

α -WISKUNDE

Alpha Wiskunde MEMORANDUM

23 Oktober 2023

Graad 12

Tyd: 3 uur

Totaal: 200 punte

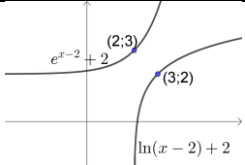
VRAAG 1 [30 Punte]

Vraag	Antwoord			
1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D
11	A	B	C	D
12	A	B	C	D
13	A	B	C	D
14	A	B	C	D
15	A	B	C	D

Vraag 15

$$\frac{d}{dx}(y) = \frac{d}{dx}(e^{x \ln x}) = e^{x \ln x} \left(\frac{x}{x} + \ln x \right) = y(1 + \ln x)$$

VRAAG 2 [20 Punte]

2.1		
(a)	$f(2) = e^0 + 2 = 3$	✓ Antwoord (1)
(b)	$x = e^{y-2} + 2; e^{y-2} = x - 2$ $y - 2 = \ln(x - 2)$ $f^{-1}(x) = \ln(x - 2) + 2$	✓ Ruil x en y ✓ Antwoord (2)
(c)		✓ Eksponent ✓ Ln ✓✓✓✓ Punte (6)
(d)	Reflekteer om x -as Transleer 6 eenhede op	✓ Reflekteer ✓ Transleer (2)
(e)	Stel $f(x) = \ln(x - 2) + 2 + e^{x-2} - 4$ Dan $f'(x) = \frac{1}{x-2} + e^{x-2}$ $x_{n+1} = x_n - \frac{\ln(x_n - 2) + e^{x_n-2} - 2}{\frac{1}{x_n - 2} + e^{x_n-2}}$ $x \approx 2,7993$	✓ Stel ✓ Differentieer ✓ Newton ✓ Antwoord ✓ Regte des. (5)
2.2	$f(g(x)) = e^{2\ln(x-2)} - 16 = 0$ $e^{2\ln(x-2)} = 16$ $2 \ln(x - 2) = \ln 16$ $(x - 2)^2 = 16$ $x - 2 = 4$ $x = 6$	✓ $f(g(x))$ ✓ Kry ln ✓ formule ✓ Antwoord (4)

VRAAG 3 [16 Punte]

3.1		
(a)	$r = 2$ $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $2\text{cis}\left(\frac{2\pi}{3}\right)$	✓ Antwoord ✓✓ Hoek (3)
(b)	$\left(2\text{cis}\left(\frac{2\pi}{3}\right)\right)^7$ $= 128\text{cis}\left(\frac{14\pi}{3}\right)$ $= -64 + 64\sqrt{3}i$	✓ Vervang ✓ 128 en Hoek ✓ Antwoord (3)
(c)	$z^2 = 16\text{cis}\left(\frac{7\pi}{6}\right)$ Hoeke: $\frac{7\pi}{6}; \frac{19\pi}{6}$ $z = 4\text{cis}\frac{7\pi}{12}$ of $4\text{cis}\left(\frac{19\pi}{12}\right)$ OF $z_k = 16^{\frac{1}{2}} \left[\text{cis}\left(\frac{\frac{7\pi}{6} + 2k\pi}{2}\right) \right]$ vir $k=0; 1$ $z = 4\text{cis}\frac{7\pi}{12}$ of $4\text{cis}\left(\frac{19\pi}{12}\right)$	✓ Poolvorm ✓ Hoeke ✓ 4 ✓✓ Hoeke (5)
3.2	$x = -2 \pm 3i$ $(x + 2)^2 = -9$ Dus $x^2 + 4x + 13$ is 'n faktor $2x^3 + 7x^2 + 22x - 13 = (x^2 + 4x + 13)(2x - 1)0$ $x = \frac{1}{2}$	✓ ander nulpunt ✓✓ Faktor ✓ Faktoriseer ✓ Antwoord (5)

VRAAG 4 [20 Punte]

