α-WISKUNDE/ MATHEMATICS

Rekordeksamen/*Preliminary Exam* 2024 **Tyd/ Time: 3 uur/*hours***
Graad/*Grade* 12 **Totaal/ Total: 200 PUNTE/*MARKS***

VRAAG/*QUESTION* 1 [30 PUNTE/*MARKS*]

1.1	**A**	**B**	**C**	**D**
1.2	**A**	**B**	**C**	**D**
1.3	**A**	**B**	**C**	**D**
1.4	**A**	**B**	**C**	**D**
1.5	**A**	**B**	**C**	**D**
1.6	**A**	**B**	**C**	**D**
1.7	**A**	**B**	**C**	**D**
1.8	**A**	**B**	**C**	**D**
1.9	**A**	**B**	**C**	**D**
1.10	**A**	**B**	**C**	**D**
1.11	**A**	**B**	**C**	**D**
1.12	**A**	**B**	**C**	**D**
1.13	**A**	**B**	**C**	**D**
1.14	**A**	**B**	**C**	**D**
1.15	**A**	**B**	**C**	**D**

VRAAG/*QUESTION* 2 [16 PUNTE/*MARKS*]

2.1a	$a = 2\checkmark\text{cis}\left(\frac{7\pi}{6}\right)\checkmark$ of/*or* $2\text{cis}\left(\frac{-5\pi}{6}\right)$ $b = 3\text{cis}(\pi)\checkmark$	1: 2 1: $\frac{7\pi}{6}$ OF/*OR* $\frac{-5\pi}{6}$ 1: $3\text{cis}(\pi)$ **[3]**
2.1b	$9\checkmark\text{cis}(2\pi)\checkmark$	1: 9 1: 2π **[2]**
2.1c	$\sqrt{a} = \sqrt{2}\checkmark\text{ cis}\left(\frac{7\pi}{12}\right)\checkmark$ en/*and* $\sqrt{a} = \sqrt{2}\text{ cis}\left(\frac{19\pi}{12}\right)\checkmark$ of/*or* $\sqrt{a} = \sqrt{2}\text{ cis}\left(\frac{-5\pi}{12}\right)$ en/*and* $\sqrt{a} = \sqrt{2}\text{ cis}\left(\frac{7\pi}{12}\right)$	1: $\sqrt{2}$ 1: $\frac{7\pi}{12}$ 1: $\frac{19\pi}{12}$ OF/*OR* $\frac{-5\pi}{12}$ **[3]**
2.2a	$y = e^0 - 3\checkmark$ $= -2\checkmark$	1: Vervang/*Substitute* 1: Antwoord/*Answer* **[2]**
2.2b	Horisontaal/*Horizontal*$\checkmark$ $y = -3\checkmark$	1: Horisontaal/*Horizontal* 1: $y = -3$ **[2]**
2.2c	$x = e^{2y} - 3\checkmark$ $\therefore 2y = \ln(x+3)\checkmark$ $\therefore p = \frac{1}{2}\checkmark$ en/*and* $g(x) = x + 3\checkmark$	1: Ruil x en y/*Swop* x *and* y 1: Omskakel na ln/ *Convert to* ln 1: $p = \frac{1}{2}$ 1: $g(x) = x + 3$ **[4]**

VRAAG/*QUESTION* 3 [20 PUNTE/*MARKS*]

