

α -MATHEMATICS

Grade 11 Alpha Mathematics June Exam 2024

Grade 11

Examinator: M. Botha

Moderator: P. Marx

Time: 2 ½ hours

Total: 150 marks

INSTRUCTIONS

Read the following instructions carefully before attempting the questions:

1. This paper consists of **8** pages, an answer sheet and 2 diagram sheets.
2. Answer ALL **7** questions. Answer only questions 5A **OR** question 5B, **NOT** both.
3. Number answers according to the numbering scheme of the paper.
4. Non-programmable calculators may be used, unless otherwise stated.
5. Round all answers to two decimal places.
6. Clearly indicate all necessary calculations, diagrams, graphs etc. that you used to determine your answers.
7. Full marks will not necessarily be awarded to answers only.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers should be given in radians if necessary.
10. Write neatly and legibly.
11. **Enjoy!**

Question 1 – Multiple Choice**[20 marks]**This question must be answered **on the Answer Sheet**.Each question only has **ONE** correct answer which counts **TWO (2)** marks each.Mark the correct answer with an **X** on the Answer Sheet.

1.1 Solve for x , in $|x - 2| = 2$

- (A) $x = 0$
- (B) No Solution
- (C) $x = 0$ or $x = 4$
- (D) $x \in \mathbb{R}$

1.2 $f(x) = x + 2x^3 - 3x^4 + 30$. Which of the following statements are correct?

- (A) -1 is a root
- (B) $x + 2$ is a factor of f
- (C) $x^2 - 4$ is a factor of f
- (D) 2 is a root of f

1.3 $|2x - 5| < -3$.

- (A) $-1 < x < 1$
- (B) $x < -1$ or $x > 1$
- (C) $x \in \mathbb{R}$
- (D) No Solution

1.4 $K(x) = |x + 4| + 12$. Which of the following statements are correct?

- (A) The salient point is $(4; 12)$
- (B) K has no x -intercept(s)
- (C) The y -intercept is $(0; 12)$
- (D) K has no y -intercepts(s)