4.1	$x \geq 0$ of $x < 0$ $x^2 - 3x + 2 = 0$ of $-x^2 - 3x = -2$ $(x - 1)(x - 2) = 0$ of $x^2 + 3x - 2 = 0$ $x = 1$ of $x = 2$ 0,56 of -3,56 n.v.t	✓ Beperkings ✓✓ Vergelykings ✓✓✓✓ Antwoorde (7)
4.2	Stel $n = 1$: Lk = 4 RK = 4 Dus waar vir $n = 1$ Aanvaar waar vir $n = k$: $4 + 9 + \dots + (5k - 1) = \frac{5k^2 + 3k}{2}$ Stel $n = k + 1$: LK = $\frac{5k^2 + 3k}{2} + 5k + 4$ $= \frac{5k^2 + 3k + 10k + 8}{2}$ $= \frac{5k^2 + 13k + 8}{2}$ RK = $\frac{5(k + 1)^2 + 3(k + 1)}{2}$ $= \frac{5K^2 + 10k + 5 + 3k + 3}{2}$ $= \frac{5k^2 + 13k + 8}{2}$ Dus waar vir $n = k + 1$. Volgens die beginsel van wiskundige induksie waar vir alle natuurlike getalle. OF Breuk : $\frac{(5k+8)(k+1)}{2}$	✓ $n=1$ ✓ Aanvaar en stelling ✓✓ $n=k+1$ LK ✓ Vereenvoudig ✓ RK ✓ Vereenvoudig ✓ Stelling (8)
4.3	$n = 12, r = 5, n - r = 7$ $\binom{12}{5} (2x)^7 \left(-\frac{1}{3x}\right)^5$ $= 792 \times 128x^7 \times \left(-\frac{1}{243x^5}\right)$ $= -\frac{11\,264}{27} x^2$	✓✓ ✓ Elkeen ✓ Vereenvoudig ✓✓ Antwoord (5)

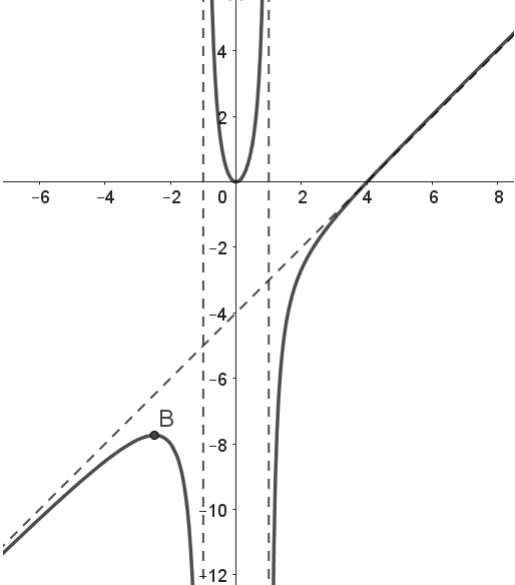
VRAAG 5 [18 Punte]

