α-Mathematics

Graad 11 Alpha Wiskunde
Grade 11 Alpha Mathematics
June 2024

MEMORANDUM

Totaal / *Total*: 150 punte / *marks*

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Hierdie memorandum bestaan uit 14 bladsye. /
This memorandum consists of 14 pages.

Vraag / *Question* 1 [20 punte / *marks*]

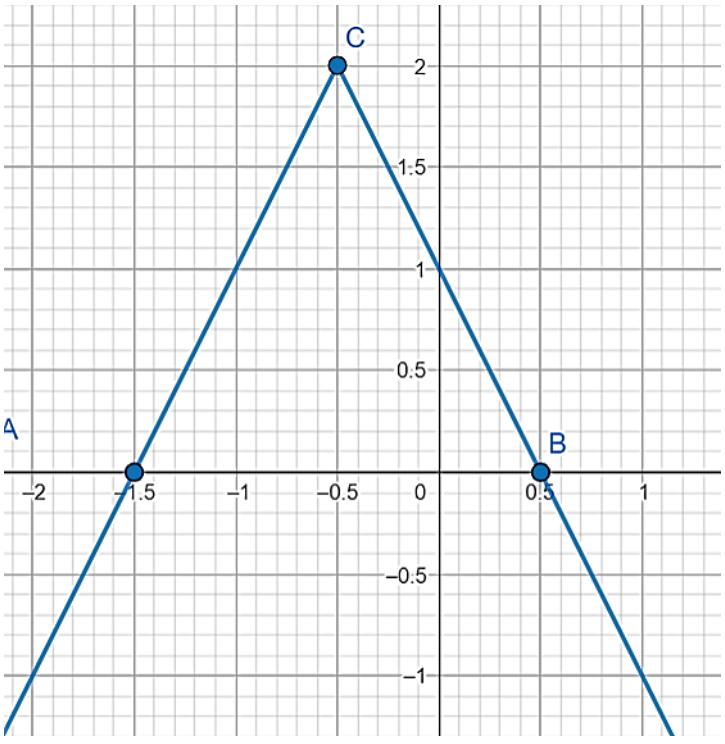
1.1	A	B	**C**	D
1.2	A	B	C	**D**
1.3	A	B	C	**D**
1.4	A	**B**	C	D
1.5	A	B	**C**	D
1.6	A	B	**C**	D
1.7	A	B	C	**D**
1.8	A	**B**	C	D
1.9	**A**	B	C	D
1.10	**A**	B	C	D

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	C	$\|x-2\|=2 \Longrightarrow x-2=\pm 2 \Longrightarrow x=0$ of $x=4$	**2**
1.2	D	$f(2)=0$	**2**
1.3	D	$\|x\| \geq 0, \forall x \in \mathbb{R}$	**2**
1.4	B	Knakpunt/salient point is (-4,12). $K(x)=0$ no solution/geen oplos.	**2**
1.5	C	Radius verdubbel/ doubled: $A=\frac{1}{2}r^2\theta \rightarrow \frac{1}{2}(2r)^2\theta=4A$ Angle halfved: $A=\frac{1}{2}r^2\theta \rightarrow \frac{1}{2}r^2\left(\frac{\theta}{2}\right)=\frac{A}{2}$ Result: $A \rightarrow 4 \times \frac{1}{2}A=2A$	**2**
1.6	C	Faktoriseer / Factorise	**2**
1.7	D	Komplekse faktore in pare/ Complex factors in pairs Faktorstelling / Factor Theorem	**2**
1.8	B	Ruil x en y. Swap x and y	**2**
1.9	A	$n+1$ terms	**2**
1.10	A	$P(x)$ ongedef/ undefined for $x=2$	**2**