1.5 Given a sector with area A . If the radius is doubled and the angle is halved, what is the area of the new sector?

- (A) A
- (B) $\frac{A}{2}$
- (C) $2A$
- (D) $4A$

1.6 If $\frac{-2x-1}{(x^2-3x)(x+1)}$, is decomposed using partial fractions we find:

(A) $\frac{Ax+B}{(x^2-3x)} + \frac{C}{(x+1)}$

(B) $\frac{1}{x} + \frac{1}{x-3} + \frac{1}{x+1}$

(C) $\frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1}$

(D) $\frac{A}{(x^2-3x)} + \frac{B}{(x+1)}$

1.7 If $x = -3 - i$ is a root of $g(x)$, then

(A) $x - 3 + i$ is a factor

(B) $x - 3 - i$ is a factor

(C) g has atleast 3 factors

(D) $x + 3 - i$ is a factor

1.8 The inverse function of $h(x) = \cos(2x + 1)$ is

(A) $\text{bgcos}(2x + 1)$

(B) $\frac{\text{bgcos}(x)-1}{2}$

(C) $\text{bgcos}\left(\frac{x}{2} - 1\right)$

(D) $\frac{\text{bgcos}(x-1)}{2}$

1.9 How many terms are in the binomial expansion of $(x + \frac{2}{x})^6$?

(A) 7

(B) 6

(C) 8

(D) 4

1.10 The root(s) of $P(x) = \frac{x^2-4}{x-2}$ are

(A) $x = -2$

(B) $x = 2$

(C) $x = 2$ and $x = -2$

(D) $x = 4$

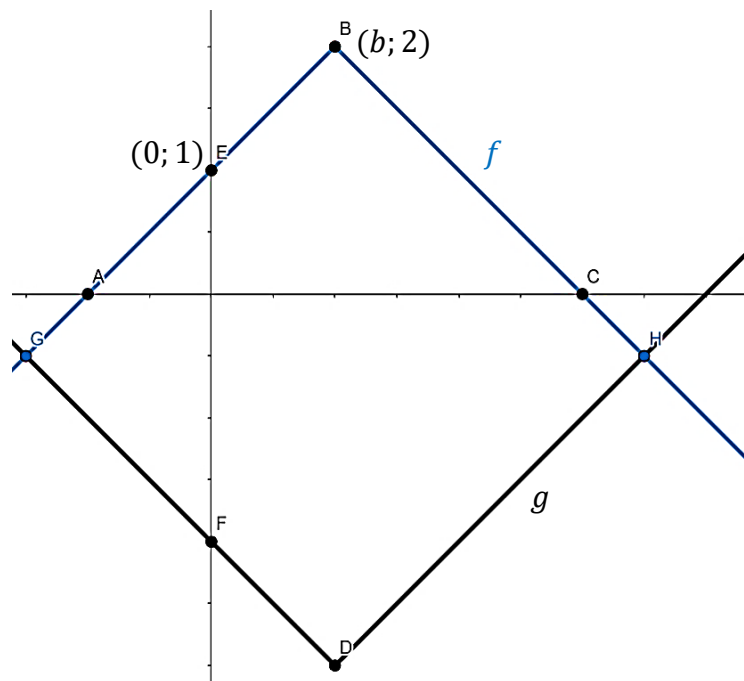
Question 2 - Absolute Values**[37 marks]**2.1 Solve for x , (with $x \in \mathbb{R}$), in:

2.1.1 $|x - 2| = -1$ (2)

2.1.2 $|x + 3| = x$ (5)

2.1.3 $|x^2 - 4| = 3x$ (8)

2.2 In the diagram below, the graph of $f(x) = -|x - b| + a$ as well as the graph of $g(x) = |x - c| + d$ are sketched. The points E (0; 1) and B (b ; 2) are also indicated.

2.2.1 Determine (with reasons) the values of a and b (4)2.2.2 If $a = 2$ and $b = 1$ (this does not necessarily follow from question 2.2.1).

Determine the coordinates of A and C. (5)

2.2.3 If $g(x) = -f(x) - 1$, determine the coordinates of D (c ; d) and F. (4)2.2.4 For which values of x is $f(x) > 1$? (2)2.3 Given $y = -|2x + 1| + 2$

2.3.1 Write down the coordinate of the salient point. (2)

2.3.2 Sketch the graph of $y = -|2x + 1| + 2$.

Indicate, clearly, **all** intercepts with the axes, the salient point and all points of interest. Use **DIAGRAM SHEET 1** for the sketch.

(5)

Question 3 – Partial Fractions**[18 marks]**

3.1 Decompose $\frac{2x^3 - 2x^2 + x - 1}{x^2(x^2 + 1)}$ into partial fractions. (10)

3.2 Match the fraction in **COLUMN A** with the decomposition in **COLUMN B**.

A letter may be used more than once.

Write down only the question number and letter, for example: “3.2.5 G” (8)

COLUMN A		COLUMN B	
3.2.1	$\frac{3x + 2}{x^3 + x^2}$	A	$\frac{A}{x^2} + \frac{B}{x + 1}$
3.2.2	$\frac{3x + 2}{x^4 + x^2}$	B	$\frac{A}{x^2} + \frac{B}{x} + \frac{C}{x + 1}$
3.2.3	$\frac{3x + 2}{x^2(x + 1)}$	C	$\frac{A}{x + 1} + \frac{B}{x - 1}$
3.2.4	$\frac{3x + 2}{x^3 + x}$	D	$\frac{A}{x^2} + \frac{Bx + C}{x^2 + 1}$
		E	$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}$
		F	$\frac{A}{x} + \frac{Bx + C}{x^2 + 1}$

Question 4 - Polynomials**[15 marks]**

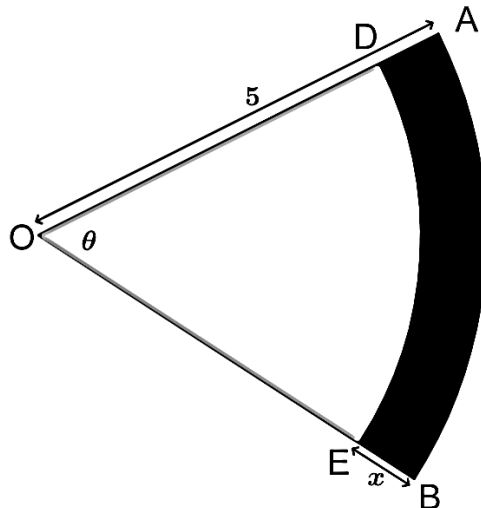
4.1 It is given that $P(x) = x^3 - 2x^2 + 12$, $Q(x) = 3x^3 - 4x + 8$,
 $S(x) = x^2 + 2x + 10$ and $R(x) = x^2 + 3x - 4$.

Determine the roots of the following polynomials in $\mathbb{C}[x]$:

4.1.1 $S(x)$, if one of the roots is $x = -1 + 3i$ (2)

4.1.2 $K(x) = S(x) \cdot R(x)$ (4)

4.1.3 $H(x) = P(x) - Q(x)$ if one of the factors is $x + 1$ (9)

Question 5**[15 Marks]**ANSWER ONLY **ONE** OF THE FOLLOWING **TWO** QUESTIONS.**A. Trigonometry and Radians***Don't do this question, if you have done question 5B.*5. In the diagram AOB is a sector, of a circle with center O and radius $r = 5 \text{ cm}$.In the sector $A\hat{O}B = \theta$ and $EB = x$. The area DABE is shaded.5.1 Show that the area of the shaded region is given by $\frac{x(10-x)\theta}{2}$. (5)5.2 If $\theta = \frac{8}{100}$ determine the value(s) of x for which the area of the shaded region is 1 cm^2 is. (4)5.3 If $x = 2 \text{ cm}$ and $\theta = \frac{8}{100}$ (*this is not a follow up of question 5.2*), determine the perimeter of the shaded region. (6)**OR****B. Vectors***Don't do this question, if you have done question 5A.*5. The vectors $x = 1\mathbf{i} - 1\mathbf{j} + 1\mathbf{k}$, $y = 1\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $z = 1\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ are given.5.1 Determine the unit vector of z . (2)5.2 Determine the angle between x and y . (5)5.3 Determine the area of the parallelogram that is formed by vectors x and y with the **determinant method**. Which shape is it? (8)

Question 6 – Mathematical Induction**[20 marks]**6.1 Write out the expansion of the expression below, with $k = 5$ and simplify fully.

$$\sum_{i=2}^k (-1)^i (2i)$$

(7)

6.2.1 Prove that $\frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \dots + \frac{1}{4} \left(\frac{3}{4}\right)^{n-1} = 1 - \left(\frac{3}{4}\right)^n$ for all $n \in \mathbb{N}$ by use of Mathematical Induction. (11)

6.2.2 David wants to prove that

$$|x_1 + x_2 + x_3 + \dots + x_n| \leq |x_1| + |x_2| + \dots + |x_n| \text{ for all } n \in \mathbb{N}$$

It is given that,

$ x + y \leq x + y $	(*)
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Complete the missing steps in David's proof (shown below), by using the given statement (*) as well as the assumption.

David's Proof

For $n = 1$: LHS= $|x_1| \leq |x_1|$ =RHS

\therefore LHS \leq RHS for $n = 1$

Assumption: Assume that the statement is true for $k \in \mathbb{N}$, i.e:

Assume that $|x_1 + x_2 + \dots + x_k| \leq |x_1| + |x_2| + \dots + |x_k|$

Induction step:

We want to prove that: $|x_1 + x_2 + \dots + x_{k+1}| \leq |x_1| + |x_2| + \dots + |x_{k+1}|$

LHS= $|x_1 + x_2 + \dots + x_{k+1}|$

=...

=...

=...

\therefore LHS \leq RHS for $n = k + 1$

Conclusion:

Thus, the statement is true for all $n \in \mathbb{N}$, by the principal of mathematical induction.

(2)

Question 7 – Binomial Theorem, Power Series & Inverses [25 marks]

7.1 Determine the coefficient of x^2 in the expansion of $\left(x - \frac{3}{x^3}\right)^6$. (6)

7.2 Write the first three terms in the power series of

$$\frac{1}{2+4x}$$

as well as the values of x for which the expansion holds. (6)

7.3 If $f(x) = 2 \tan(x - 1)$

7.3.1 Determine the inverse of f (5)

7.3.2 Sketch the functions $f^{-1}(x)$ and $|f^{-1}(x)|$ on **DIAGRAM SHEET 2**

Clearly indicate the functions, as well as **all** intercepts and asymptotes.

HINT: Sketch f^{-1} , and use the definition of absolute value. (8)

- END OF THE PAPER-

ALPHA WISKUNDE FORMULA PAGE

ALGEBRA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & \text{as } x \geq 0 \\ -x & \text{as } x < 0 \end{cases}$$

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots ; \text{mits } |x| < 1$$

VECTORS:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

CALCULUS:

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

$$V = \pi \int_a^b [f(x)]^2 dx \quad a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

TRIGONOMETRY:

$$\text{In a sector: } s = r\theta \quad \text{en } A = \frac{1}{2}r^2\theta$$

$$\text{Identities: } \sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x \quad \cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

TABLE WITH DERIVATIVES:

$F(x)$	$F'(x)$
ax^n	nax^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \cdot \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$

$F(x)$	$F'(x)$
$\operatorname{bgsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arcsin} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\operatorname{bgcos} x$	$\frac{1}{x^2+1}$
$\operatorname{arccos} x$	$\frac{1}{x^2+1}$
$\operatorname{bgtan} x$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\operatorname{arctan} x$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
$f(x) \cdot g(x)$	$f'[g(x)] \cdot g'(x)$
$\frac{f(x)}{g(x)}$	
$f[g(x)]$	

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ANSWER SHEET

Name and Surname: _____

Question Total	1 [20]	2 [37]	3 [18]	4 [15]	5 [15]	6 [20]	7 [25]
Learner Total							

TOTAL 150

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1

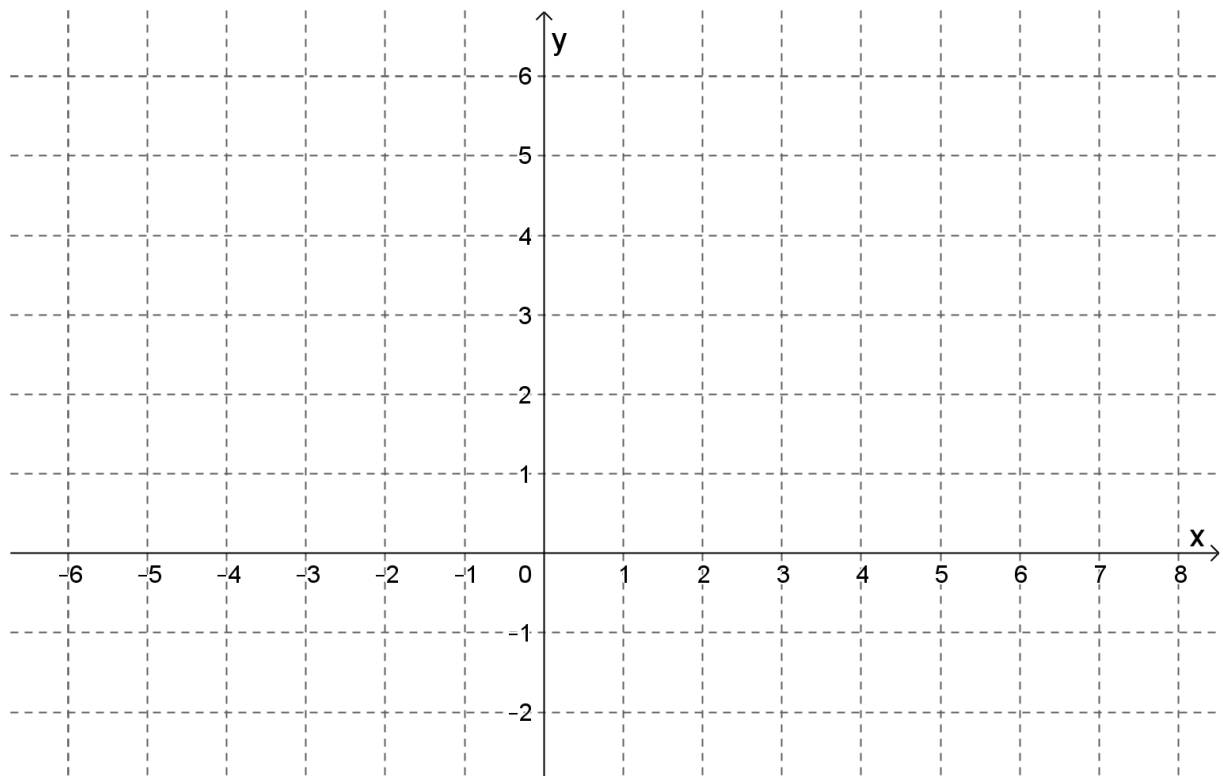


DIAGRAM SHEET 2