5.1	<p>Puntproduk: $1.4 + 2a - 18 = 0$ $2a = 14$, dus $a = 7$</p>	<p>✓ Puntproduk ✓ Antwoord</p>	(2)
5.2	<p>Grootte: $\sqrt{16 + 4 + 9} = \sqrt{29}$ $\theta = \text{bgcos}\left(\frac{2}{\sqrt{29}}\right) = 1,19$</p>	<p>✓ Grootte ✓ Antwoord</p>	(2)
5.3 (a)	$\begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -1 & -3 \end{vmatrix}$ $= i(-6 - 4) - j(-3 + 8) + k(-1 - 4)$ $= -10i - 5j - 5k$ <p>OF $-5(2; 1; 1)$ OF $(-10 - 5; -5)$</p>	<p>✓ determinant ✓✓✓ Bereken matriks ✓ Antwoord</p>	(5)
5.3 (b)	<p>Oppervl $= \sqrt{10^2 + 5^2 + 5^2} = 5\sqrt{6}$ $(12,25)$</p>	<p>✓ Vervang ✓ Antwoord</p>	(2)
5.4	<p>Oppervl sektor: $\frac{1}{2}r^2 \cdot \frac{\pi}{6} = \frac{\pi r^2}{12}$ Oppervl driehoek: $\frac{1}{2}CO \cdot CA$ $= \frac{1}{2}r \cos\left(\frac{\pi}{6}\right)r \sin\frac{\pi}{6}$ $= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}r^2$ $= \frac{\sqrt{3}r^2}{8}$</p> <p>Oppervl geskakeerde deel: $\frac{\pi r^2}{12} - \frac{\sqrt{3}r^2}{8} = \frac{2\pi - 3\sqrt{3}}{6}$ $\frac{r^2(2\pi - 3\sqrt{3})}{24} = \frac{2\pi - 3\sqrt{3}}{6}$ $r^2 = \frac{24}{6} = 4$ $r = 2$</p>	<p>✓ Sektor ✓ CO ✓ CA ✓ Driehoek ✓ Geskakeerde deel ✓ Vergelyking ✓ Antwoord</p>	(7)

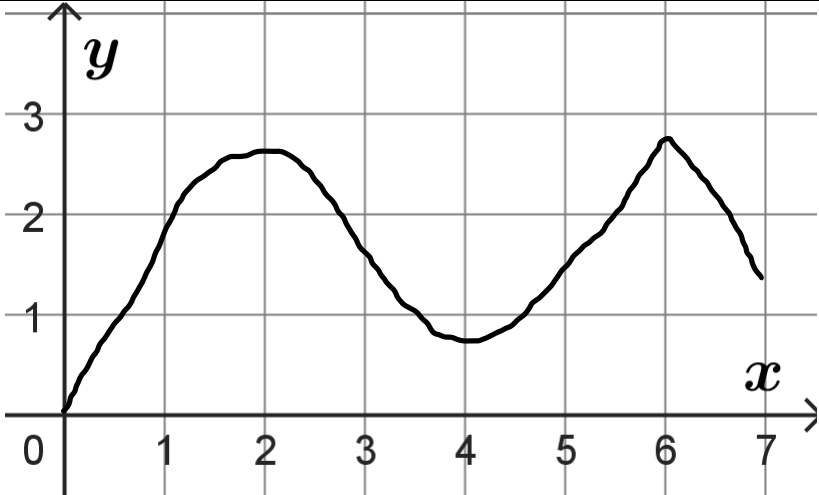
VRAAG 6 [19 Punte]

6.1		
(a)	$f'(x) = \begin{cases} 2e^{2x} & \text{as } x \leq 0 \\ 2 & \text{as } 0 < x < 1 \\ \frac{1}{1+x^2} & \text{as } x \geq 1 \end{cases}$	<p>✓✓✓ Antwoord</p> <p>(3)</p>
(b)	$\lim_{x \rightarrow 0^-} f'(x) = 2$ $\lim_{x \rightarrow 0^+} f'(x) = 2$ <p>Ja, differensieerbaar by $x = 0$</p>	<p>✓ Limiet links</p> <p>✓ Limiet regs</p> <p>✓ Gevolgtrekking</p> <p>(3)</p>
(c)	$\lim_{x \rightarrow 1^-} f(x) = 1 + 2 = 3$ $f(1) = \lim_{x \rightarrow 1^+} f(x) = (1) + 3 - \frac{\pi}{4} = 3$ <p>Dus is</p> $f(1) = \lim_{x \rightarrow 1} f(x)$ <p>Dus kontinu by $x = 1$</p>	<p>✓ Limiet links</p> <p>✓ Limiet regs</p> <p>✓ Rede</p> <p>(3)</p>
(d)	$\lim_{x \rightarrow 1^-} f'(x) = 2$ $\lim_{x \rightarrow 1^+} f'(x) = \frac{1}{2}$ <p>Nee, nie differensieerbaar by $x = 0$ nie.</p>	<p>✓ Limiet links</p> <p>✓ Limiet regs</p> <p>✓ Gevolgtrekking</p> <p>(3)</p>
6.2 (a)	$f'(x) = \frac{\pi \cdot 5^{\pi x} \cdot \ln 5 \cdot (\ln(5x) + \pi) - 5^{\pi x} \cdot \frac{1}{x}}{(\ln(5x) + \pi)^2}$	<p>✓ Kwos.r</p> <p>✓✓ Afgeleide van $5^{\pi x}$</p> <p>✓ Afgeleide van noemer</p> <p>(4)</p>
(b)	$g'(x) = -\operatorname{cosec}(\sqrt{5x}) \cot(\sqrt{5x}) \cdot \sqrt{5} \cdot \frac{1}{2} x^{-\frac{1}{2}}$	<p>✓✓✓</p> <p>(3)</p>