Vraag / *Question* 2 **[37 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1.1	$\|x-2\| \geq 0$ for all $x \in \mathbb{R}$ ∴ geen oplossing / no solution	✓Rede/ Reason ✓No solution **OF/OR** ✓✓ No soluton **Max/ Maksimum: 1** **(if values for x are given)** **[2 punte / *marks*]**
2.1.2	**As $x+3 \geq 0$:** $x+3=x$ ∴ geen oplos. / no sol **As $x+3 \leq 0 \Rightarrow x \leq -3$:** $x+3=-x$ $x=\frac{3}{2} \nleq -3$ (n.v.t / NA) ∴ geen oplossing / no solution	✓$x+3 \geq 0$ of/or $x \geq -3$ ✓$x+3=x$ / no sol. ✓$x+3=-x$ ✓n.v.t / NA ✓Geen oplossing **[5 punte / *marks*]**
2.1.3	$\|x^2-4\|=3x$ *Kwadratiese ongelykhede / Quadraitic inequalities* As/ If $x^2-4 \geq 0$: ⟹As/ If $x \leq -2$ of $x \geq 2$ dan: $x^2-4=3x$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ ∴ $x=4$ of/or $x=-1$ (n.v.t / NA) As/ If $x^2-4<0$ ⟹ As/ If $-2 \leq x \leq 2$ dan: $x^2-4=-3x$ $x^2+3x-4=0$ $(x+4)(x-1)=0$ ∴ $x=-4$ (n.v.t / NA) of/or $x=1$ ∴ $x=1$ of $x=4$	✓$x \leq -2$ of/or $x \geq 2$ ✓$x^2-4=3x$ ✓$x=4$ ✓$x=-1$ n.v.t / NA ✓$-2 \leq x \leq 2$ ✓$x^2+3x-4=0$ ✓$x=1$ ✓$x=-4$ n.v.t / NA **[8 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.2.1	$a = 2$ (knakpunt / salient point) OF / OR $f(b) = 2 \Rightarrow a = 2$ $\therefore f(x) = -\lvert x - b\rvert + 2$ $f(0) = -\lvert 0 - b\rvert + 2 = 1 \Rightarrow \lvert b\rvert = 1 \Rightarrow b = \pm 1$ $b \geq 0 \Rightarrow b = 1$	✓$a = 2$ ✓ $f(0) = 1$ ✓$b = \pm 1$ ✓ $b = 1$ **[4 punte / *marks*]**
2.2.2	$f(x) = -\lvert x - 1\rvert + 2 = 0$ $\Rightarrow \lvert x - 1\rvert = 2$ **As $x - 1 \geq 0$:** $x - 1 = 2$ $\therefore x = 3$ **As $x - 1 \leq 0$:** $x - 1 = -2$ $x = -1$ $A(-1; 0)$ $B(3; 0)$	 ✓$f(x) = 0$ /simplify ✓$x - 1 = 2$ ✓$x = -1$ ✓A ✓B **[5 punte / *marks*]**
2.2.3	**D:** $g(x) = -f(x) - 1$ $\lvert x - c\rvert + d = -(-\lvert x - 1\rvert + 2) - 1$ $\lvert x - c\rvert + d = \lvert x - 1\rvert - 3$ $\Rightarrow c = 1$ $d = -3$ $\therefore D(1; -3)$ **F:** $g(0) = -f(0) - 1$ $= -1 - 1 = -2$ $\therefore F(0; -2)$	✓In vervang/ substitute ✓$D(1, -3)$ ✓ y-asnit / itercept or $g(0)$ ✓$F(0; -2)$ **[4 punte / *marks*]**
2.2.4	Algebraïes / algebraic: $-\lvert x - 1\rvert + 2 > 1 \Rightarrow \lvert x - 1\rvert < 1$ **OF/OR** Inspeksie / By inspection: $0 < x < 2$	✓ 0 ✓2 **[2 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.3.1	$y = -\|2x + 1\| + 2$ $y = -2\left\|x + \left(\frac{1}{2}\right)\right\| + 2$ ∴ Knakpunt/ Salient Point: $(-\frac{1}{2}; 2)$	✓✓Knakpunt /salient point $\left(-\frac{1}{2}; 2\right)$ **[2 punte / *marks*]**
2.3.2	<u>y-afsnit / intercept:</u> $y = -\|2x + 1\| + 2$ $= -\|1\| + 2$ $= 1 \quad \therefore (0; 1)$ <u>x-afsnit / intercept:</u> $0 = -\|2x + 1\| + 2$ $\|2x + 1\| = 2$ $\Rightarrow x = \frac{1}{2}$ of $x = -\frac{3}{2}$ $(\frac{1}{2}; 0)$ en $(-\frac{3}{2}; 0)$  	Bewerkings word nie gemerk nie / working not marked. ✓Knakpunt ✓y-afsnit / intercept ✓✓ x-afsnitte / intercepts ✓Vorm / Form **Maximum / Maksimum: 2** **(if intercepts aren't indicated / as afsnitte nie benoem is nie)** **[5 punte / *marks*]**