3.1a	$1 = \left\lvert\frac{k}{-6}\right\rvert$ ✓ $\therefore k = 6$✓	1: Vervang/*Substitute* 1: Antwoord/*Answer* **[2]**
3.1b	✓$0 = 2\lvert x+1\rvert - 2$ $x + 1 = 1$ of/*or* $x + 1 = -1$✓ $\therefore \text{A}(-2; 0)$✓	1: $= 0$ 1:Vergelykings/*Equations* 1: $\text{A}(-2; 0)$ **[3]**
3.1c	✓ $\text{B}(-1; -2)$✓	1: -1 1: 2 **[2]**
3.1d	$\lvert x+1\rvert < 3$✓ ✓ $\therefore -4 < x < 2$✓	1: Vereenvoudig/*Simplify* 1: $-4 < x$ 1: $x < 2$ **[3]**
3.1e	$-\frac{6}{x}$✓ $= -2(x+1) - 2$✓ $x = 1$ n.v.t./*n.a.* ✓ $\therefore \text{C}(-3; 2)$✓✓	2: Korrekte vergelykings/ *Correct equations* 1: $x = 1$ n.v.t./*n.a.* 2: $\text{C}(-3; 2)$ **[5]**
3.2	$x = \sqrt{2}$ is ook 'n nulpunt/*is a zero as well*✓ $\therefore (x - \sqrt{2})(x + \sqrt{2})$ ✓ $= x^2 - 2$✓ $\therefore (x-2)(x^2-2)$✓ $= x^3 - 2x^2 - 2x + 4$ $\therefore p = 4$✓	1: $x = \sqrt{2}$ 1: $(x - \sqrt{2})(x + \sqrt{2})$ 1: $x^2 - 2$ 1: $(x-2)(x^2-2)$ 1: $p = 4$ **[5]**

VRAAG/*QUESTION* 4 [20 PUNTE/*MARKS*]

4.1a	$= \binom{4}{0}(1)^4(2x)^0 + \binom{4}{1}(1)^3(2x)^1 + \binom{4}{2}(1)^2(2x)^2 + \cdots$ ✓ ✓ ✓ ✓ $= 1 + 8x + 24x^2 + \cdots$	1: Brei uit/*Expand* 1: 1 1: $8x$ 1: $24x^2$ **[4]**
4.1b	$n = \frac{1}{4}$ ✓ ✓ ✓ ✓ $1 + \frac{x}{2} - \frac{3x^2}{8} + \cdots$	1: $n = \frac{1}{4}$ 1: 1 1: $\frac{x}{2}$ 1: $-\frac{3x^2}{8}$ **[4]**
4.1c	$(1 + 8x + 24x^2 + \cdots)\left(1 + \frac{x}{2} - \frac{3x^2}{8} + \cdots\right)$ ✓ $-\frac{3x^2}{8} + 4x^2 + 24x^2$ ✓ ∴Koëffisiënt is/*Coeffisient is* $27\frac{5}{8}$ of/*or* 27,625 ✓	1: Vermenigvuldig/*Multiply* $(1 + 2x)^4\ (1 + 2x)^{\frac{1}{4}}$ 1: Vereenvoudig/*Simplify* 1: Antwoord/*Answer* **[3]**
4.2	Stel/*Set* $n = 1$: LK/*LHS* $= \frac{3}{4}$ RK/*RHS* $= \frac{3}{4}$ ∴ Die bewering is waar as/*The statement is true for* $n = 1$ ✓ Aanvaar die bewering is waar vir /*Accept the statement is true for* $n = k$: $\frac{3}{4} + \frac{3}{2} + 3 + \cdots + \left(\frac{3}{4}\right)(2^{k-1}) = -\left(\frac{3}{4}\right)(1 - 2^k)$ ✓ Beskou nou/*Consider* $n = k + 1$: LK/*LHS* $= -\left(\frac{3}{4}\right)(1 - 2^k)$ ✓ $+ \left(\frac{3}{4}\right)(2^k)$ ✓ $= -\frac{3}{4}(1 - 2^k - 2^k)$ ✓ $= -\left(\frac{3}{4}\right)(1 - 2^{k+1})$ ✓ RK/*RHS* $= -\left(\frac{3}{4}\right)(1 - 2^{k+1})$ ✓ $LK = RK$ en die bewering is dus waar vir $n = k + 1$/ $LHS = RHS$ *and the statement is therefore true for* $n = k + 1$. Volgens die beginsel van wiskundige induksie is die bewering dus waar vir alle $n \in \mathbb{N}$ /*By the principle of mathematical induction the statement is true for all* $n \in \mathbb{N}$ ✓✓	1: Bewys waar vir n = 1/ *Prove true for n = 1* 1: Aanvaar waar vir/ Accept true for $n = k$ 1: Vervang/ *Substitute* 1: $(k + 1)$de term/$(k + 1)$*th term* 2: Vereenoudig LK *Simplify LHS* 1: Rk Vervang/*Substitute* 2: Afleiding/*Deduction* **[9]**

