

Alpha Mathematics
Grade 12 Exam Paper
October 23 2015

Total: 200 marks

Time: 3 hours

INSTRUCTIONS

1. Number the answers exactly the way the questions are numbered.
2. Work must be neat and legible.
3. Non-programmable calculators may be used.
4. The diagrams in the question paper are not necessarily drawn to scale.
5. All angles are given in radians. Answers must also be given in radians if necessary.
6. Unless otherwise indicated, all answers, where necessary, must be given correct to two decimal figures.
7. This question paper consists of 8 pages and 3 formula sheets.
8. Section A Question 1 consists of 10 multiple choice questions. Answer this on the answering sheet.
9. In Section B, all necessary calculations must be shown clearly.

Section A

Answer this question on the answering sheet by making X on A, B, C of D.

QUESTION 1

1.1 Given:

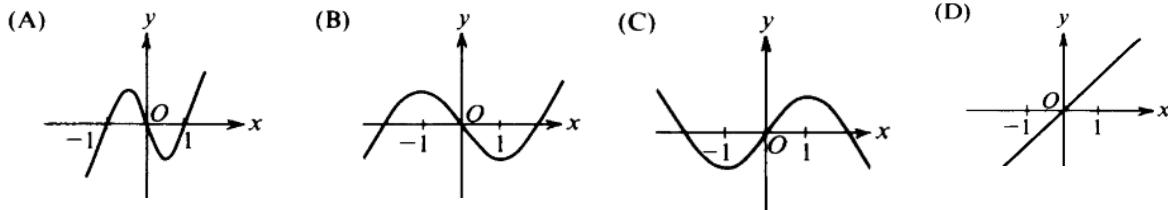
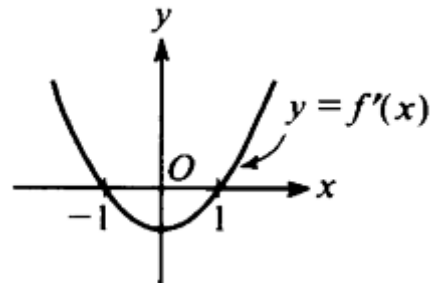
$$\lim_{x \rightarrow a} f(x) = f(a)$$

This means that at $x = a$ the function

- (A) is always continuous. (B) is always differentiable.
 (C) has a removable discontinuity. (D) has a jump discontinuity.

1.2 The sketch on the right shows the graph of $y = f'(x)$, the derivative of $y = f(x)$.

Which of the following can be a sketch of $y = f(x)$?



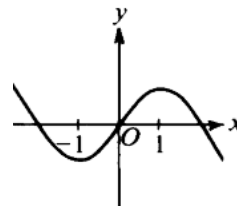
1.3 For which values of x will the function $f(x) = x^2 - 4x - 5$ be decreasing?

- A $x < -1$ or $x > 5$ B $-1 < x < 5$
 C $x < 2$ D $x > 2$

1.4 The graph of $y = f(x)$ is shown on the right.

Then $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$ at $x =$

- (A) -1 (B) 0 (C) 1 (D)



1.5 The graph of $\frac{(x-1)(x^2-5x+6)}{(x-1)(x+2)} = f(x)$ will have at $x = 1$

- (A) a x -intercept (B) a vertical asymptote
 (C) a turning point (D) an undefined point

1.6 If $|x| = 2x - 1$, then $x =$

- (A) $\frac{1}{3}$ (B) 1 (C) 1 of $\frac{1}{3}$ (D) no solution.

1.7 The inverse function of $f(x) = \sin(2x)$ will be defined if the domain of f is restricted to $x \in$

- (A) $[-\frac{\pi}{4}; \frac{\pi}{4}]$ (B) $[-\frac{\pi}{2}; \frac{\pi}{2}]$ (C) $[-\pi; \pi]$ (D) $[-2\pi; 2\pi]$

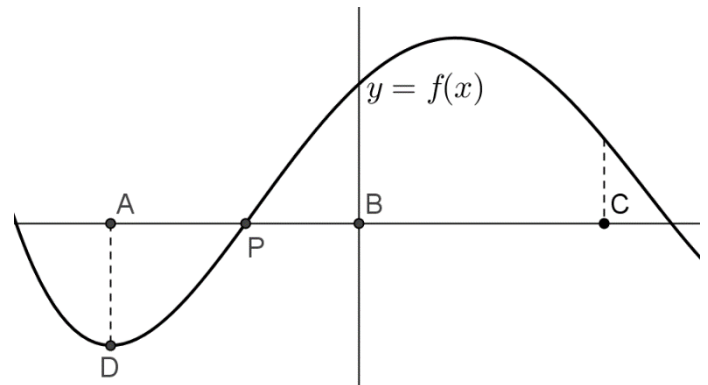
1.8 If a tangent is drawn to $y = e^x$ at any point $(x; y)$, then the gradient of this tangent will be:

- (A) 1 (B) 0 (C) y (D) x

1.9 The graph of $y = f(x)$ is shown.
D is the minimum turning point of f .

Newton's method must be used to find the x -intercept at P. As starting place to find this x -value, the following point(s) can be used:

- (A) A (B) B
(C) C (D) A, B or C.



1.10 If $y = \ln\left(\frac{1}{x}\right)$, then $\frac{dy}{dx} =$

- (A) $\frac{1}{x}$ (B) x (C) $-\frac{1}{x^2}$ (D) $-\frac{1}{x}$

[20]

Section B**QUESTION 2**

- 2.1 When a camera takes a photo with a flash, the charge on the battery can be determined with the equation $L(t) = m(1 - e^{-\frac{t}{a}})$, where m is the maximum capacity of the battery and t the time it takes in seconds. After 4,6 seconds the charge falls to 90% of the original charge., thus $\frac{L(t)}{m} = 0,9$. Calculate the value of a . Give the answer as a whole number. (4)
- 2.2 Given: $f(x) = |\ln(x + 1) - 1|$
- (a) Determine the x - and y -intercepts of f as well as the equation of the asymptote. Give the answers correct to one decimal digit if necessary. (7)
- (b) Make a sketch of the graph of $y = f(x)$ and show the information obtained from **QUESTION 2.2 (a)** clearly. (5)
- 2.3 $x^4 - 6x^3 + 5x^2 + 10x + 2$ has a zero at $x = 2 + \sqrt{5}$, factorize the expression fully in $\mathbb{Q}[x]$. (7)
- [23]**

QUESTION 3

- 3.1
- (a) Determine and simplify the first two terms of $(2x - x^2)^6$ (4)
- (b) Determine and simplify the first three terms of $\sqrt{1-x}$ (6)
- (c) Hence determine the coefficient of x^7 in the expansion of $\sqrt{1-x} \cdot (2x - x^2)^6$ (3)
- 3.2 Use mathematical induction and prove that
- $$6 + 3 + \frac{3}{2} + \dots + 6 \left(\frac{1}{2}\right)^{n-1} = 12\left(1 - \left(\frac{1}{2}\right)^n\right) \quad (11)$$
- [24]**

QUESTION 4

4.1 The following system of equations are given:

$$(a + 1)x - y = 11 \quad \text{and} \quad ax + y = 9$$

(a) Write the system in matrix form. (3)

(b) Use Cramer's method and determine the value of x in terms of a . Show clearly the matrixes that you use. (8)

(c) Hence determine the value of a if it is given that $x = 4$. (3)

4.2 Use de Moivre's theorem and show that $(-1 + i)^8$ is real. The angles must be in terms of π and the calculations must be done in polar form. (6)

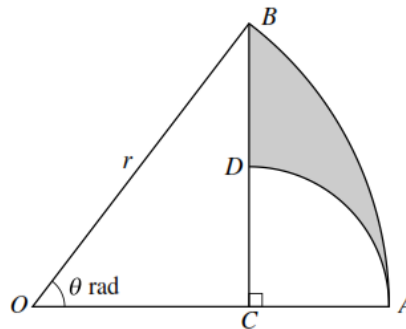
[20]

QUESTION 5

5.1 The diagram shows a sector OAB of a circle with radius r .

Angle AOB is θ radians. The point C on OA is such that $BC \perp OCA$.

The point D is on BC and the centre of the arc AD is C .



(a) Show that $BD = r \cos \theta + r \sin \theta - r$ (5)

(b) Determine the circumference of the shaded area if $\theta = 1$ and $r = 12$. (4)

5.2

(a) Determine the inverse function of $f(x) = 2 \sin \left(x - \frac{\pi}{3} \right) + 1$ (4)

(b) Determine the equations of the asymptotes of $y = \arctan x - \frac{\pi}{4}$ (3)

[16]

QUESTION 6

6.1 Use differentiation rules and differentiate. It is not necessary to simplify the answer.

(a) $f(x) = \arcsin(e^x)$ (3)

(b) $y = (x^4 - 2x)^{10} \times \cot(2x)$ (5)

6.2 Use implicit differentiation and determine $\frac{dy}{dx}$ if $\sin\left(\frac{x}{y}\right) = y^2 - \pi$ (7)

6.3 $f(x) = \sqrt{1-x^2} + x \cdot \arcsin x$

(a) Show that $f'(x) = \arcsin x$ (6)

(b) Hence determine $\int \arcsin(2x) dx$ (3)

[24]**QUESTION 7**

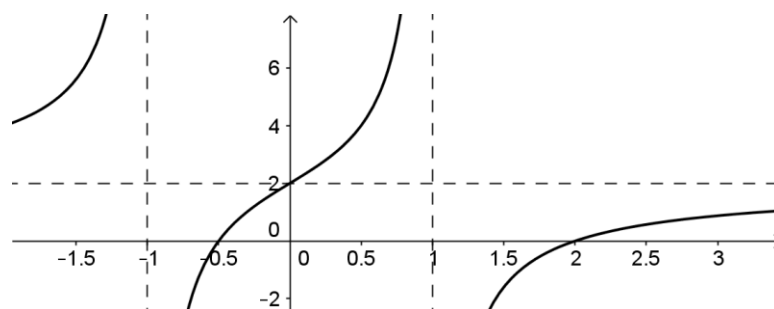
7.1 $f(x) = \begin{cases} x^2 + 1 & \text{as } x \leq 0 \\ \cos x & \text{as } 0 \leq x \leq 1 \\ \ln x & \text{as } x > 1 \end{cases}$

(a) Write down the x -value where f has a jump discontinuity and motivate your answer algebraically. (3)

(b) Is f differentiable in this point? Motivate your answer. (2)

7.2 The function $f(x) = x^3 + ax^2 + bx$ has a point of inflection that is also a stationary point at $x = -1$. Determine the values of a and b . (6)

7.3 The sketch shows the graph of $f(x) = \frac{(ax+b)(x+c)}{x^2-d}$.
 f has vertical asymptotes at $x = \pm 1$ and a horizontal asymptote at $y = 2$.
 f has x -intercepts at $-0,5$ and 2 .



Give the values of a, b, c and d . (4)

[15]

QUESTION 8

Determine the following integrals:

8.1 $\int (\tan^2 x + \sec x \cdot \tan x) dx$ (4)

8.2 $\int \frac{1}{\sqrt{9-16x^2}} dx$ (5)

8.3 $\int \frac{2}{(x-1)(2x-1)} dx$ by decomposing $\frac{2}{(x-1)(2x-1)}$ into partial fractions. (8)

8.4 Use factor (integration by parts) integration and determine $\int x \cdot e^x dx$ (6)

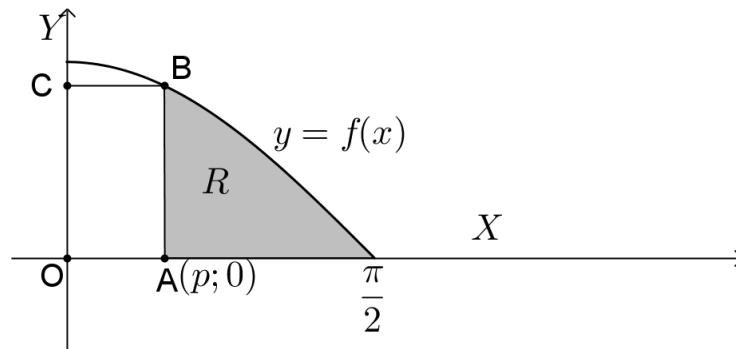
[23]**QUESTION 9**

9.1 The sketch shows the graph of $y = \cos x$, for $0 \leq x \leq \frac{\pi}{2}$

A rectangle $OABC$ is drawn with $A(p; 0)$ and B a point on the graph.

A and C are points on the axes, as shown, and O is the origin.

The shaded area R is included by the graph and the lines $x = p$ and $y = 0$.



(a) Determine the area of R in terms of p . (4)

(b) This area R is equal to the area of the rectangle $OABC$.

Show that $p = \frac{1-\sin p}{\cos p}$ (2)

9.2 Use Newton's method to determine the zero of $f(x) = \tan(2x) + x - 1$ between $x = 0$ and $x = 1$. Use $x = 0,5$ as initial value.

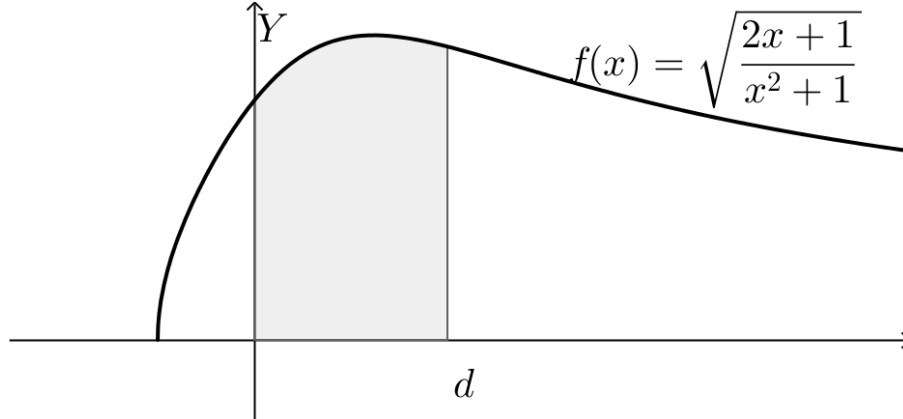
Give the answer correct to 5 decimal digits and show clearly how you used Newton's method. (7)

[16]

QUESTION 10

10.1 Use a Riemann sum and determine the value of $\int_0^2 (2x - x^2) dx$ (12)

10.2 The graph of $f(x) = \sqrt{\frac{2x+1}{x^2+1}}$ is shown. The area between the lines $x = 0$ and $x = d$ is shaded.



This area rotates around the x -axis. Determine the volume of the solid of revolution in terms of d .

(10)

[22]

TOTAL: 200