Vraag / *Question* 3 **[18 punte / *marks*]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
3.1	$\frac{2x^3-2x^2+x-1}{x^2(x^2+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$ $2x^3-2x^2+x-1=Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2$ **Substitute $x=0$:** $\underline{\boldsymbol{x=0}}$**:** $B=-1$ **Look at coefficients / substitute in any x:** $\underline{\boldsymbol{x^2}}$**:** $-2=B+D\Rightarrow D=-2-B\Rightarrow D=-1$ $\underline{\boldsymbol{x}}$**:** $A=1$ $\underline{\boldsymbol{x^3}}$**:** $A+C=2\Rightarrow C=2-A\Rightarrow C=1$ $\therefore \frac{2x^3-2x^2+x-1}{x^2(x^2+1)}=\frac{1}{x}-\frac{1}{x^2}+\frac{x-1}{x^2+1}$	✓✓Ontbind / Decompose ✓KGC / LCD ✓$B=-1$ ✓ $D=-1$ ✓$A=1$ ✓Bewerking / Workings ✓$C=1$ ✓✓ Antwoord **[10 punte / *marks*]**
3.2	3.2.1 B 3.2.2 E 3.2.3 B 3.2.4 F	✓✓ Per antwoord/ per answer **[8 punte / *marks*]**

Vraag / *Question* 4 **[15 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1	**4.1.1** $S(x) = x^2 + 2x + 10$ $x = -1 + 3i$ 'n nulpunt/ a root $\Rightarrow x = -1 - 3i$ 'n wortel/ a root (wortestelling / root theorem) $\therefore x = -1 \pm 3i$	✓✓.Antwoord/ Answer **[2 punte / *marks*]**
	4.1.2 $K(x) = S(x).R(x)$ $= (x^2 + 2x + 10)(x^2 + 3x - 4)$ $= (x + 1 - 3i)(x + 1 + 3i)(x + 4)(x - 1)$ $x = -1 \pm 3i$ of /or $x = 1$ of/ or $x = -4$	✓ Sibstitusie / Substitution ✓ Komplekse Wortels/ Complex Roots ✓✓ Reële Wortels / Real Roots **[4 punte / *marks*]**
	4.1.3 $H(x) = P(x) - Q(x)$ $= (x^3 - 2x^2 + 12) - (3x^3 - 4x + 8)$ $= -2x^3 - 2x^2 + 4x + 4$ $x + 1$ a factor, thus $x = -1$ a root Met langdeling of inspeksie / With long division or inspection: $H(x) = -(x + 1)(2x^2 - 4)$ $= -2(x^2 - 2)(x + 1)$ Faktoriseer verder (kwadratiese formule) / Factor further (may use quadratic formula): $x^2 - 2 = (x - \sqrt{2})(x + \sqrt{2})$ Dus/ thus $x = \pm\sqrt{2}$ of / or $x = -1$	✓ Substitute ✓ $H(x)$ ✓ Wortel/ Root ✓✓ Faktore / Factors ✓✓ Factor further ✓✓ Alle wortels / All roots **[9 punte / *marks*]**

