α-Mathematics

Graad 11 Alpha Mathematics
Finale Eksamen / Final Exam 2024

MEMORANDUM

Totaal / *Total*: 150 punte / *marks*

Eksaminator / *Examiner*: Marco Botha
Moderator: Petrolene Marx

Hierdie memorandum bestaan uit 12 bladsye. /
This memorandum consists of 12 pages.

Vraag / *Question* 1 [20 punte / *marks*]

1.1	A	B	C	**D**
1.2	A	**B**	C	D
1.3	A	B	**C**	D
1.4	A	B	C	**D**
1.5	A	B	**C**	D
1.6	A	**B**	C	D
1.7	A	B	C	**D**
1.8	**A**	B	C	D
1.9	A	B	C	**D**
1.10	A	B	**C**	D

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	D	$\|x-2\|=2 \Longrightarrow x-2=\pm 2 \Longrightarrow x=0$ of $x=4$	**2**
1.2	B	$\|x\| \geq 0$, f is kontinu by $x=a$ indien $\lim_{x\to a} f(x)=f(a)$, $\|x\|$ is nie differensieërbaar by $x=0$ nie	**2**
1.3	C	$\|x+2\|=1$, so $x+2=1$ of/or $x+2=-1$	**2**
1.4	D	$\frac{1}{\sqrt{1-(3x)^2}}.(3)$	**2**
1.5	C	Horisontale lyn het helling 0.	**2**
1.6	B	Faktoriseer / Factorise	**2**
1.7	D	Teorie / Theory	**2**
1.8	A	$f(x)=\frac{1}{2}$ tussen/between $x=2$ en $x=3$, en / and $\int_2^3 \frac{1}{2}dx=\frac{1}{2}$	**2**
1.9	D	Faktoriseer/ Factorise $x^2(x^2-1)=x^2(x-1)(x+1)$	**2**
1.10	C	$r=4, \binom{10}{4}x^{10-4}(-2)^4=3360x^6$	**2**

Vraag / *Question* 2 **[18 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1.1	$\|4x-2\|-2 \Longrightarrow 4\left\|x-\frac{1}{2}\right\|-2$ $\therefore\ (\frac{1}{2};-2)$	✓Herskryf/ Rewrite ✓$(\frac{1}{2};-2)$ **[2 punte / *marks*]**
2.1.2	$f(x)=\|4x-2\|-2=0$ $\Longrightarrow \|2x-1\|=1$ **As/ if $4x-2\geq 0$ i.e. $x\geq\frac{1}{2}$:** $4x-2=2$ $\therefore x=1$ **As/ if $4x-2<0$ i.e. $x<\frac{1}{2}$:** $4x-2=-2$ $\therefore x=0$	✓$x\geq\frac{1}{2}$ of $4x-2\geq 0$ ✓$x=1$ ✓$x=0$ **[3 punte / *marks*]**
2.1.3	$\|f(x)\|=0 \Longrightarrow f(x)=0$ $\therefore x=0$ of / or $x=1$ soos tevore / as before	✓$f(x)=0$ ✓Same solutions as in 2.1.2 / Selfde oplossing as in 2.1.2 **[2 punte / *marks*]**