VRAAG 7 [21 Punte]

7.1	$y = 0$ $x^2(x - 4) = 0$ $x = 0 \text{ of } x = 4$	✓ y ✓ Faktore ✓ x (3)
7.2	Vertikaal: $x^2 - 1 = (x - 1)(x + 1)$ dus $x = -1$ en $x = 1$ Skuins: $y = x - 4$	✓✓ Vertikaal ✓✓ Skuins (4)
7.3	$\frac{x^3 - 4x^2}{x^2 - 1} = x - 4$ $\frac{x^2(x - 4)}{x^2 - 1} = x - 4$ $x = 4$	✓ Stel gelyk ✓ Vereenvoudig ✓ Antwoord (3)
7.4	$f'(x) = 0$ $(3x^2 - 8x)(x^2 - 1) - (x^3 - 4x^2)(2x) = 0$ $3x^4 - 3x^2 - 8x^3 + 8x - 2x^4 + 8x^3 = 0$ $x(x^3 - 3x + 8) = 0$ $x = 0$ $(0; 0)$	✓ Afgeleide=0 ✓✓ Differensieer ✓ Vereenvoudig ✓ Antwoord (5)
7.5		✓ Afsnitte ✓ Vert asimpt ✓ Skuins asimpt ✓ Min draaip ✓ Maks draaip ✓✓✓ Elke deel se vorm (6)

VRAAG 8 [18 Punte]

8.1		
(a)	$x = 2$ en $x = 4$	✓✓Antwoorde (2)
(b)	$x = 2$: Maksimum, want $f''(x)$ negatief $x = 4$: Minimum, want $f''(x)$ positief	✓✓ Maks en motivering ✓✓Min en motivering (4)
(c)	$x = 3$	✓ Antwoord (1)
(d)		✓ Begin punt ✓✓Draaipunte ✓Knak by 6 ✓Bokant x-as (5)
8.2	$3x^2 + 3y^2y' = 6y + 6x \cdot y'$ $y' = \frac{6y - 3x^2}{3y^2 - 6x}$ $m = \frac{6(3) - 3(3^2)}{3(3^2) - 6(3)}$ $= -1$ $y - 3 = -(x - 3)$ $y = -x + 6$	✓✓✓Diff ✓Afgeleide ✓m ✓ Vergelyking (6)

VRAAG 9 [24 Punte]

9.1	$\Delta x_i = \frac{2}{n}; x_i = \frac{2}{n} \cdot i$ $f(x_i) = 3\left(\frac{2}{n} \cdot i\right)^2 + \frac{2}{n} \cdot i = \frac{12}{n^2} i^2 + \frac{2i}{n}$ $\sum_{i=1}^n \left(\frac{12}{n^2} i^2 + \frac{2i}{n}\right) = \frac{12}{n^2} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + \frac{2}{n} \left(\frac{n^2}{2} + \frac{n}{2}\right)$ $= 4n + 6 + \frac{2}{n} + n + 1$ <p>basis x hoogte</p> $= \frac{2}{n} \times \left(5n + 7 + \frac{2}{n}\right) = 10 - \frac{14}{n} + \frac{4}{n^2}$ $\int_0^2 (3x^2 + x) dx = \lim_{n \rightarrow \infty} \left(10 - \frac{14}{n} + \frac{4}{n^2}\right) = 10$	<ul style="list-style-type: none"> ✓ Δx_i ✓ x_i ✓ $f(x_i)$ ✓ sigma i^2 ✓ sigma i ✓ Vereenvoudig ✓ bh ✓ lim ✓ Antwoord <p style="text-align: right;">(9)</p>
9.2	<p>(a)</p> $\frac{2x-3}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$ $2x-3 = A(x-2) + B$ <p>Stel $x = 2$: $1 = B$</p> <p>x: $2 = A$</p> $\int \left(\frac{2}{x-2} + \frac{1}{(x-2)^2}\right) dx$ $= \int \left(\frac{2}{x-2} + (x-2)^{-2}\right) dx$ $= 2 \ln(x-2) - (x-2)^{-1} + k$	<ul style="list-style-type: none"> ✓ Regte vorm ✓ x kgv ✓ A en B ✓✓ Antwoorde <p style="text-align: right;">(5)</p>
	<p>(b)</p> $\int (2 \sin^2 4x + 3^{\pi x}) dx = \int \left(\frac{1}{2}[1 - \cos(8x)] + 3^{\pi x}\right) dx$ $= \frac{1}{2} \left[x - \frac{1}{8} \sin(8x)\right] + \frac{3^{\pi x}}{\pi \cdot \ln 3} + k$	<ul style="list-style-type: none"> ✓ Formule ✓✓ Antwoord <p style="text-align: right;">(5)</p>
	<p>(c) Stel $f(x) = \log x$, dan is $f'(x) = \frac{1}{x \ln 10}$</p> <p>Stel $g'(x) = x^{-2}$, dan is $g(x) = -\frac{1}{x}$</p> $\int \frac{(\log x)}{x^2} dx = \log x \left(-\frac{1}{x}\right) + \int \frac{1}{x \ln 10} \cdot \frac{1}{x} dx$ $= -\frac{\log x}{x} + \frac{1}{\ln 10} \int x^{-2} dx$ $= -\frac{\log x}{x} + \frac{1}{\ln 10} (-x^{-1}) + k$	<ul style="list-style-type: none"> ✓ f ✓ g' ✓ 1e stap ✓ Integraal reg ✓ Antwoord <p style="text-align: right;">(5)</p>

VRAAG 10 14 Punte

10.1	$V = \pi \int_3^4 (4 - x) dx - \pi \int_3^4 (16 - 8x + x^2) dx$ $V = \pi \int_3^4 (-12 + 7x - x^2) dx$ $= \pi \left(-12x + \frac{7x^2}{2} - \frac{x^3}{3} \right) \Big _3^4$ $= \pi \left(-48 + 56 - \frac{64}{3} \right) - \pi \left(-36 + \frac{63}{2} - 9 \right)$ $= \frac{\pi}{6}$	<p>✓ Regte formule</p> <p>✓ Funksies apart</p> <p>✓ Funksies gekwadreer</p> <p>✓ Vereenvoudig</p> <p>✓ Integreer</p> <p>✓ Vervang</p> <p>✓ Antwoord</p> <p>(7)</p>
10.2	$\int_0^a \frac{2 \operatorname{bgsin}(x)}{\sqrt{1-x^2}} dx = \frac{2(\operatorname{bgsin}(x))^2}{2} \Big _0^a$ $= (\operatorname{bgsin}(a))^2 = \frac{\pi^2}{36}$ $\operatorname{bgsin}(a) = \frac{\pi}{6}$ $a = \sin \frac{\pi}{6}$ $a = \frac{1}{2}$	<p>✓✓✓ Integraal</p> <p>✓✓ Vervang</p> <p>✓ Vierkantsw</p> <p>✓ a</p> <p>✓ Antwoord</p> <p>(7)</p>