α-WISKUNDE

Graad 11 Alpha Mathematics
Final Exam 2024

Grade 11 **Time: 2 ½ hours**

Examinator: M. Botha **Total: 150 marks**

Moderator: P. Marx

INSTRUCTION

Read the following instructions carefully before attempting the questions:

1. This paper consists of 7 pages, an answer sheet and a formula page.
2. Answer ALL 8 questions.
3. Number answers according to the numbering scheme of the paper.
4. Non-programmable calculators may be used, unless otherwise stated.
5. Round all answers to two decimal places.
6. Clearly indicate all necessary calculations, diagrams, graphs etc. that you used to determine your answers.
7. Full marks will not necessarily be awarded to answers only.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers should be given in radians if necessary.
10. Write neatly and legibly.
11. Enjoy!

Question 1 – Multiple Choice Questions [20 marks]

This question must be answered **on the answer sheet**.

Each question has only **ONE** correct answer and counts two (2) marks.

Mark the correct answer with an **X** on the Answer sheet.

1.1 If $x = 2 + 3i$ is a root of f, then

(A) $f(-2 + 3i) = 0$

(B) $x - 2$ is a factor

(C) $f(0) = 2 + 3i$

(D) $x - 2 - 3i$ is a factor

1.2 Which of the following statements, are always true?

1	$\|x\| > 0$
2	f is continuous at $x = a$ if $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$
3	$\|x\|$ is everywhere differentiable
4	$\binom{n}{0} = 1$ for any $n \in \mathbb{N}$

(A) Only 1 and 4

(B) Only 4

(C) Only 1

(D) Only 2 and 3

1.3 Solve for x in $|x + 2| - 1 = 0$.

(A) $x = -1$

(B) $x \in \mathbb{R}$

(C) $x = -1$ of $x = -3$

(D) No solution for x

1.4 The derivative of $arcsin(3x)$ is

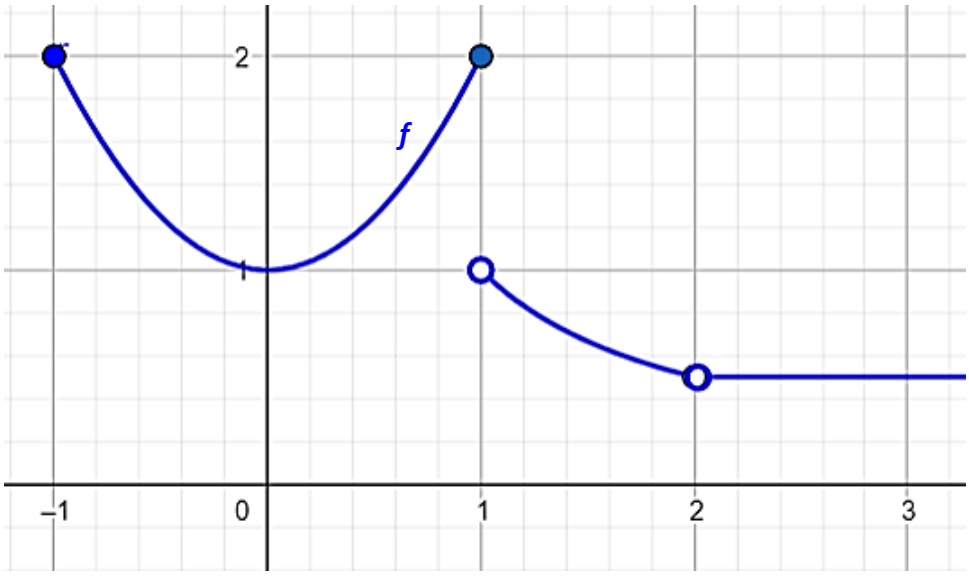
(A) $\frac{3}{\sqrt{1-3x^2}}$

(B) $\frac{9}{\sqrt{1-9x^2}}$

(C) $\frac{9}{\sqrt{1-x^2}}$

(D) $\frac{3}{\sqrt{1-9x^2}}$

Use the graph of $f(x)$ below, fo question 1.5 to 1.7:



1.5 $f'(3)$

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) Does not exist

1.6 $f(1)$

(A) 1

(B) 2

(C) Does not exist

(D) 0

1.7 Which of the following is true?

(A) The limit at $x = 2$ does not exist

(B) f is differentiable at $x = 2$

(C) There is a removable discontinuity at $x = 1$

(D) There is a jump discontinuity at $x = 1$

1.8 $\int_2^3 f(x)dx$ with $f(x) = \frac{1}{2}$

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{4}{3}$

(D) 2

1.9 If $\frac{2}{x^2(x^2-1)}$ is decomposed into partial fractions we find:

(A) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x-1)}+\frac{D}{(x^2-1)}$

(B) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x-1)}+\frac{D}{(x-1)^2}$

(C) $\frac{A}{x^2}+\frac{B}{(x-1)}+\frac{C}{(x+1)}$

(D) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x-1)}+\frac{D}{(x+1)}$

1.10 The fifth term of $(x-2)^{10}$ is

(A) $13440x^4$

(B) $3360x^4$

(C) $3600x^6$

(D) $13440x^6$

Question 2 - Absolute Value [18 marks]

2.1 Given $f(x) = |4x - 2| - 2$

2.1.1 Write down the coordinate of the salient point. (2)

2.1.2 Solve for x if $f(x) = 0$. (3)

2.1.3 Subsequently (or otherwise) solve for x in $|f(x)| = 0$ (2)

2.2 Solve for x, $x \in \mathbb{R}$, in:

2.2.1 $|x - 4| < 3$ (3)

2.2.2 $|x - 5| + 1 = 0$ (2)

2.2.2 $|4x - 2| = x^2 + 1$. Leave your answer in rootform. (6)

Question 3 – Polinomials & Induction [17 marks]

3.1 If $x = 2\sqrt{5}$ is a root of $f(x) = x^3 + 5x^2 - 20x - 100$, factorise $f(x)$ fully in $\mathbb{R}[x]$. (5)

3.2 Subsequently (or otherwise) factorise $g(x) = f(x - 1)$ in $\mathbb{R}[x]$. (2)

3.3 Use mathematical induction to prove that

$$\sum_{r=1}^{n} (r^2 - r) = \frac{n^3 - n}{3}$$

(10)

Question 4 – Partial Fractions & Functions [21 marks]

4.1 Decompose $\frac{x-2}{x^3+x}$ into partial fractions. (8)

4.2 If $h(x) = \begin{cases} 3x^2 - 12 & if\ x < -1 \\ -x - k & if\ x \geq -1 \end{cases}$

4.2.1 Determine the value of k such that h is continuous at $x = -1$. (5)

4.2.2 Is h differentiable at $x = -1$? Motivate fully. (3)

4.2.3 If $k = -3$ (this is not a follow up of question 4.2.1), sketch the graph of $h(x)$ on the **DIAGRAMSHEET.** (5)

Question 5 – Vectors, Trigonometry & Binomial Theorem [21 marks]

5.1 If $\boldsymbol{u} = \boldsymbol{i} - 2\boldsymbol{j}$, $\boldsymbol{v} = 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = 4\boldsymbol{i} - 3\boldsymbol{k}$

5.1.1. Determine the angle between $\boldsymbol{u}$ and $\boldsymbol{w}$ (5)

5.1.2. Determine the area of the parallelogram formed by $\boldsymbol{u}$ and $\boldsymbol{v}$. (7)

5.2 Given a sector with radius r, angle θ, area A and arclength s.

Give all answers in terms of A and s.

5.2.1 If the radius is doubbled, what is the new arclength? (2)

5.2.2 If the angle is halved, and the radius is doubled, what is the new area of the sector? (2)

5.3 Determine the coefficient of the x^4 term in $(2 - 3x^2)^{12}$ (5)

Question 6 – Differentiation [21 marks]

6.1 Determine the following:

6.1.1 $\frac{d}{dx}[sin^3(4x)]$ (4)

6.1.2 $D_x[cos(4x).(4x^3 + \sqrt{x})]$ (6)

6.1.3 $\frac{d}{dx}\left[\frac{3x^2}{(5x^2-3)^2}\right]$ (6)

6.2 Consider, $2x^3 - 16x - 46 = 0$. Use $x_1 = 2$ and determine the root of the equation accurate to 5 decimal places by using Newton's Method. (5)

Question 7 – Integration [17 marks]

7.1 Determine the following integrals

7.1.1 $\int \frac{1}{\sqrt{-x^2+2x}}dx$ (5)

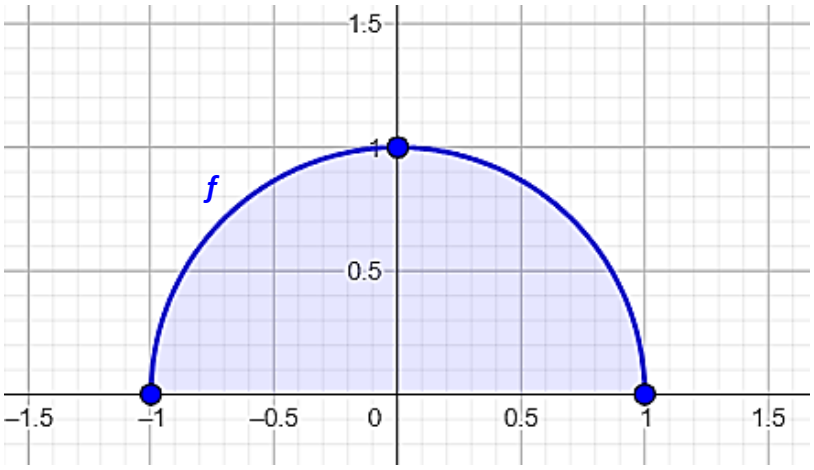
7.1.2 $\int sec^2(2x) + sec(-x).tan(-x)dx$ (4)

7.1.3 Determine k if $\int_0^{\frac{k}{3}} \frac{1}{(3x)^2+1}\ dx = \frac{\pi}{12}$ (5)

7.2 Determine $f(x)$ if $f(x)$ passes through the point $(0; 2)$ and $f'(x) = -4x^3 + 5$. (3)

Question 8 – Area & Volume [15 marks]

8.1 $f(x) = \sqrt{1 - x^2}$ is plotted in the diagram below.



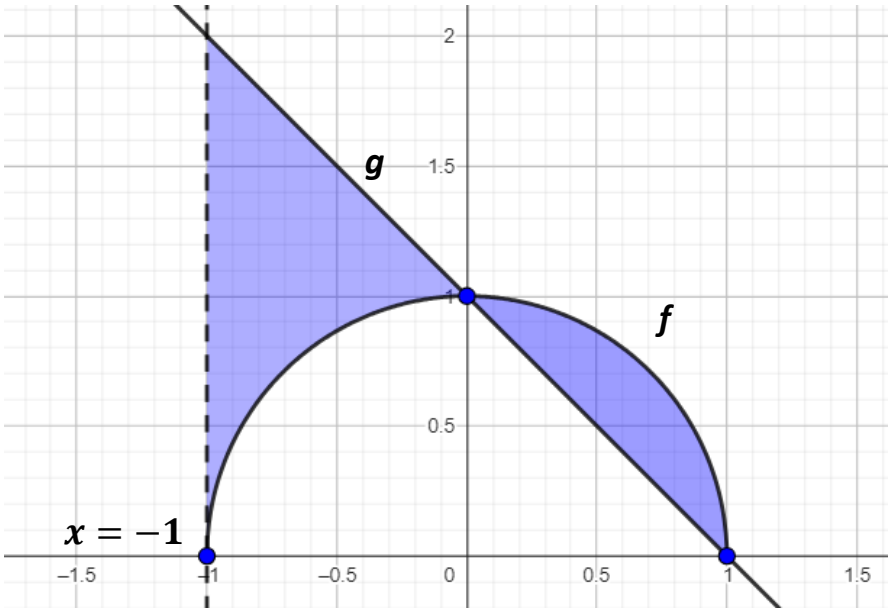
8.1.1 Determine the volume of the rotating body that is formed when $f(x)$ is rotated about the x-axis between $x = -1$ and $x = 1$. (6)

8.1.2 Which shape is the rotating body? Does your answer in question 8.1.1 agree with what you already know about this shape? (2)

8.1.3 Determine the area of the shaded region above. (2)

Hint: You don't have to integrate.

8.2 If $g(x) = -x + 1$ and $f(x) = \sqrt{1 - x^2}$ with $x = -1$ in the diagram below.



Determine an expression for computing the area between the two graphs. You don't have to simplify. **(Don't compute the integrals!)** (5)

- END OF THE PAPER -

ALPHA MATHEMATICS FORMULA PAGE

ALGEBRA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad |x| = \begin{cases} x & \text{as} \quad x \geq 0 \\ -x & \text{as} \quad x < 0 \end{cases}$$

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r \qquad (1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \text{ ; mits } |x| < 1$$

VECTORS:

$$\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta \qquad \boldsymbol{a} \bullet \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$$

CALCULUS:

$$\int_a^b x^n \, dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b \qquad V = \pi \int_a^b [f(x)]^2 \, dx \qquad a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

TRIGONOMETRY:

In a sector: $s = r\theta$ en $A = \frac{1}{2} r^2 \theta$

Identities: $\sin^2 x + \cos^2 x = 1$ $\quad \tan^2 x + 1 = \sec^2 x$ $\quad \cot^2 x + 1 = \text{cosec}^2 x$

$$\text{cosec}\, x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

area ΔABC $= \frac{1}{2} ab.\sin C$

TABLE WITH DERIVATIVES:

$F(x)$	$F'(x)$
ax^n	nax^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \,.\tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \,.\cot x$

$F(x)$	$F'(x)$
bgsin x arcsin x	$\frac{1}{\sqrt{1-x^2}}$
bgcos x arccos x	$\frac{-1}{\sqrt{1-x^2}}$
bgtan x arctan x	$\frac{1}{x^2+1}$
$f(x).g(x)$	$f'(x).g(x) + f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x).g(x) - f(x).g'(x)}{[g(x)]^2}$
$f[g(x)]$	$f'[g(x)].g'(x)$

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ANSWER SHEET

Name and Surname: _____

Question Total	1 [20]	2 [18]	3 [17]	4 [21]	5 [21]	6 [21]	7 [17]	8 [15]
Student Mark								

TOTAL 150

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAMSHEET [Question 4.2.3]