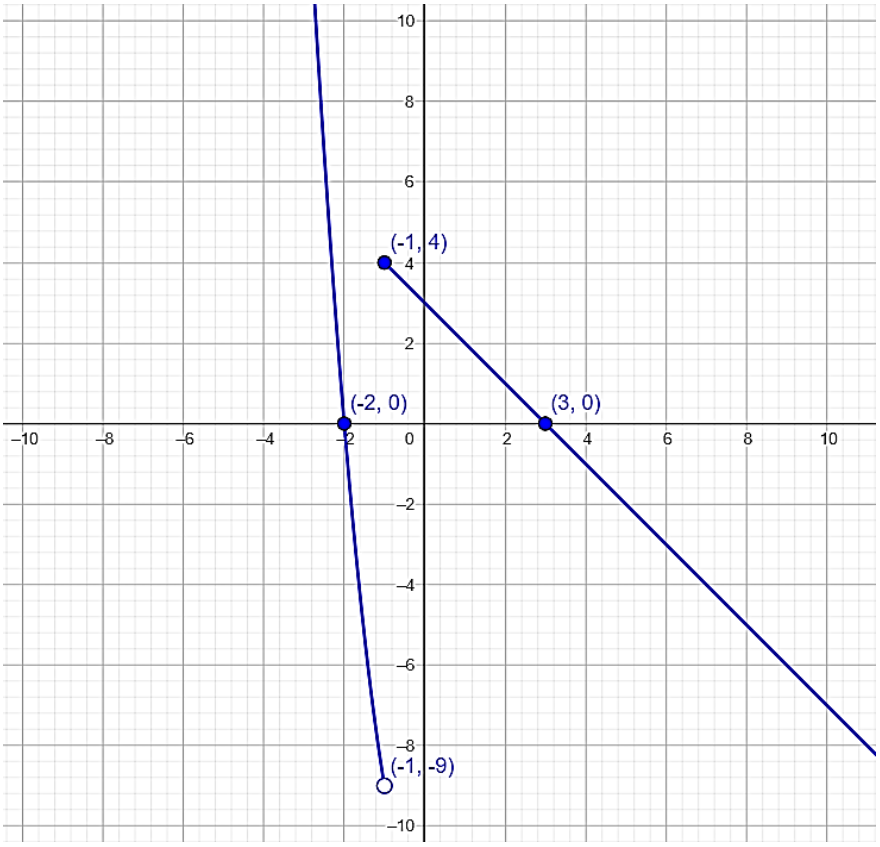
2.2.1	$\|x-4\| < 3$ **As $x \geq 4$**: $\|x-4\| = x-4 < 3 \Rightarrow x < 7$ $\therefore 4 \leq x < 7$ **As $x < 4$**: $\|x-4\| = -x+4 < 3 \Rightarrow -x < -1 \Rightarrow x > 1$ $\therefore 1 < x < 4$ thus $1 < x < 7$ **OF / OR** $-3 < x-4 < 3$ $1 < x < 7$	✓ $x \geq 4$ of/or $x < 4$ ✓ $4 \leq x < 7$ ✓ $1 < x < 4$ **OF / OR** ✓ $-3 < x-4 < 3$ ✓✓ $1 < x < 7$ **[3 punte / *marks*]**
2.2.2	$\|x-5\| = -1$ ∴ geen oplos. / no sol	✓ $\|x-5\| = -1$ ✓ Geen oplossing **[2 punte / *marks*]**
2.2.3	$\|4x-2\| = x^2+1$ As/ If $x \geq \frac{1}{2}$: $x^2+1 = 4x-2$ $x^2-4x+3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3$ of/or $x = 1$ As/ If $x < \frac{1}{2}$: $x^2+1 = -4x+2$ $x^2+4x-1 = 0$ $\therefore x = -2+\sqrt{5}$ of/or $x = -2-\sqrt{5}$	✓ $x \leq \frac{1}{2}$ of/or $x < \frac{1}{2}$ ✓ $x^2+1 = 4x-2$ ✓ $x = 3$ ✓ $x = 1$ ✓ $x^2+4x-1 = 0$ ✓ $x = -2 \pm \sqrt{5}$ **[6 punte / *marks*]**

