

α MATHEMATICS

Alpha Mathematics PRELIMINARY EXAM

September 2024
Grade 12

Time: 3 hours
Total: 200 marks

INSTRUCTIONS AND INFORMATION

Please read the following instructions carefully before answering any questions:

1. Write your name on the answer sheet.
2. Answer all 11 questions of this paper.
3. This question paper consists of nine pages, a formula sheet of three pages and an answer sheet of three pages.
4. Question 1 consists of 15 multiple choice questions. Answer this on the answer sheet.
5. Non-programmable calculators may be used unless otherwise indicated.
6. Unless indicated otherwise, all answers, where applicable, must be given correctly to **two decimal places**.
7. The diagrams in the question paper are not necessarily drawn to scale.
8. **All angles are given in radians.** Answers must be given in radians where applicable.
9. All necessary calculations must be shown. The correct answer on its own will not necessarily lead to full marks.
10. Write neatly and legibly.

QUESTION 1 [30 MARKS]

- The following questions have only ONE correct answer.
- Only mark the correct letter, A, B, C or D, with an X (cross) on the answer sheet provided.
- Each question counts 2 marks.

$$1.1 \quad \frac{(3\text{cis}\frac{\pi}{4})(2\text{cis}\frac{-\pi}{2})}{4\text{cis}(\frac{-3\pi}{4})} =$$

(A) $-\frac{2}{3}$

(B) $-\frac{3}{2}$

(C) $\frac{2i}{3}$

(D) $\frac{3i}{2}$

1.2 If $f(x) = \ln|x + 1|$ is reflected around the x-axis and shifted 3 units to the right, what will be the equation of the new graph, g ?

(A) $g(x) = \ln|x - 2|$

(B) $g(x) = \ln\left|\frac{1}{x-2}\right|$

(C) $g(x) = \ln|x + 4|$

(D) $g(x) = \ln\left|\frac{1}{x+4}\right|$

1.3 When will the expansion of $\frac{1}{3-x}$ converge?

(A) $|x| < 3$

(B) $|x| < \frac{1}{3}$

(C) $|x| > 3$

(D) $|x| > \frac{1}{3}$

1.4 The asymptotes of $y = 2\arctan\left(x - \frac{\pi}{4}\right)$ are

(A) $y = \pm\frac{\pi}{2}$

(B) $y = \frac{\pi}{4}$ and $y = -\frac{3\pi}{4}$

(C) $y = \pm\pi$

(D) $y = \frac{-\pi}{4}$ and $y = \frac{3\pi}{4}$

1.5 If \mathbf{a} , \mathbf{b} , \mathbf{c} and \mathbf{d} are vectors, which one of the following calculations is not possible?

(A) $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{d})$

(B) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

(C) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

(D) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

1.6 The solution of $2 - |-x + 1| < 6$ is

(A) $-8 < x < 4$

(B) $x \in \mathbb{R}$

(C) $-4 < x < 8$

(D) No solution

1.7 Which of the following will be a tangent of $y = \frac{1}{\sqrt{x}}$ at $x = 4$?

(A) $y = \frac{x}{4} + \frac{1}{4}$

(B) $y = -\frac{x}{4}$

(C) $y = \frac{x}{2}$

(D) $y = -\frac{x}{16} + \frac{3}{4}$

1.8 Given: $f(x) = \begin{cases} \frac{(x+2)(x-2)}{x+2} & \text{if } x < 2 \\ c & \text{if } x = 2 \\ -x^2 + 4 & \text{if } x > 2 \end{cases}$

For which value of c will f be continuous?

(A) $c = 2$

(B) $c = -2$

(C) $c = 0$

(D) $c = 4$

1.9 The function $y = f(x)$ has a maximum at $x = 1$. Which of the following statements is true?

(A) $f'(1) > 0$

(B) $f''(1) > 0$

(C) $f'(1) < 0$

(D) $f''(1) < 0$

1.10 The area under the graph of $y = \cos(3x)$ rotates around the x -axis. Which of the following can be used to determine the volume of the rotating body?

(A) $\frac{\pi}{2} \int [1 + \cos(3x)] dx$

(B) $\frac{\pi}{2} \int [1 + \cos(6x)] dx$

(C) $\int [1 + \cos(3x)] dx$

(D) $\int \cos(3x) dx$

1.11 If $F(x) = (f \circ g)(x) = \sqrt{9x^2 + 9} - x^2 - 1$, then

(A) $f(x) = \sqrt{9x^2 + 9}$ and $g(x) = x^2 - 1$

(B) $f(x) = 3\sqrt{x} - x$ and $g(x) = x^2 + 1$

(C) $f(x) = x^2 - 1$ and $g(x) = 9x$

(D) Not one of the above

1.12 The system of equations $2x + y - 3 = 0$ and $-x = 3y - 4$ can be solved with Cramer's rule. To calculate y , which one of the following should be used?

(A) $\frac{\begin{vmatrix} 2 & 3 \\ -1 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ -1 & -3 \end{vmatrix}}$

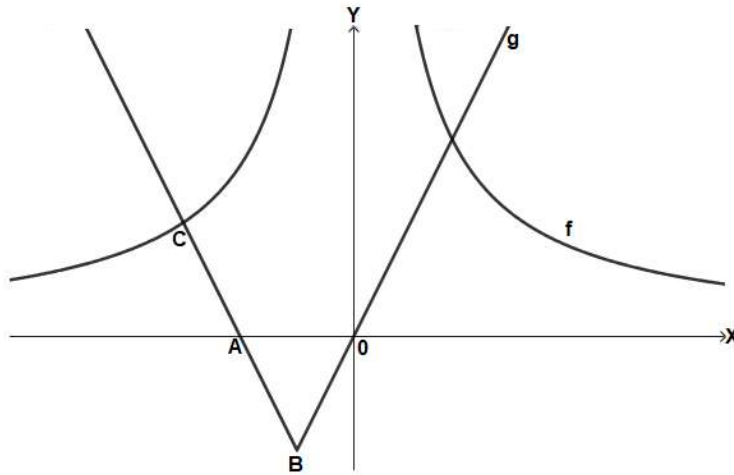
(B) $\frac{\begin{vmatrix} 3 & 1 \\ -4 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ -1 & -3 \end{vmatrix}}$

(C) $\frac{\begin{vmatrix} 2 & 1 \\ -1 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ -1 & -4 \end{vmatrix}}$

(D) $\frac{\begin{vmatrix} 2 & 1 \\ -1 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ -4 & -3 \end{vmatrix}}$

QUESTION 3 [20 MARKS]

3.1 The graphs of $f(x) = \left|\frac{k}{x}\right|$ and $g(x) = 2|x + 1| - 2$ are given below.



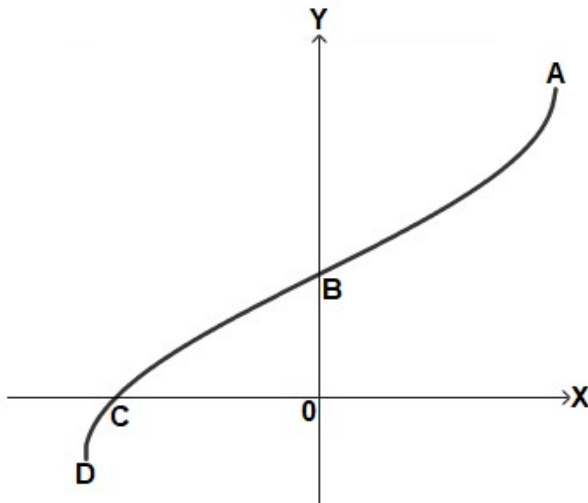
- a) Determine the value of k if $(-6; 1)$ is a point on f . (2)
 - b) Determine the coordinates of A, one of the x -intercepts of g . (3)
 - c) Determine the coordinates of B, the salient point of g . (2)
 - d) Determine x if $g(x) - 4 < 0$. (3)
 - e) Determine the coordinates of C, one of the intersects of f and g . (5)
- 3.2 The function $h(x) = x^3 - 2x^2 - 2x + p$ has zeros $x = -\sqrt{2}$ and $x = 2$. Determine the value of p . (5)

QUESTION 4 [20 MARKS]

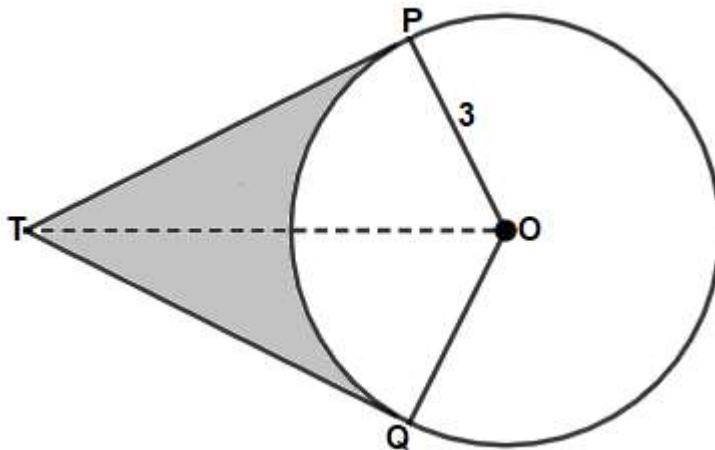
- 4.1
 - a) Determine the first three terms of $(1 + 2x)^4$. (4)
 - b) Determine the first three terms of $\sqrt[4]{1 + 2x}$. (4)
 - c) Hence, determine the coefficient of x^2 in $(\sqrt[4]{1 + 2x})(1 + 2x)^4$. (3)
- 4.2 Use mathematical induction to prove that the following statement is true for all $n \in \mathbb{N}$: $\frac{3}{4} + \frac{3}{2} + 3 + \dots + \left(\frac{3}{4}\right)(2^{n-1}) = -\left(\frac{3}{4}\right)(1 - 2^n)$. (9)

QUESTION 5 [17 MARKS]

- 5.1 The graph of $g(x) = \arcsin(px) + \frac{\pi}{3}$ is given below. The graph intersects the x - and y -axis at C and B respectively. The end points of the graph are at $D(-2; -\frac{\pi}{6})$ and A.



- Show that $p = \frac{1}{2}$. (3)
 - Determine the coordinates of B, the y -intercept of g . (2)
 - Determine the domain of g . (2)
- 5.2 In the diagram below, O is the center of a circle with a radius of 3 cm. P and Q are points on the circumference and the tangents PT and QT intersect at T.



- Calculate the size of \hat{POT} , if the arc length of $PQ=6\text{cm}$. (2)
- Determine the length of PT .
Give your answer correct to three decimal places. (2)
- Calculate the area of sector POQ . (2)
- Determine the area of the shaded part. Use $PT=4,7\text{cm}$. (4)

QUESTION 6 [12 MARKS]

6.1 Given: Vectors $\mathbf{A} = (2; 1; -2)$ and $\mathbf{B} = (3; -1; p)$

- a) The distance between \mathbf{A} and \mathbf{B} is 3 units. One possible value of p is 0. Calculate the other possible value of p . (3)
- b) Determine the angle between the vector \mathbf{A} and the z -axis if $|\mathbf{A}| = 3$. (2)
- c) Are \mathbf{A} and \mathbf{B} perpendicular to each other? Use $p = 0$ and motivate your answer. (3)

6.2 Vector $-i + 5j + 2k$ is perpendicular to the vectors $\mathbf{p} = ai - j + 3k$ and $\mathbf{q} = 2i + k$. Determine the value of a . (4)

QUESTION 7 [15 MARKS]

7.1 When will a function f have the following discontinuity at $x = a$? Illustrate your answer with a sketch.

- a) Jump discontinuity (2)
- b) Removable discontinuity (Name two conditions) (4)

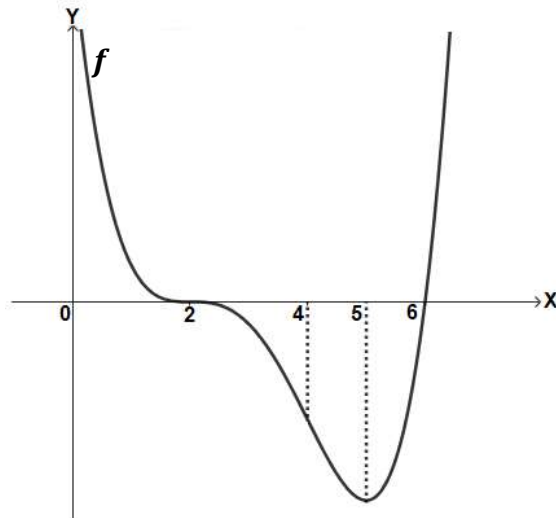
7.2 Given: $f(x) = \begin{cases} 2x^3 - 1 & \text{as } x < 1 \\ 1 & \text{as } x = 1 \\ -x + 2 & \text{as } x > 1 \end{cases}$

The function f is continuous at $x = 1$. Determine algebraically if f is differentiable at $x = 1$. (4)

7.3 Determine $\frac{dy}{dx}$ if $y = [\log_5(\operatorname{cosec}(2x))]^3$ (5)

QUESTION 8 [19 MARKS]

- 8.1 The graph of function f is given below. The function has stationary points at $x = 2$ en $x = 5$ and inflection points at $x = 2$ and $x = 4$. The x -intercepts are at $x = 2$ en $x = 6$.



- a) For which x -value(s) will (5)
- (i) $f'(x) > 0$
 - (ii) f' have a stationary point
 - (iii) f' have an x -intercept
- b) Use the answer sheet and sketch a possible graph of f'' . (6)
- 8.2 Use implicit differentiation to determine $\frac{dy}{dx}$ if $\arccos(y) = x^3y^3$ (8)

QUESTION 9 [19 MARKS]

- 9.1 Given: $f(x) = \frac{x^2+x-2}{x}$
- a) Determine the asymptotes of the function and classify the asymptotes. (4)
 - b) It is given that $f(-2) = f(1) = 0$. The function increases for all $x \in \mathbb{R}$ and does not have any stationary points. Use the answer sheet and draw a clear sketch of f . (6)
- 9.2 Given: $h(x) = 3^{-2x} + \pi x$
- a) The function h has a minimum in the interval $[-1; 0]$. Determine an equation with which the x -coordinate of this stationary point can be calculated. (4)
 - b) Use **Newton's method** and determine this x -value accurately to four decimal places. Use $x = -0,2$ as a first approximation. Clearly show how you use Newton's method. (5)

QUESTION 10 [16 MARKS]

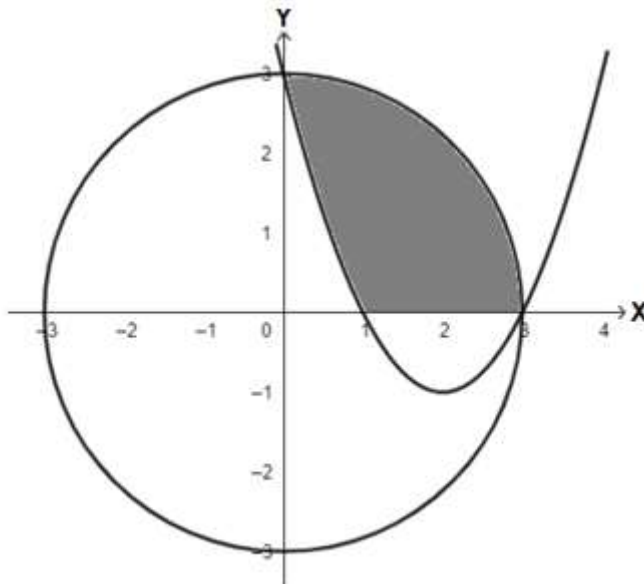
10.1 Determine the following integrals:

a) $\int \sec^2 x \cdot \tan^5 x \, dx$ (2)

b) $\int \frac{e}{1+3x} \, dx$ (2)

10.2 Use integration by parts and determine $\int (5x - 3)e^{-2x} \, dx$ (5)10.3 a) Decompose $\frac{4x^2-3x+1}{3x(1+4x^2)}$ into partial fractions. (4)

b) Hence, determine $\int \frac{4x^2-3x+1}{3x(1+4x^2)} \, dx$ (3)

QUESTION 11 [16 MARKS]11.1 Use a Riemann sum to calculate the value of $\int_1^4 (-x + x^2) \, dx$ (9)11.2 The graphs of $x^2 + y^2 = 9$ and $y = x^2 - 4x + 3$ is given below.

The graphs intersect at $(0; 3)$ and $(3; 0)$. The function of g intersects the x -axis at $x = 1$ and $x = 3$. Determine the area of the shaded area. Give your answer as a decimal number.

(7)

Total: 200