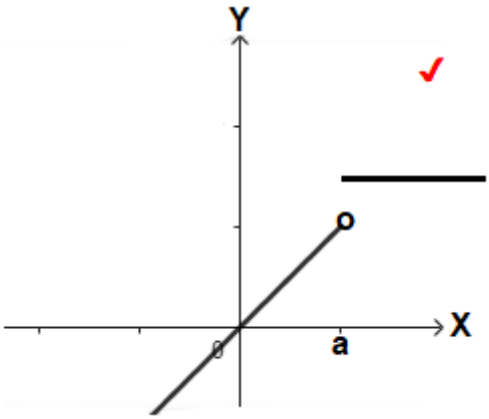
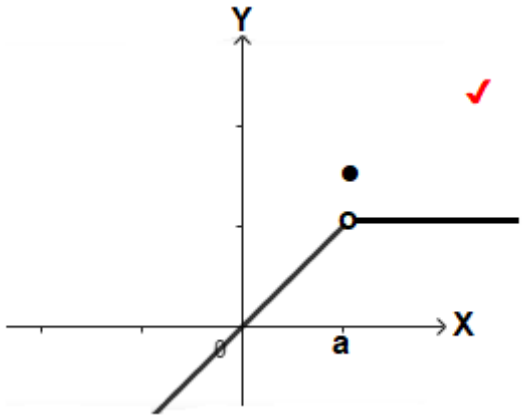
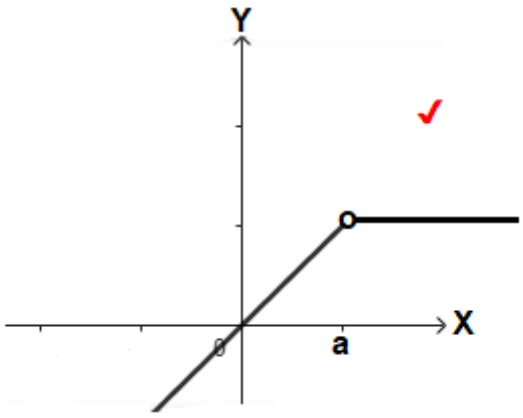
VRAAG/*QUESTION* 5 [17 PUNTE/*MARKS*]

5.1a	$-\frac{\pi}{6} = \text{bgsin}(-2p) + \frac{\pi}{3}$ of/*or* $-\frac{\pi}{6} = \arcsin(-2p) + \frac{\pi}{3}$ ✓ $\therefore \sin\left(-\frac{\pi}{2}\right) = -2p$ ✓ $\therefore p = \frac{1}{2}$ ✓	1: Vervang/ *Substitute* 1: Vereenvoudig/ *Simplify* 1: Antwoord/*Answer* **[3]**
5.1b	$y = \text{bgsin}(0) + \frac{\pi}{3}$ of/*or* $y = \arcsin(0) + \frac{\pi}{3}$ ✓ $\therefore \text{B}\left(0; \frac{\pi}{3}\right)$ ✓	1: Vervang/ *Substitute* 1: Antwoord/*Answer* **[2]**
5.1c	$-1 \leq \frac{x}{2} \leq 1$ ✓ $\therefore -2 \leq x \leq 2$ ✓	1: $-1 \leq \frac{x}{2} \leq 1$ 1: Antwoord/*Answer* **[2]**
5.2a	$\text{P}\hat{\text{O}}\text{Q} = \frac{6}{3} = 2$ ✓ $\therefore \text{P}\hat{\text{O}}\text{T} = 1$ ✓	1: $\text{P}\hat{\text{O}}\text{Q}$ 1: $\text{P}\hat{\text{O}}\text{T}$ **[2]**
5.2b	$\tan \text{P}\hat{\text{O}}\text{T} = \frac{\text{PT}}{3}$ ✓ $\therefore \text{PT} = 4{,}672$ ✓	1: $\tan \text{P}\hat{\text{O}}\text{T}$ 1: Antwoord/*Answer* **[2]**
5.2c	Opp/*Area* $= \frac{1}{2}(3)^2.2$ ✓ $= 9\text{cm}^2$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* **[2]**
5.2d	Opp/*Area* ΔPTO $= 7{,}05\text{cm}^2$ ✓ Opp/*Area* PTQO $= 14{,}10\text{cm}^2$ ✓ Opp ingekleurde gedeelte/*Area shaded part* $= 14{,}10 - 9$ ✓ $= 5{,}10\text{cm}^2$ ✓	1: Opp/*Area* ΔPTO 1: Opp/*Area* PTQO 1: Vereenvoudig/ *Simplify* 1: Antwoord/*Answer* **[4]**

VRAAG/*QUESTION* 6 [12 PUNTE/*MARKS*]

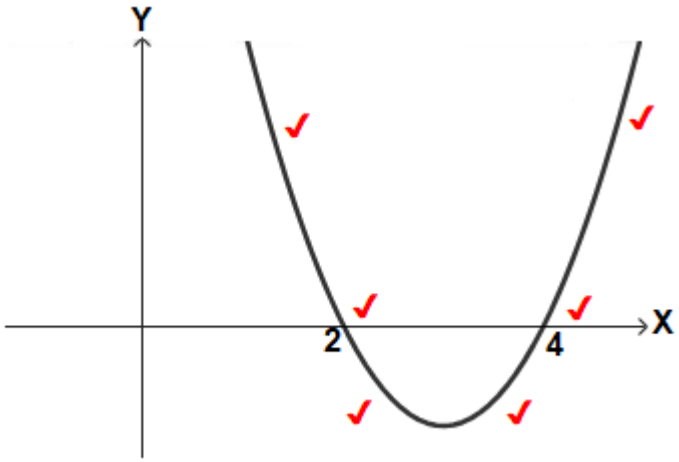
6.1a	$\sqrt{(3-2)^2+(-1-1)^2+(p+2)^2}=3$✓ $\therefore p(p+4)=0$✓ $\therefore p=0$ of/*or* $p=-4$✓	1: Formule/*Formula* 1: Vereenvoudig/ *Simplify* 1: Antwoord/*Answer* **[3]**
6.1b	$\gamma=\text{bgcos}\left(-\frac{2}{3}\right)$ of/*or* $\gamma=\arccos\left(-\frac{2}{3}\right)$✓ $=2{,}30$✓	1: $\text{bgcos}/\arccos$ 1: Antwoord/*Answer* **[2]**
6.1c	$\mathbf{A}\bullet\mathbf{B}=2(3)+1(-1)+(-2)(0)$ ✓ $=5\neq 0$✓ Nee **A** en **B** is nie loodreg op mekaar nie/*No **A** and **B** are not perpendicular*✓	1: Formule/*Formula* 1: $5\neq 0$ 1: Afleiding/*Deduction* **[3]**
6.2	$\begin{vmatrix} i & j & k \\ a & -1 & 3 \\ 2 & 0 & 1 \end{vmatrix}$ ✓ $=i(-1)-j(a-6)+k(2)$ ✓ $\therefore -a+6=5$✓ $\therefore a=1$✓	1: Matriks/*Matrix* 1: Vereenvoudig/ *Simplify* 1: Stel gelyk/*Set equal* 1: Antwoord/*Answer* **[4]**