Vraag / *Question* 5 (A) [15 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$A = \frac{1}{2}OA^2\theta - \frac{1}{2}OD^2\theta$ $OD = OA - x = 5 - x$ $\therefore A = \frac{1}{2}5^2\theta - \frac{1}{2}(5-x)^2\theta$ $= \frac{1}{2}\left(25 - (25 - 10x + x^2)\right)\theta$ $= \frac{1}{2}(10x - x^2)\theta$ $= \frac{1}{2}x(10 - x)\theta$	✓Formule /Formula ✓$5 - x$ ✓vervang/ substitute ✓verenvoudig / simplify ✓antwoord /answer **[5 punte / *marks*]**
5.2	$\frac{x(10-x)\theta}{2}$ $= \frac{x(10-x)}{25} = 1$ $x^2 - 10x + 25 = 0$ $(x-5)^2 = 0$ $x = 5$	✓vervang/substitute $\boldsymbol{\theta}$ ✓stel gelyk aan 1/ set equal to 1 ✓ Vereenvoudig/ Simplify ✓ $\boldsymbol{x = 5}$ **[4 punte / *marks*]**
5.3	$P_{DABE} = DA + AB + EB + DE$ $DA = EB = x = 2$ $AB = OA.\theta$ $= 5\left(\frac{8}{100}\right) = 0.4$ $DE = OE.\theta$ $= (3)\left(\frac{8}{100}\right) = 0.24$ $P_{DABE} = DA + AB + EB + DE$ $= 2 + 0.4 + 2 + 0.24$ $= 4.64$ cm	✓ formule of opstel / formula or setup ✓✓ $\boldsymbol{AB}$ ✓✓ $\boldsymbol{DE}$ ✓ Finale antwoord / Final answer **[6 punte / *marks*]**

Vraag / *Question* 5 (B) [15 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$\boldsymbol{z} = 1\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ $\lvert\boldsymbol{z}\rvert = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$ Eenheidsvektor / unit vector: $\frac{\boldsymbol{z}}{\lvert\boldsymbol{z}\rvert} = \frac{1}{\sqrt{14}}\boldsymbol{i} + \frac{2}{\sqrt{14}}\boldsymbol{j} + \frac{3}{\sqrt{14}}\boldsymbol{k}$ OF / OR $(\frac{1}{\sqrt{14}}; \frac{2}{\sqrt{14}}; \frac{3}{\sqrt{14}})$	✓Grootte / Magnitude ✓ Antwoord / Answer **[2 punte / *marks*]**
5.2	$\mathbf{x} \bullet \mathbf{y} = \lvert\boldsymbol{x}\rvert\lvert\boldsymbol{y}\rvert \cos\theta$ $\mathbf{x} \bullet \mathbf{y} = (1)(1) + (-1)(2) + (1)(1) = 0$ $\lvert\boldsymbol{x}\rvert = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$ $\lvert\boldsymbol{y}\rvert = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$ $cos\theta = \frac{0}{\sqrt{6}\sqrt{3}} = 0$ $\theta = \frac{\pi}{2}$ rad	✓✓ puntproduk/ dot product ✓✓ Grootte / Magnitudes ✓$\boldsymbol{\theta} = \frac{\pi}{2}$ **[5 punte / *marks*]**
5.3	$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$ $= \big((-1)(1) - (2)(1)\big)\boldsymbol{i} - \big((1)(1) - (1)(1)\big)$ $+\big((1)(2) - (-1)(1)\big)\boldsymbol{k}$ $= -3\boldsymbol{i} + \boldsymbol{3k}$ Oppervlakte / Area: $\lvert -3\boldsymbol{i} + 3\boldsymbol{k}\rvert = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}\ eenhende^2$ Vorm / Shape: Reghoek (Parallelogram met 90° hoek) / Rectangle (Parallelogram with 90° angle)	✓ Determinant ✓✓✓ Compute ✓ Answer ✓ Compute area ✓✓ Rectangle Maks / Max: 4 marks (if the determinant method was not used) **[8 punte / *marks*]**