Vraag / *Question* 3 **[17 punte / *marks*]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
3.1	$x = 2\sqrt{5}$ 'n nulpunt / a zero, then/ dan $x = -2\sqrt{5}$ ook 'n nulpunt / also a zero Faktore / Factors $(x - 2\sqrt{5})(x + 2\sqrt{5}) = x^2 - 20$ By inspection or long division / deur inspeksie of lang deel $f(x) = x^3 + 5x^2 - 20x - 100$ $= (x^2 - 20)(x + 5)$ $f(x) = (x - 2\sqrt{5})(x + 2\sqrt{5})(x + 5)$	✓ $x = -2\sqrt{5}$ ✓$x^2 - 20$ ✓✓$(x + 5)$ ✓ Finale antwoord/ Final Answer **[5 punte / *marks*]**
3.2	$g(x) = f(x - 1)$ $= (\boldsymbol{x - 1} - 2\sqrt{5})(\boldsymbol{x - 1} + 2\sqrt{5})(\boldsymbol{x - 1} + 5)$	✓✓ Use answer in 3.1 / gebruik antwoord in 3.1 **[2 punte / *marks*]**
3.3	**Vir $n = 1$:** LHS=$(1)^2 - (1) = 0$ RHS=$\frac{(1)^3-(1)}{3} = 0$ ∴LHS=RHS vir $n = 1$ **Aanname:** Aanvaar dat $$\sum_{r=1}^{k}(r^2 - r) = \frac{k^3 - k}{3}$$ **Induksie stap:** Om te bewys: $$\sum_{r=1}^{k+1}(r^2 - r) = \frac{(k+1)^3 - (k+1)}{3}$$ LHS=$\frac{k^3-k}{3} + [(k+1)^2 - (k+1)]$ $= \frac{k^3 - k + 3(k+1)^2 - 3(k+1)}{3} = \frac{k^3 + 2k + 3k^2}{3}$ $= \frac{k(k^2 + 3k + 2)}{3} = \frac{k(k+2)(k+1)}{3}$ RHS=$\frac{(k+1)^3-(k+1)}{3} = \frac{(k+1)^3-(k+1)}{3} = \frac{(k+1)((k+1)^2-1)}{3} = \frac{k(k+2)(k+1)}{3}$ ∴LHS=RHS vir $n = k + 1$ **Slot:** Deur die beginsel van wiskundige induskie is die stelling waar vir alle $n \in \mathbb{N}$.	✓✓$n = 1$ ✓✓Aanname ✓ $\frac{k^3-k}{3}$ ✓$(k+1)^2 - (k+1)$ ✓✓Bewerkings ✓ LHS=RHS ✓ Conclusion / Slot **[10 punte / *marks*]**

Vraag / *Question* 4 [21 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1	$\frac{x-2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ $x-2 = A(x^2+1) + (Bx+C)x$ $x-2 = (A+B)x^2 + Cx + A$ $\underline{\boldsymbol{x=0}}$: $-2 = \mathrm{A}$ $\underline{\boldsymbol{x^2}}$: $0 = A + B \Longrightarrow B = -(-2) = 2$ $\underline{\boldsymbol{x}}$: $C = 1$ $$\frac{x-2}{x^3+x} = \frac{-2}{x} + \frac{2x+1}{x^2+1}$$	✓ Ontbind/ Decompose ✓ Herskryf/ Rewrite ✓ $\boldsymbol{A = -2}$ ✓✓ $\boldsymbol{B = 2}$ ✓✓ $\boldsymbol{C = 1}$ ✓ Final answer/ Finale antwoord **[8 punte / *marks*]**
4.2.1	Left and righthand limits at $x = -1$: $\lim_{x\to -1^-} h(x) = \lim_{x\to -1^-} 3x^2 - 12 = 3(-1)^2 - 12 = -9$ $\lim_{x\to -1^+} h(x) = \lim_{x\to -1^-} -x - k = 1 - k$ Set limits equal / stel limiete gelyk: $\therefore 1 - k = -9$ $\therefore\ k = 10$	✓✓ linker limiet/ lefthand limit ✓✓ regter lmiet/ righthand limit ✓ **10** **[5 punte / *marks*]**
4.2.2	Nee / No $\lim_{x\to -1^-} h'(x) = \lim_{x\to -1^-} 6x = -6$ $\lim_{x\to -1^+} h'(x) = \lim_{x\to -1^-} -1 = -1$ Limiete van hellings nie gelyk / Limits of derivatives not equal	✓ Nee / No ✓✓ Limiet / Limits **[3 punte / *marks*]**

4.2.3	$h(x) = \begin{cases} 3x^2 - 12 & as\ x < -1 \\ -x + \mathbf{3} & as\ x \geq -1 \end{cases}$ 	✓ $(-\mathbf{2};\mathbf{0})$ ✓ $(-\mathbf{1};\mathbf{4})$ ✓ $(\mathbf{3};\mathbf{0})$ ✓ $(-\mathbf{1};-\mathbf{9})$ ✓ Shape / Vorm **[5 punte / *marks*]**

Vraag / *Question* 5 **[21 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1.1	$\mathbf{u}\bullet\mathbf{w} = \lvert\boldsymbol{u}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$ $\mathbf{u}\bullet\mathbf{w} = (1)(4)+(-2)(0)+(0)(-3) = 4$ $\lvert\boldsymbol{u}\rvert = \sqrt{1^2+(-2)^2+0^2} = \sqrt{5}$ $\lvert\boldsymbol{w}\rvert = \sqrt{4^2+0^2+(-3)^2} = 5$ $cos\theta = \frac{4}{5\sqrt{5}}$ $\theta = 1.20$ rad	✓✓ punt product / dot product ✓ Magnitudes / Grootte ✓Bewerking / Compute ✓1.205 **[5 punte / *marks*]**
5.1.2	$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & -2 & 0 \\ 0 & 2 & 1 \end{vmatrix}$ $= \big((-2)(1)-(0)(2)\big)\boldsymbol{i} - \big((1)(1)-(0)(0)\big)\boldsymbol{j}$ $+\big((1)(2)-(0)(-2)\big)\boldsymbol{k}$ $= -2\boldsymbol{i} - \mathbf{1}\boldsymbol{j} + \mathbf{2}\boldsymbol{k}$ Oppervlakte / Area: $\lvert -2\boldsymbol{i} - 1\boldsymbol{j} + 2\boldsymbol{k}\rvert = \sqrt{(-2)^2+(-1)^2+2^2} = \sqrt{9} = 3 eenhende^2$	✓ Determinant ✓✓ Compute ✓✓ Answer ✓✓ Compute area **[7 punte / *marks*]**
5.2.1	$(2\mathrm{r})\theta = 2(\mathrm{r}\theta) = 2\mathrm{s}$	✓✓ $\mathbf{2}\boldsymbol{s}$ **[2 punte / *marks*]**
5.2.2	$\frac{1}{2}(2r)^2(\frac{\theta}{2}) = 2(\frac{1}{2}r^2\theta) = 2A$	✓✓ $\mathbf{2}\boldsymbol{A}$ **[2 punte / *marks*]**
5.2	$(2-3x^2)^{12}$ met/ with $r = 2$: $\binom{12}{2}(2)^{12-2}(-3x^2)^2$ $= \binom{12}{2}2^{10}(-3x)^2$ Koëffisiënt / Coefficient: $\binom{12}{2}(2)^{10}(-3)^2$ =608256 ∴ Koëffisiënt / Coefficient is 608256	✓✓$\boldsymbol{r = 2}$ ✓Working / Bewerking ✓✓Answer /Antwoord **[5 punte / *marks*]**

Vraag / *Question* 6 [21 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1.1	$\frac{d}{dx}[sin^3(4x)] = 3sin^2(4x).(cos(4x).(4))$	✓ ✓ $3sin^2(4x)$ ✓ $cos(4x)$ ✓ 4 **[4 punte / *marks*]**
6.1.2	$D_x[cos(4x).(4x^3+\sqrt{x})]$ $= (-\sin(4x).(4)).(4x^3+\sqrt{x}) + cos(4x).(12x^2+\frac{1}{2}x^{\frac{-1}{2}})$	✓✓$-\sin(4x).(4)$ ✓$(4x^3+\sqrt{x})$ ✓ $cos(4x).$ ✓✓ $(12x^2+\frac{1}{2}x^{\frac{-1}{2}})$ **[6 punte / *marks*]**
6.1.3	$\frac{d}{dx}\left[\frac{3x^2}{(5x^2-3)^2}\right]$ $= \frac{(6x)(5x^2-3)^2 - [2(5x^2-3)(10x)](3x^2)}{(5x^2-3)^4}$	✓✓ $(6x)(5x^2-3)^2$ ✓✓ $2(5x^2-3)(10x)$ ✓ $(3x^2)$ ✓$(5x^2-3)^4$ **[6 punte / *marks*]**
6.2	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $f(x) = 2x^3 - 16x - 46$ $f'(x) = 6x^2 - 16$ $x_1 = 2$ $x_2 = 9.75$ $x \approx 3.75775$	✓ f' ✓✓ formula / substitute ✓ berekening ✓ 3.75775 **[5 punte / *marks*]**

Vraag / *Question* 7 **[17 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1.1	Vierkantvolttoing / Complete the square: $-x^2+2x=-(x^2-2x)=-(x^2-2x+(1)^2-(1)^2)$ $=-((x-1)^2-1)=1-(x-1)^2$ $\int\frac{1}{\sqrt{-x^2+2x}}dx=\int\frac{1}{\sqrt{1-(x-1)^2}}dx=bgsin(x-1)+k$	✓✓ Vierkantsvoltooing / Complete the square ✓ Herskryf integraal / Rewrite integral ✓✓ Answer / Antwoord **[5 punte / *marks*]**
7.1.2	$\int sec^2(2x)+sec(-x).tan(-x)dx$ $=\frac{tan(2x)}{2}-sec(-x)+k$	$\frac{tan(2x)}{2}$✓✓$-sec(-x)$✓$+k$✓ **[4 punte / *marks*]**
7.1.3	$\int_0^{\frac{k}{3}}\frac{1}{(3x)^2+1}\,dx=\frac{\pi}{12}$ $\left[\frac{arctan(3x)}{3}\right]_0^{\frac{k}{3}}=\frac{\pi}{12}$ $[arctan(3x)]_0^{\frac{k}{3}}=\frac{\pi}{4}$ $arctan(k)-0=\frac{\pi}{4}$ $\therefore k=tan(\frac{\pi}{4})=1$	✓✓$\frac{arctan(3x)}{3}$ ✓ bewerking / working ✓ $arctan(k)=\frac{\pi}{4}$ ✓ $k=1$ **[5 punte / *marks*]**
7.2	$f(x)=\int f'(x)dx=\int(-4x^3+5)dx=-\frac{4x^4}{4}+5x+k$ $\therefore f(x)=-x^4+5x+k$ $f(0)=-0^4+5(0)+k=2\implies k=2$ $\therefore f(x)=-x^4+5x+2$	✓Integreer / integrate ✓Bepaal konstant / determine constant ✓ $f(x)$ **[3 punte / *marks*]**

Vraag / *Question* 8 **[15 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1.1	$V = \pi \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx = \pi \int_{-1}^{1} 1 - x^2 \, dx$ $\pi \left[x - \frac{x^3}{3}\right]_{-1}^{1}$ $= \frac{4}{3}\pi$	✓Formule / In vervang ✓ Integral / integraal ✓✓Integrate/ Inetgreer ✓Calculation / Bewerking ✓ Antwoord / Answer **[6 punte / *marks*]**
8.1.2	'n Sfeer / a Sphere Ja, 'n sfeer se volume is gegee deur $V = \frac{4}{3}\pi r^3$ en $r = 1$ en gee dus dieselfde antwoord. / Yes, a sphere has volume $V = \frac{4}{3}\pi r^3$ and $r = 1$ which gives the same answer.	sfeer /sphere✓ rede/ reason✓ **[2 punte / *marks*]**
8.1.3	It is a half circle / Dit is 'n halwe sirkel, $A = \frac{1}{2}(\pi(1)^2) = \frac{\pi}{2}$	✓✓ antwoord **[2 punte / *marks*]**
8.2	$\int_{-1}^{0}[(-x+1) - \sqrt{1-x^2}\,]dx$ ✓✓ $+ \int_{0}^{1}[\sqrt{1-x^2} - (-x+1)\,]dx$ ✓✓ Heeltemal korrek / entirely correct ✓	**[5 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -