α-WISKUNDE/ MATHEMATICS

Junie/ June 2024
Graad/ Grade 12

Tyd/ Time: 3 uur/*hours*
Totaal/ Total: 200 PUNTE/ *MARKS*

VRAAG/*QUESTION* 1 [30 PUNTE/*MARKS*]

1.1	A	B	**C**	D
1.2	A	**B**	C	D
1.3	A	B	C	**D**
1.4	**A**	B	C	D
1.5	A	**B**	C	D
1.6	**A**	B	C	D
1.7	A	B	C	**D**
1.8	A	B	**C**	D
1.9	A	B	C	**D**
1.10	A	B	**C**	D
1.11	**A**	B	C	D
1.12	A	**B**	C	D
1.13	A	B	**C**	D
1.14	A	**B**	C	D
1.15	A	B	C	**D**

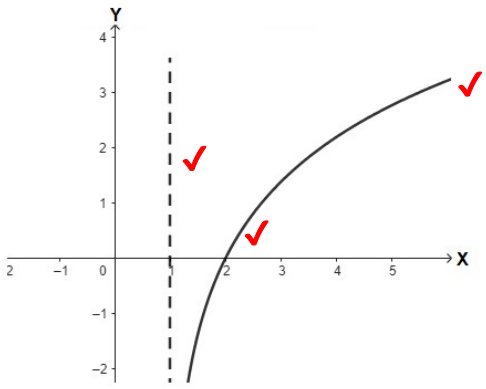
VRAAG/*QUESTION* 2 [20 PUNTE/*MARKS*]

2.1a	✓ ✓ $p = 2\text{cis}\left(\frac{2\pi}{3}\right)$ ✓ ✓ $k = 2\text{cis}\left(-\frac{\pi}{4}\right)$	1: 2 1: $\frac{2\pi}{3}$ 1: 2 1: $-\frac{\pi}{4}$ **[4]**
2.1b	✓ ✓ $p.k = 4\text{cis}\left(\frac{5\pi}{12}\right)$	1: 4 1: $\frac{5\pi}{12}$ **[2]**
2.1c	✓ ✓ $p^3 = 8\text{cis}(2\pi)$	1: 8 1: 2π **[2]**
2.1d	$p^3 = 8\big(\cos(2\pi) + i\sin(2\pi)\big)$✓ $= 8 + 0i$ $\therefore a = 8$✓ en/*and* $b = 0$✓	1: $8\big(\cos(2\pi) + i\sin(2\pi)\big)$ 1: $a = 8$ 1: $b = 0$ **[3]**
2.2a	$m(i) = i^4 + i^3 - 5i^2 + i - 6$✓ $= 1 - i + 5 + i - 6$✓ $= 0$✓	1:Vervang/*Substitute* 1:Vereenvoudig/*Simplify* 1:=0 **[3]**
2.2b	$\frac{x^4 + x^3 - 5x^2 + x - 6}{x^2 + 1} = x^2 + x - 6$✓ $\therefore m(x) = (x^2 + 1)(x^2 + x - 6)$✓ ✓ ✓ ✓ ✓ $= (x + i)(x - i)(x + 3)(x - 2)$	1:Langdeling/*Long division* 1:Faktore/*Factors* $(x^2 + 1)$ en/and $(x^2 + x - 6)$ 4:Faktore/*Factors* **[6]**

VRAAG/*QUESTION* 3 [17 PUNTE/*MARKS*]

3.1	$\begin{vmatrix} a & -2 \\ -1 & a \end{vmatrix} = a^2 - 2$✓ ✓ $\begin{vmatrix} 5 & -2 \\ -4 & a \end{vmatrix} = 5a - 8$✓ ✓ $\therefore \frac{5a-8}{a^2-2} = 1$✓ ✓ $a = 2$ of/*or* $a = 3$✓	1: $a^2 - 2$ 1: Matriks/*Matrix* 1: $5a - 8$ 1: Toon/*Show* Cramer 1: $= 1$ 1: $a = 2$ 1: $a = 3$ **[7]**
3.2a	✓ $0 = 2x - 1 - 1$ $\therefore x = 1$✓	1:$= 0$ 1:Antwoord/*Answer* **[2]**
3.2b	$2x - 1 - 1 = x$✓ $\therefore x = 2$✓ $\therefore A(2;2)$✓	1:Vergelykings gelyk/*Equations equal* 1: $x = 2$ 1: $A(2;2)$ **[3]**
3.2c	✓ ✓ $D\left(\frac{1}{2};-1\right)$	1:$\frac{1}{2}$ 1:-1 **[2]**
3.2d	$\lvert 2x - 1 \rvert < 4$ $\therefore -5 < 2x - 1 < 5$✓ ✓ ✓ $\therefore -2 < x < 3$	 1:Vereenvoudig/*Simplify* 1: $x > -2$ 1: $x < 3$ **[3]**

VRAAG/*QUESTION* 4 [15 PUNTE/*MARKS*]

4.1	$f(-3) = \text{bgcos}\left(e^{2(-3)+5}\right)$ of/*or* $\arccos\left(e^{2(-3)+5}\right)$ ✓ $= 1{,}19$✓	1:Vervang/*Substitute* 1: Antwoord/*Answer* **[2]**
4.2	$e^x = e^{2x} - 12$✓ $(e^x - 4)(e^x + 3) = 0$✓ $\therefore e^x = 4$ ✓ of/or $e^x = -3$ $x = \ln 4$ ✓ geen oplossing/*no solution*✓ $x = 1{,}39$ ✓	1:Vereenvoudig/*Simplify* 1:Faktore/*Factors* 1: $e^x = 4$ 1: $x = \ln 4$ 1: $x = 1{,}39$ 1: $e^x = -3$ geen oplossing/ *no solution* **[6]**
4.3	$x = \frac{e^{3y}}{2} + 1$✓ $\therefore 2(x-1) = e^{3y}$✓ $\therefore 3y = \ln(2x-2)$✓ $\therefore y = \frac{1}{3}\ln(2x-2)$✓	1:Ruil $x$en y/*Swop* x *and* y 1:Vereenvoudig/*Simplify* 1: Vereenvoudig/*Simplify* 1: Antwoord/*Answer* **[4]**
4.4		1:Asimptoot/*Asymptote* $x = 1$ 1:$x = 2$ 1:Vorm/*Form* **[3]**

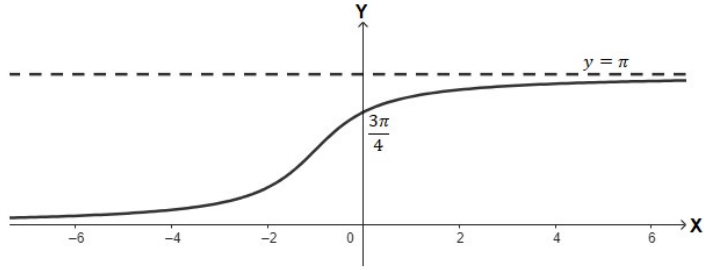
VRAAG/*QUESTION* 5 [15 PUNTE/*MARKS*]

5.1	$\binom{10}{r}(x^{10-r})(2x^{-2})^r$ ✓ $\therefore x^{10-r}.x^{-2r} = x^{-2}$ ✓ $\therefore r = 4$ ✓ $\therefore \binom{10}{4}(x^6)(2x^{-2})^4$ ✓ $= 210x^6.16x^{-8}$ ✓ $= 3360x^{-2}$ Koëffisiënt is/*Coefficient is* 3360 ✓	1:Formule/*Formula* 1:Vergelyking/*Equation* 1: $r = 4$ 1: Formule/*Formula* 1:Vereenvoudig/*Simplify* 1:Antwoord/*Answer* **[6]**
5.2	Stel/*Set* $n = 1$: LK/*LHS* $= \frac{1}{2}$ RK/*RHS*$= \frac{1}{2}$ $\therefore$ Die bewering is waar as/*The statement is true for* $n = 1$ ✓ Aanvaar die bewering is waar vir/*Accept the statement is true for* $n = k$: $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$ ✓ Beskou nou/*Consider* $n = k + 1$: LK/*LHS* $= \frac{k}{k+1}$ ✓ $+ \frac{1}{(k+1)(k+2)}$ ✓ $= \frac{k(k+2)+1}{(k+1)(k+2)}$ ✓ $= \frac{k+1}{k+2}$ ✓ RK/*RHS* $= \frac{k+1}{k+2}$ ✓ Volgens die beginsel van wiskundige induksie is die bewering dus waar vir alle $n \in \mathbb{N}$ /*By the principle of mathematical induction, the statement is true for all* $n \in \mathbb{N}$ ✓✓	1: Bewys n = 1/*Prove n = 1* 1: Aanvaar/*Accept* $n = k$ 1: Vervang/*Substitute* 1: $(k+1)$de term/ $(k+1)$*th term* 2: Vereenoudig LK/ *Simplify LHS* 1: RK vervang/ *RHS Substitute* 2: Afleiding/*Deduction* **[9]**

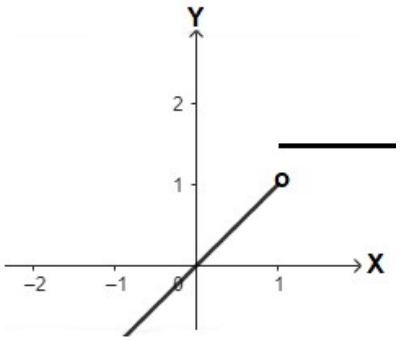
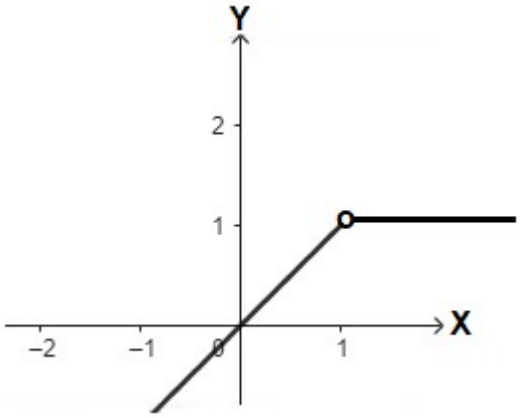
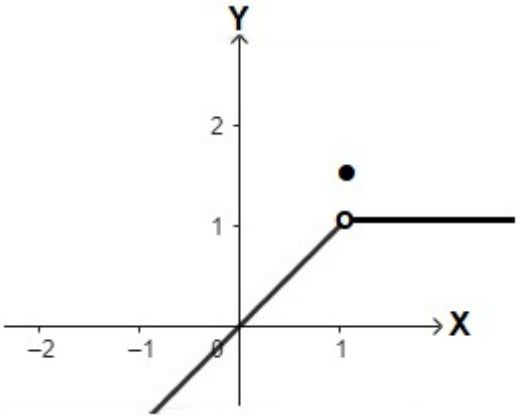
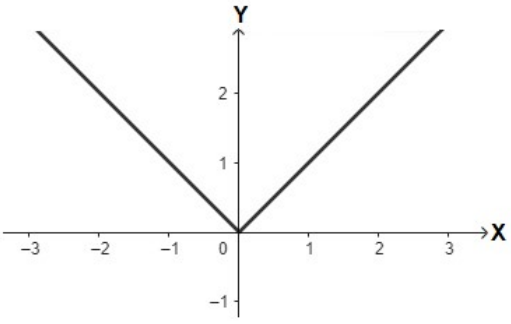
VRAAG/*QUESTION* 6 [17 PUNTE/*MARKS*]

6.1a	PT= $2\left(\frac{\pi}{3}\right) = \frac{2\pi}{3}$ ✓ Omtrek/*Circumfrence* $= 4 + \frac{2\pi}{3}$ ✓	1:PT 1:Antwoord/*Answer* **[2]**
6.1b	Opp/*Area* CPT $= \frac{1}{2}.2^2\left(\frac{\pi}{3}\right)$ ✓ $= \frac{2\pi}{3}$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* **[2]**
6.1c	Opp/*Area* $= \frac{1}{2}.4^2\sin\left(\frac{\pi}{3}\right)$ ✓ $= 4\sqrt{3}$ ✓ ∴ Opp/*Area* PABT $= 4\sqrt{3} - \frac{2\pi}{3}$ ✓ $= 4{,}83$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* 1: Opp/*Area* 1: Antwoord/*Answer* **[4]**
6.2a	$\lvert\boldsymbol{a}\rvert = \sqrt{2^2 + (-1)^2 + (-2)^2}$ ✓ $= 3$ ✓ ∴ Eenheidsvektor/*Unit vector*: $\frac{2}{3}i - \frac{1}{3}j - \frac{2}{3}k$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* 1: Antwoord/*Answer* **[3]**
6.2b	$\gamma = \text{bgcos}\left(-\frac{2}{3}\right)$ of/*or* $\gamma = \arccos\left(-\frac{2}{3}\right)$ ✓ $= 2{,}30$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* **[2]**
6.2c	$\begin{vmatrix} i & j & k \\ 2 & -1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$ ✓ $= i(0+2) - j(0+2) + k(2+1)$ ✓ $= 2i - 2j + 3k$ ✓ En/*And* $-2i + 2j - 3k$ ✓	1: Formule/*Formula* 1:Vereenvoudig/*Simplify* 1: Antwoord/*Answer* 1: Antwoord/*Answer* **[4]**

VRAAG/*QUESTION 7* **[14 PUNTE/*MARKS*]**

7.1		1:$y = \pi$ 1:$y = 0$ 1:$x = \frac{3\pi}{4}$ 1:Vorm/*Form* **[4]**
7.2a	$f'(x) = \cos\left(x - \frac{\pi}{6}\right)$ ✓ ✓ $f''(x) = -\sin\left(x - \frac{\pi}{6}\right) = 0$✓ $\therefore x = \frac{\pi}{6}$✓ Buigpunt/*Point of inflection*: $\left(\frac{\pi}{6}; 0\right)$ ✓	1:$f'(x)$ 1:$f''(x)$ 1:$= 0$ 1: $x = \frac{\pi}{6}$ 1: $\left(\frac{\pi}{6}; 0\right)$ **[5]**
7.2b	Stel/*Let* $x = 0$: $f''(0)$✓ $= -\sin\left(0 - \frac{\pi}{6}\right)$ $= \frac{1}{2} > 0$✓ Stel/*Let* $x = \frac{\pi}{3}$ $f''\left(\frac{\pi}{3}\right)$ ✓ $= -\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$ $= \frac{-1}{2} < 0$✓ $f''(0) > 0$ en/and $f''\left(\frac{\pi}{3}\right) < 0$ $\therefore \left(\frac{\pi}{6}; 0\right)$ is 'n buigpunt/*point of inflection*✓	1: $f''(0)$ 1: $\frac{1}{2} > 0$ 1: $f''\left(\frac{\pi}{3}\right)$ 1: $-\frac{1}{2} < 0$ 1:Afleiding/Deduction **[5]**

VRAAG/*QUESTION* 8 [17 PUNTE/*MARKS*]

8.1a		2: Een voorbeeld van 'n sprongdiskontinuïteit/*One example of a jump discontinuity* **[2]**
8.1b	 	2x2:Twee verskillende voorbeelde van 'n verwyderbare diskontinuïteit/*Two different examples of a removable discontinuity* **[4]**
8.1c		2:Een voorbeeld van 'n kontinue funksie wat nie differensieerbaar is nie/*One example of a continuous, but non-differentiable function* **[2]**

8.2a	$f'(x) = 3\tan^2(x^2)$✓ $.\sec^2(x^2)$✓ $2x$✓ $+\frac{1}{7}x^{-2}$ ✓	1: $3\tan^2(x^2)$ 1: $\sec^2(x^2)$ 1: $2x$ 1: $\frac{1}{7}x^{-2}$ **[4]**
8.2b	$\frac{dy}{dx} = \frac{2}{1+4x^2}$✓ $+ \frac{\cos x}{\sin x}$✓$.\ln 10$✓	1:2 1: $\frac{1}{1+4x^2}$ 1: $\cos x$ 1: $\frac{1}{\sin x}$ 1: $\frac{1}{\ln 10}$ **[5]**

VRAAG/*QUESTION* 9 [18 PUNTE/*MARKS*]

9.1a	$x < 0$✓ of/*or* $x > 2$✓	1: $x < 0$ 1: $x > 2$ **[2]**
9.1b	✓ ✓ $0 < x < 2$	1: $x > 0$ 1: $x < 2$ **[2]**
9.1c	$x = 0$✓ of/*or* $x = 2$✓	1: $x = 0$ 1: $x = 2$ **[2]**
9.2a	✓ ✓ ✓ ✓ ✓ $e^y.\frac{dy}{dx}.\sqrt{x} + e^y.\frac{1}{2}x^{-\frac{1}{2}} = 2ey.\frac{dy}{dx}$ ✓ $\frac{dy}{dx} = \frac{-e^y.\frac{1}{2}.x^{-\frac{1}{2}}}{e^y\sqrt{x} - 2ey}$✓	1: e^y 1: $\frac{dy}{dx}$ 1: Produkreël/*Product rule* 1: $\frac{1}{2}x^{-\frac{1}{2}}$ 1: $2ey$ 1: $\frac{dy}{dx}$ 1: $-e^y.\frac{1}{2}.x^{-\frac{1}{2}}$ 1: $e^y\sqrt{x} - 2ey$ **[8]**
9.2b	$m = \frac{-e.\frac{1}{2}.1}{e-2e}$ ✓ $= \frac{1}{2}$ ✓ $\therefore y - 1 = \frac{1}{2}(x-1)$✓ $\therefore y = \frac{x}{2} + \frac{1}{2}$ ✓	1: Vervanging/*Substitution* 1: Antwoord/*Answer* 1: Vergelyking/ *Equation* 1: Antwoord/*Answer* **[4]**

VRAAG/*QUESTION* 10 [19 PUNTE/*MARKS*]

10.1a	✓Vertikaal/*Vertical*: $x = 0$✓ ✓Horisontaal/*Horizontal*: $y = 1$✓	1: Vertikaal/*Vertical* 1: $x = 0$ 1: Horisontaal/*Horizontal* 1: $y = 1$ **[4]**
10.1b	$\frac{x^2 + x + 1}{x^2} = 1$✓ $\therefore x^2 + x + 1 = x^2$✓ $\therefore x = -1$✓	1: $= 1$ 1:Vereenvoudig/*Simplify* 1:Antwoord/*Answer* **[3]**
10.1c	f styg as $f'(x) > 0$ $f'(x) = \frac{(2x+1)x^2 - (x^2+x+1)(2x)}{x^4}$✓ > 0 $\therefore x^2 + 2x < 0$✓ $\therefore -2 < x < 0$✓✓	 1:$f'(x)$ 1:Vereenvoudig/*Simplify* 1:$x > -2$ 1:$x < 0$ **[4]**
10.2a	$\frac{dy}{dx} = 2^{3x-1}$✓$.3$✓$\ln 2$✓ $+ \frac{1}{x+1}$✓	1: 2^{3x-1} 1: 3 1: $\ln 2$ 1: $\frac{1}{x+1}$ **[4]**
10.2b	Stel/*Let* $h(x) = 2^{3x-1} + \ln(x+1) - \ln 2$✓ $\therefore x_{n+1} = x_n - \frac{2^{3x_n-1} + \ln(x_n+1) - \ln 2}{2^{3x_n-1}.3\ln 2 + \frac{1}{x_n+1}}$✓ $\therefore x \approx 0{,}09187$✓✓	1:$h(x) = f(x) - g(x)$ 1:Newton 2:Antwoord/*Answer* **[4]**

VRAAG/*QUESTION* 11 [18 PUNTE/*MARKS*]

11.1a	$\frac{5^{7x}\checkmark}{7\checkmark\ln 5\checkmark} - \frac{e^{2x}\checkmark}{2\checkmark} + k$	1:5^{7x} 1:7 1:ln5 1:e^{2x} 1:$\frac{1}{2}$ **[5]**
11.1b	$-\frac{3\checkmark(4x-1)^{-1}\checkmark}{4\checkmark} + \ln(x+1)\checkmark + k$	1:-3 1:$\frac{1}{4}$ 1:$(4x-1)^{-1}$ 1: $\ln(x+1)$ **[4]**
11.1c	$e.\text{bgtan}x\checkmark - 2x^{\frac{1}{2}}\checkmark + x\ln 5\checkmark + k$ of/*or* $e.\arctan x - 2x^{\frac{1}{2}} + x\ln 5 + c$	1: $e.\text{bgtan}x$ of/*or* $e.\arctan x$ 1: $x^{\frac{1}{2}}$ 1: $x\ln 5$ **[3]**
11.2	Vol$= \pi\int_p^0 \left(\sqrt{x+\pi}\right)^2 dx\checkmark$ $= \pi\int_p^0 (x+\pi)dx\checkmark$ $= \pi\left[\frac{x^2}{2}+\pi x\right]\Big\|_p^0\checkmark$ $= \pi\left[0-\left(\frac{p^2}{2}+p\pi\right)\right]\checkmark$ $= -\frac{p^2\pi}{2} - p\pi^2\checkmark\checkmark$	1:Formule/*Formula* 1:Vereenvoudig/*Simplify* 1:Integreer/*Integrate* 1:Vervang/*Substitute* 2:Antwoord/*Answer* **[6]**