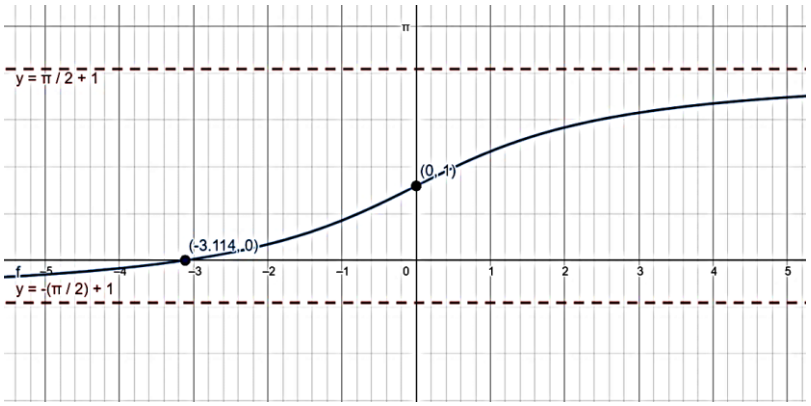
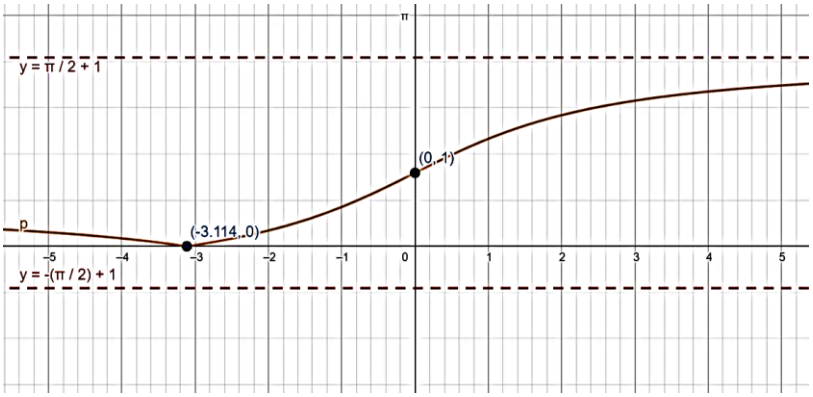
Vraag / *Question* 6 [20 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1	$\sum_{i=2}^{5}(-1)^i(2i)$ $=(-1)^2(2(2))+(-1)^3(2(3))+(-1)^4(2(4))+(-1)^5(2(5))$ $=4-6+8-10$ $=-4$	✓ Vervang /substitute $k=5$ ✓begin by / start with $i=2$ ✓eindig by /end at $i=5$ ✓✓vereenvoudig / simplify ✓✓antwoord / answer **[7 punte / *marks*]**
6. 2.1	**Vir $n=1$:** LHS= $\frac{1}{4}\left(\frac{3}{4}\right)^{1-1}=\frac{1}{4}$ RHS=$1-\left(\frac{3}{4}\right)^1=\frac{1}{4}$ ∴ LHS=RHS vir / for $n=1$ **Aanname / Assumption:** Aanvaar dat / Assume that $\frac{1}{4}+\frac{3}{16}+\frac{9}{64}+\cdots+\frac{1}{4}\left(\frac{3}{4}\right)^{k-1}=1-\left(\frac{3}{4}\right)^k$ **Induksie stap / Induciton:** Om te bewys / R.T.P: $\frac{1}{4}+\frac{3}{16}+\frac{9}{64}+\cdots+\frac{1}{4}\left(\frac{3}{4}\right)^{(k+1)-1}=1-\left(\frac{3}{4}\right)^{k+1}$ LHS= $\frac{1}{4}+\frac{3}{16}+\frac{9}{64}+\cdots+\frac{1}{4}\left(\frac{3}{4}\right)^{(k+1)-1}$ $=\frac{1}{4}+\frac{3}{16}+\frac{9}{64}+\cdots+\frac{1}{4}\left(\frac{3}{4}\right)^{(k)-1}+\frac{1}{4}\left(\frac{3}{4}\right)^{(k+1)-1}$ $=1-\left(\frac{3}{4}\right)^k+\frac{1}{4}\left(\frac{3}{4}\right)^k$ **[anname/ assumption]** $=1-\frac{3}{4}\left(\frac{3}{4}\right)^k$ $=1-\left(\frac{3}{4}\right)^{k+1}$ =RHS ∴LHS=RHS vir $n=k+1$ **Slot:** Deur die beginsel van wiskundige induskie is die stelling waar vir alle $n\in\mathbb{N}$. / By the principal of mathematical induction the statement is true for all $n\in\mathbb{N}$.	✓✓bewys vir $n=1$ ✓✓aanvaar vir / assume for $n=k$ ✓ Expand summation ✓✓geruik / use assumption ✓✓vereenvoudig / simplify ✓✓sluit af / conclude **[11 punte / *marks*]**

6.2.2	LHS= $\lvert x_1 + x_2 + \cdots + x_{k+1}\rvert$ $= \lvert x_1 + x_2 + \cdots + x_k + x_{k+1}\rvert$ $= \lvert (x_1 + x_2 + \cdots + x_k) + x_{k+1}\rvert$ $\leq \lvert \boldsymbol{x_1 + x_2 + \cdots + x_k}\rvert + \lvert x_{k+1}\rvert$ **[Gegee]** $\leq \lvert \boldsymbol{x_1}\rvert + \lvert \boldsymbol{x_2}\rvert + \cdots + \lvert \boldsymbol{x_k}\rvert + \lvert x_{k+1}\rvert$ [Aanname/Assumption] =RHS ∴LHS≤RHS	✓geruik gegewe/ use given ✓ gebruik aanname / use assumption **[2 punte / *marks*]**

Vraag / *Question* 7 **[25 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	$\left(x-\frac{3}{x^3}\right)^6$ $\binom{6}{r}(x)^{6-r}(-3x^{-3})^r$ $=\binom{6}{r}1^{6-r}x^{6-r}(-3)^r(x^{-3r})$ $\therefore 6-r-3r=2 \Longrightarrow -4r=-4 \Longrightarrow r=1$ Koëffisiënt / Coefficient: $\binom{6}{1}(1)^{6-1}(-3)^1$ $=6(-3)=-18$ ∴ Koëffisiënt / Coefficient is -18	✓Vervang / Substitute in formula ✓Vereenvoudig / Simplify ✓✓ $r=21$ ***Maks: 3*** ***(Indien r verkeerd)*** ✓✓Coefficient **[6 punte / *marks*]**
7.2	$\frac{1}{2+4x}=\frac{1}{2(1+2x)}=\frac{1}{2}(1+2x)^{-1}$ $\frac{1}{2}(1+2x)^{-1}=\frac{1}{2}(1+\frac{(-1)2x}{1!}+\frac{(-1)(-1-1)(2x)^2}{2!}+\cdots)$ $=\frac{1}{2}-x+2x^2+\cdots$	✓✓✓ Herskryf in korrekte vorm / Rewrite in correct form $\frac{1}{2}$✓$-x$✓$2x^2$✓ **[6 punte / *marks*]**
7.3.1	$f(x)=2\tan(x-1)$ Ruil x en y om / Interchange x and y: $x=2\tan(y-1)$ $\frac{x}{2}=\tan(y-1)$ $y-1=\arctan(\frac{x}{2})$ $y=\arctan\left(\frac{x}{2}\right)+1$ $\therefore f^{-1}(x)=\arctan\left(\frac{x}{2}\right)+1$	✓ruil om / interchange ✓✓ Vereenvoudig/ Simplify ✓ Antwoord/ Answer ✓ Herskryf / Rewrite f^{-1} **[5 punte / *marks*]**

7.3.2	Skets van f:  skets van $\|f\|$:  	✓x-afsnit /intercept ✓y-afsnit /intercept ✓✓asimptotote / asymptotes ✓✓ vorm / shape ***Maks / Max: 6 marks*** ***(Indien net f en nie $\|f\|$ geteken is /*** ***If only f and not $\|f\|$ is sketched)*** ✓✓ Absolute waarde maak negatiewe funksie waardes positief / Absolute makes negative values positive **[8 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -