α-MATHEMATICS

Grade 10 Alpha Mathematics

Final Examination 2023

Examiner: Lanice Liebenberg **Time: 2 hours**

Moderator: Rika Grobler **Total: 120**

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 8 pages and an answer sheet.
2. Answer ALL 8 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 **[20 marks]**

This question must be answered **on the answer sheet**.
Every question has **ONLY** one correct answer for TWO marks each. Mark the correct answer with an **X** on the answer sheet.

1.1 $\int a^2x^3\,da =$

A $\frac{x^3a}{1} + c$

B $\frac{x^3a^3}{3} + c$

C $\frac{x^3a^2}{3} + c$

D $\frac{a^2x^4}{4} + c$

1.2 One radian is:

A $180°$

B The angle formed when the arc length of a circle is equal to the radius.

C The arc length when the angle formed in a circle is equal to the radius.

D The radius when the arc length of a circle is equal to the angle formed.

1.3 If $x^2 + 64 = 0$, then:

A $x = \pm 8$

B $x = \pm 64i$

C $x = \pm 8i$

D No solution

1.4 If $x^3 - x^2 + 9x - 9 = 0$ the roots of the equation are:

A $x = \pm 9$; $x = 1$

B $x = \pm 3i$; $x = 1$

C $x = 0$; $x = 9$; $x = -1$

D $x = \pm 3i$; $x = -1$

1.5 Given $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$; $A^T =$

A $\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{bmatrix}$

B $\begin{bmatrix} a_{12} & a_{11} \\ a_{22} & a_{21} \\ a_{32} & a_{31} \end{bmatrix}$

C $\begin{bmatrix} a_{31} & a_{21} & a_{11} \\ a_{32} & a_{22} & a_{12} \end{bmatrix}$

D The matrix cannot be transposed.

1.6 Given

$$\frac{x-1}{x^2} = \frac{A}{x} + \frac{B}{x^2}$$

Then the values of A and B are:

A $A = 1$; $B = -1$

B $A = 2$; $B = -2$

C $A = -1$; $B = 1$

D $A = -2$; $B = -2$

1.7 The conjugate of z if $z = -i + 3$ is:

A $i = 3$

B $3 + i$

C $3 - i$

D $i = -3$

1.8 The number of rows and columns in the matrix are:

$$M = \begin{bmatrix} m_{11} & m_{12} & \dots & m_{1k} \\ m_{21} & m_{22} & \dots & m_{2k} \\ \vdots & \vdots & \dots & \vdots \\ m_{j1} & m_{j2} & \dots & m_{jk} \end{bmatrix}$$

A 3 rows and 3 columns

B j rows and k columns

C Infinitely many rows and columns

D k rows and j columns

1.9 Given vectors AB and CD. In which of the following statements will the vectors be equal?

A $|AB| = |CD|$, AB is in a southern direction and CD west.

B AB is 30m north and CD is 3m north.

C AB is 15km south east and CD is 15km north east.

D $|AB| = |CD|$, AB and CD are both in a southern direction.

1.10 $\begin{vmatrix} 1 & 2 \\ -3 & -4 \end{vmatrix} =$

A -10

B -2

C 2

D 10

Question 2 **[16 marks]**

Given $a = 2 - 3i$ and $b = 4 + 6i$.
Determine:

2.1 $a + b$ (2)

2.2 $2a.3b$ (4)

2.3 $i^{51}.a^2b$ (5)

2.4 $\frac{a}{b}$ (5)

Question 3 **[12 marks]**

Decompose the following into partial fractions:

$$\frac{-8x - 20}{(x^2 - 4)(x + 1)}$$

Question 4 **[11 marks]**

The following equations are given.

$$2x - y + 3z = 6$$
$$4x - 2y + z = 1$$
$$x + y + z = 3$$

Write the system of equations as a matrix and use Cramer's rule to find the values of x.

Question 5 **[18 marks]**

5.1 Given the vectors $a = (3; 8)$ and $b = (1; -1)$
Determine:

5.1.1 The unit vector of a. (3)

5.1.2 The angle between a and b. (5)

5.2 Given the vectors $v = (-14; -14)$ and $u = (2; -2)$.

5.2.1 Determine $v.u$ (2)

5.2.2 What conclusion can you draw regarding the vectors? (2)

5.2.3 Determine the angle between the vectors give your answer in radians. (2)

5.2.4 Determine $6v + 2u$. (4)

Question 6 **[20 marks]**

6.1 Sketch the following piecewise function on the DIAGRAM SHEET provided. (7)

$$f(x) = \begin{cases} 2x^2; & x < 2 \\ x - 2; & 2 \leq x \leq 4 \\ \frac{8}{x}; & x > 4 \end{cases}$$

6.2 Given the composite function $F(x) = \frac{2}{x+1} - \sqrt[3]{2 + x - 1} + x + 1$. (3)
Determine $f(x)$ and $g(x)$.

6.3 Given $f(x) = 2x^2 - x + 1$ and $g(x) = \sqrt{x^2 + 9}$
Determine:

6.3.1 $(f \circ g)(x)$ simplify your answer. (4)

6.3.2 $(f \circ g)(-4)$ (2)

6.3.3 $(g \circ f)(x)$ (2)

6.3.4 $(f \circ f)(x)$ (2)

Question 7 **[14 marks]**

7.1 Given

$$f(x) = 3x^5 - 7x^3 + \frac{2}{\sqrt[4]{x}} - \frac{5}{x^5}$$

Determine the following:

7.1.1 $f'(x)$ (4)

7.1.2 $\int f(x)dx$ (5)

7.2 Determine the volume of the rotating object that exists when $h(x) = (2x + 3)^4$ rotates around the x −axis between $x = 0$ and $x = 3$. (5)

Question 8 **[9 marks]**

Given $f(x) = x^3 + 3x^2 - 10x - 24$ answer the questions below.

8.1 Write down a root of f. (2)

8.2 Write down a linear factor of f. (2)

8.3 Determine a quadratic factor of f. (3)

8.4 Solve for x if it is given that $f(x) = 0$ (2)

-END OF QUESTION PAPER-

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Final Examination 2023 Answer sheet

Name and Surname: _____

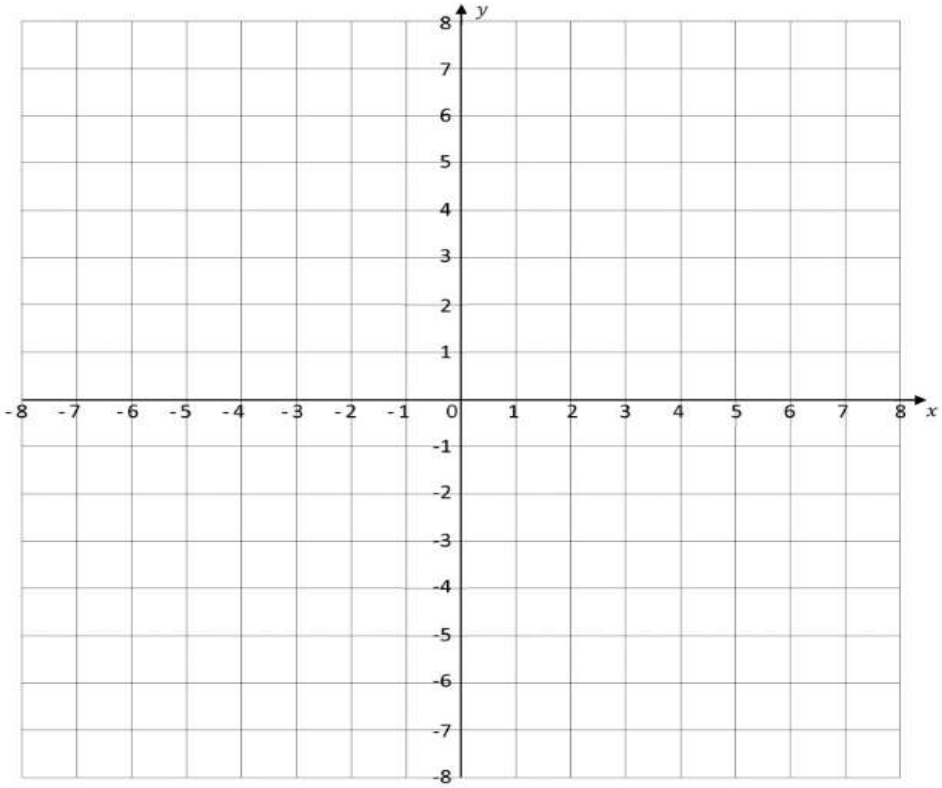
Question Total	1 [20]	2 [16]	3 [12]	4 [11]	5 [18]	6 [20]	7 [14]	8 [9]
Learner mark								

TOTAL 120

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

6.1



Formula sheet

$$\mathbf{A} = \int_{a}^{b} f(x)\,dx$$

$$\mathbf{V} = \pi \int_{a}^{b} [f(x)]^2\ dx$$

$$\mathbf{A} = \int_{a}^{b} \big(f(x) - g(x)\big)\,dx$$

$$f'(x) = nax^{n-1},$$

$$\int x^n\ dx = \frac{x^{n+1}}{n+1} + k$$

$$\int (ax + b)^n\ dx = \frac{(ax + b)^{n+1}}{(n + 1) \times a} + k$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_x \times \boldsymbol{b}_x + \boldsymbol{a}_y \times \boldsymbol{b}_y = |\boldsymbol{a}| \times |\boldsymbol{b}| \times cos\theta$$