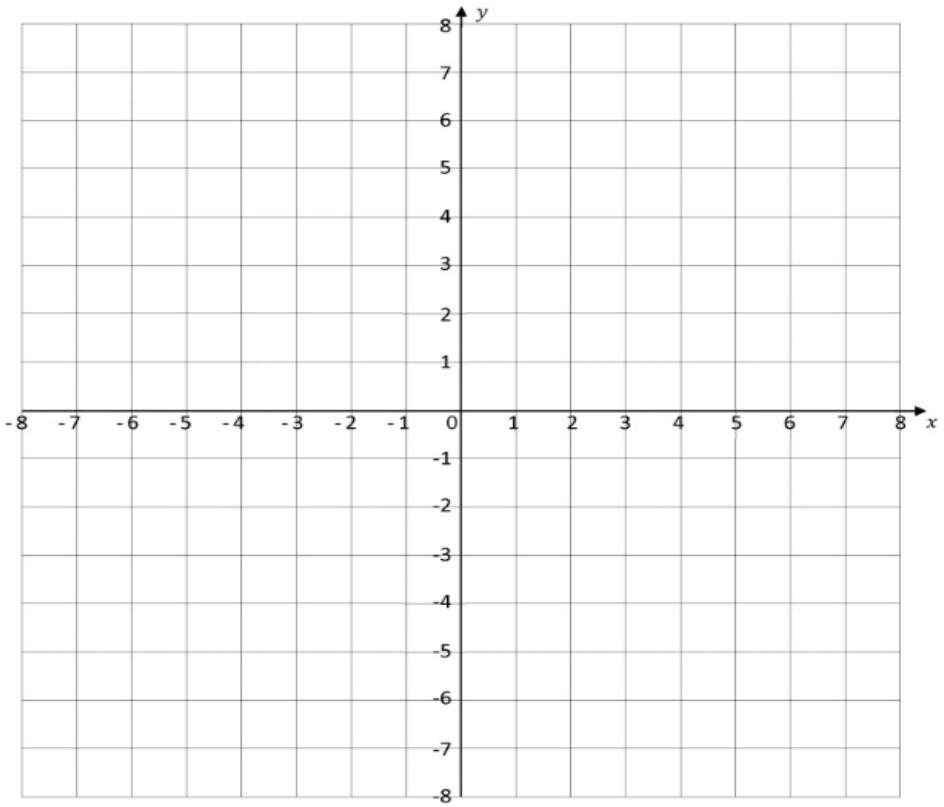
Question Total	1 [20]	2 [19]	3 [18]	4 [21]	5 [21]	6 [19]	7 [14]	8 [18]	TOTAL 150
Learner mark									

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1 Question 3.3



Question 4.3