VRAAG/*QUESTION 7* **[15 PUNTE/*MARKS*]**

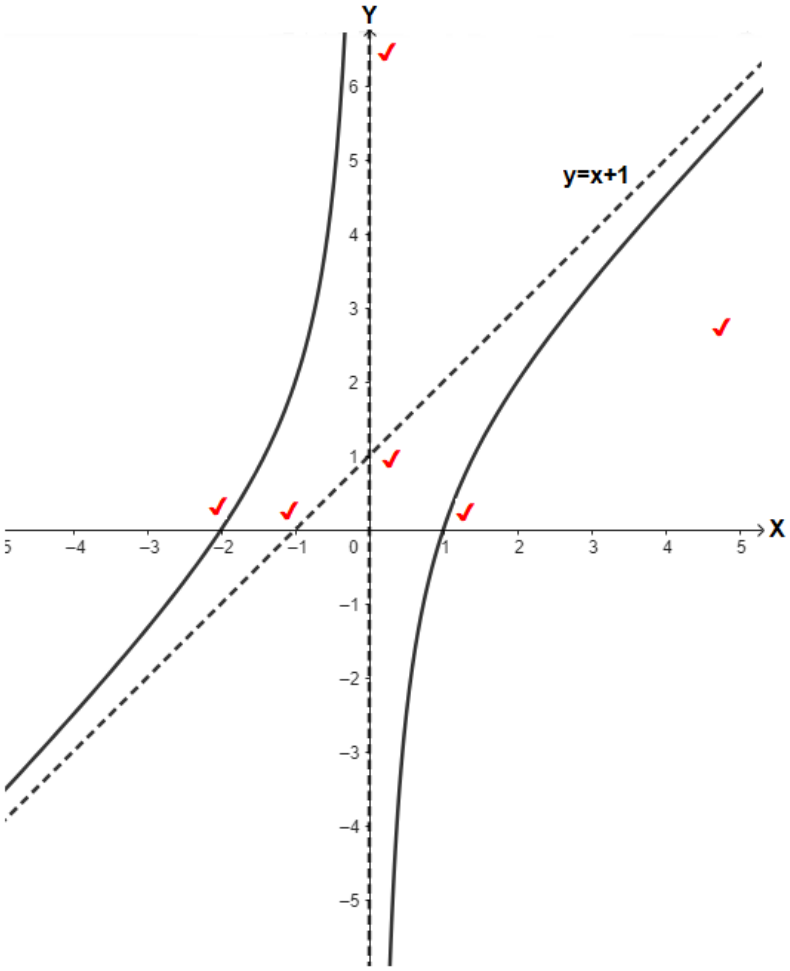
7.1a	$\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ ✓  	1: Voorwaarde/ *Condition* 1: Skets/*Sketch* **[2]**
7.1b	$\lim_{x\to a^-} f(x) \neq f(a)$ ✓  $f(a)$ is ongedefinieerd/is undefined ✓  	1: Voorwaarde/ *Condition* 1: Skets/*Sketch* 1: Voorwaarde/ *Condition* 1: Skets/*Sketch* **[4]**

7.2	$\lim_{x\to 1^-} f'(x) = 6$ ✓ $\lim_{x\to 1^+} f'(x) = -1$ ✓ $\therefore \lim_{x\to 1^-} f'(x) \neq \lim_{x\to 1^+} f'(x)$ ✓ $\therefore f$ is nie differensieerbaar by $x = 1$ nie/ *is not differentiable at* $x = 1$ ✓	1: $\lim_{x\to 1^-} f'(x)$ 1: $\lim_{x\to 1^+} f'(x)$ 1: $\neq$ 1: Afleiding/*Deduction* **[4]**
7.3	$\frac{dy}{dx} = 3\left[\log_5\left(\text{cosec}(2x)\right)\right]^2$ ✓$.\frac{-2$ ✓$\text{cosec}(2x)\cot(2x)$ ✓$}{\ln 5$ ✓$.\text{cosec}(2x)$ ✓}	1: $3\left[\log_5\left(\text{cosec}(2x)\right)\right]^2$ 1: 2 1: $-\text{cosec}(2x)\cot(2x)$ 1: $\ln 5$ 1: $\frac{1}{\text{cosec}(2x)}$ **[5]**

VRAAG/*QUESTION* 8 [19 PUNTE/*MARKS*]

8.1a(i)	$x > 5$✓	1: $x > 5$ **[1]**
8.1a(ii)	$x = 2$✓ en/*and* $x = 4$✓	1: $x = 2$ 1: $x = 4$ **[2]**
8.1a(iii)	$x = 2$✓ en/*and* $x = 5$✓	1: $x = 2$ 1: $x = 5$ **[2]**
8.1b		2: x-afsnitte/ *intercepts at* $x = 2$ en/*and* $x = 4$ 2: $f''(x) > 0$ as/*if* $x < 2$ en/*and* $x > 4$ 2: $f''(x) < 0$ as/*if* $2 < x < 4$ **[6]**
8.2	$\frac{-1}{\sqrt{1-y^2}}$✓ $\frac{dy}{dx}$✓ $= 3x^2$✓$y^3 + x^3$✓$3y^2$✓ $\frac{dy}{dx}$✓ $\therefore \frac{dy}{dx} = \frac{3x^2y^3}{\frac{-1}{\sqrt{1-y^2}} - 3x^3y^2}$✓	1: $\frac{-1}{\sqrt{1-y^2}}$ 1: $\frac{dy}{dx}$ 1: $3x^2$ 1: Produkreël/*Product rule* 1: $3y^2$ 1: $\frac{dy}{dx}$ 1: $3x^2y^3$ 1: $\frac{3x^2y^3}{\frac{-1}{\sqrt{1-y^2}}-3x^3y^2}$ **[8]**

VRAAG/*QUESTION* 9 [19 PUNTE/*MARKS*]

9.1a	Vertikaal/*Vertical*✓: $x = 0$✓ Skuins/*Oblique*✓: $y = x + 1$✓	1: Vertikaal/*Vertical* 1: $x = 0$ 1: Skuins/*Oblique* 1: $y = x + 1$ **[4]**
9.1b		2: x-afsnitte/ *intercepts* $x = -2$ en/*and* $x = 1$ 2: $y = x + 1$ 1: $x = 0$ 1: Vorm/*Form* **[6]**
9.2a	✓ ✓ ✓ $h'(x) = 3^{-2x}(\ln 3)(-2) + \pi$✓ $= 0$	1: 3^{-2x} 1: $\ln 3$ 1: -2 1: π **[4]**
9.2b	$h''(x) = 3^{-2x}(\ln 3)(-2)(\ln 3)(-2)$✓✓ $\therefore x_{n+1} = x_n - \frac{3^{-2x_n}(\ln 3)(-2) + \pi}{4(\ln 3)^2 . 3^{-2x_n}}$✓ $\therefore x \approx -0{,}1627$✓✓	2: $h''(x)$ 1: Newton 2: Antwoord/*Answer* **[5]**

VRAAG/*QUESTION* 10 [16 PUNTE/*MARKS*]

10.1a	$= \frac{1}{6}$✓$\tan^6 x$✓ $+ k$	1: $\frac{1}{6}$ 1: $\tan^6 x$ **[2]**
10.1b	$= \frac{e}{3}$✓$\ln(1+3x)$✓ $+ k$	1: $\frac{e}{3}$ 1: $\ln(1+3x)$ **[2]**
10.2	Stel/*Let* $f(x) = 5x - 3$ ✓en/*and* $g'(x) = e^{-2x}$✓ $\therefore f'(x) = 5$ en/*and* $g(x) = -\frac{1}{2}e^{-2x}$ $\therefore \int (5x-3)\mathrm{e}^{-2x}\,dx$ $= -\frac{1}{2}e^{-2x}(5x-3)$✓ $- 5.\left(-\frac{1}{2}\right)\int e^{-2x}dx$✓ $= -\frac{1}{2}e^{-2x}(5x-3) + \frac{5}{2}\left(-\frac{1}{2}\right)e^{-2x} + k$✓	1: $f(x)$ 1: $g'(x)$ 1: $-\frac{1}{2}e^{-2x}(5x-3)$ 1: $-5.\left(-\frac{1}{2}\right)\int e^{-2x}dx$ 1: Antwoord/*Answer* **[5]**
10.3a	$\frac{4x^2-3x+1}{3x(1+4x^2)} = \frac{\mathrm{A}}{3x} + \frac{\mathrm{B}x+\mathrm{C}}{1+4x^2}$✓ $4x^2 - 3x + 1 = \mathrm{A}(1+4x^2) + 3x(\mathrm{B}x+\mathrm{C})$ $\mathrm{A} = 1$✓, $\mathrm{B} = 0$✓, $\mathrm{C} = -1$✓ $\therefore \frac{4x^2-3x+1}{3x(1+4x^2)} = \frac{1}{3x} - \frac{1}{1+4x^2}$	1: Brei uit/*Expand* 1: A 1: B 1: C **[4]**
10.3b	$\int \frac{1}{3x}dx - \int \frac{1}{1+4x^2}dx$ $= \frac{1}{3}\ln x$✓ $- \frac{1}{2}$✓bgtan$(2x)$✓ $+ k$ of/*or* $\frac{1}{3}\ln x - \frac{1}{2}\arctan(2x) + c$	1: $\frac{1}{3}\ln x$ 1: $\frac{1}{2}$ 1: bgtan$(2x)$ of/*or* $\arctan(2x)$ **[3]**

VRAAG/*QUESTION* 11 [16 PUNTE/*MARKS*]

11.1	$\Delta x_i = \frac{3}{n}$ ✓ $\quad x_i = 1 + \frac{3i}{n}$ ✓ $f(x_i) = -\left(1 + \frac{3i}{n}\right) + \left(1 + \frac{3i}{n}\right)^2 = \frac{3i}{n} + \frac{9i^2}{n^2}$ ✓ $f(x_i).\Delta x_i = \frac{9i}{n^2} + \frac{27i^2}{n^3}$ ✓ $\therefore \sum_{i=1}^{n} f(x_i).\Delta x_i = \frac{9}{n^2}\sum_{i=1}^{n} i + \frac{27}{n^3}\sum_{i=1}^{n} i^2$ ✓ $= \frac{9}{n^2}\left(\frac{n^2}{2} + \frac{n}{2}\right)$ ✓ $+ \frac{27}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$ ✓ $= \frac{9}{2} + \frac{9}{2n} + 9 + \frac{27}{2n} + \frac{9}{2n^2}$ ✓ $\therefore \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i).\Delta x_i) = \frac{27}{2}$ ✓ $\int_1^4 (-x + x^2)dx = \frac{27}{2}$ **OF/*OR*** $\Delta x_i = \frac{3}{n}$ ✓ $\quad ; \quad x_i = 1 + \frac{3i}{n}$ ✓ $f(x_i) = -\left(1 + \frac{3i}{n}\right) + \left(1 + \frac{3i}{n}\right)^2 = \frac{3i}{n} + \frac{9i^2}{n^2}$ ✓ $\sum_{i=1}^{n}\left(\frac{3i}{n} + \frac{9i^2}{n^2}\right) = \frac{3}{n}\sum_{i=1}^{n} i + \frac{9}{n^2}\sum_{i=1}^{n} i^2$ ✓ $= \frac{3}{n}\left(\frac{n^2}{2} + \frac{n}{2}\right)$ ✓ $+ \frac{9}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$ ✓ $= \frac{3n}{2} + \frac{3}{2} + 3n + \frac{9}{2} + \frac{3}{2n}$ $= \frac{9n}{2} + 6 + \frac{3}{2n^2}$ ✓ $\int_1^4 (-x + x^2)dx$ = basis x hoogte $= \frac{3}{n} \times \left(\frac{9n}{2} + 6 + \frac{3}{2n^2}\right) = \frac{27}{2} + \frac{18}{n} + \frac{9}{2n^2}$ ✓ $\int_1^4 (-x + x^2)dx = \lim_{n\to\infty}\left(\frac{27}{2} + \frac{18}{n} + \frac{9}{2n^2}\right) = \frac{27}{2}$ ✓ $\therefore \int_1^4 (-x + x^2)dx = \frac{27}{2}$	1: Δx_i 1: x_i 1: $f(x_i)$ 1: $f(x_i).\Delta x_i$ 1: Sigma 1: Vervang/ *Substitute* i^2 1: Vervang/ *Substitute* i 1: Vereenvoudig/ *Simplify* 1: Antwoord/ *Answer* **[9]**

11.2	Opp van $\frac{1}{4}$sirkel/*Area of* $\frac{1}{4}$*circle* $= \frac{\pi.3^2}{4}$ ✓ $= 7{,}06\ldots$ ✓ $\int_0^1 (x^2 - 4x + 3)\,dx = \left[\frac{x^3}{3} - 2x^2 + 3x\right]\Big\|_0^1$ ✓ $= \frac{1}{3} - 2 + 3 - 0$ ✓ $= 1,\dot{3}$ ✓ ∴Opp ingekleurde deel/ *Area shaded part* $= 7{,}06\ldots - 1,\dot{3}$ ✓ $= 5{,}74$ ✓	1: Formule/*Formula* 1: Antwoord/*Answer* 1: Integreer/*Integrate* 1: Vervang/*Substitute* 1: Antwoord/*Answer* 1: Vereenvoudig/*Simplify* 1: Antwoord/*Answer* **[7]**