α-WISKUNDE/ MATHEMATICS

Rekordeksamen/*Preliminary Exam* 2023
Graad/Grade 12

Tyd/ Time: 3 uur/*hours*
Totaal/ Total: 200 PUNTE/*MARKS*

VRAAG/ *QUESTION* 1 [30 PUNTE/*MARKS*]

1.1	A	[✓ B]	C	D
1.2	A	B	C	[✓ D]
1.3	A	B	[✓ C]	D
1.4	A	B	[✓ C]	D
1.5	[✓ A]	B	C	D
1.6	A	[✓ B]	C	D
1.7	[✓ A]	B	C	D
1.8	A	[✓ B]	C	D
1.9	A	B	C	[✓ D]
1.10	A	B	[✓ C]	D
1.11	[✓ A]	B	C	D
1.12	A	B	C	[✓ D]
1.13	[✓ A]	B	C	D
1.14	A	B	[✓ C]	D
1.15	A	[✓ B]	C	D

VRAAG/ *QUESTION* 2 [20 PUNTE/*MARKS*]

2.1a	$I=10^{-12}(e^{0.1(110)})$ ✓ $=5{,}99\times10^{-8}\ \text{watt/m}^2$ ✓	1: Vervang/*Substitute* 1:Antwoord/*Answer* **[2]**
2.1b	✓ $8{,}1\times10^{-9}=10^{-12}e^{0.1x}$ $8100=e^{0.1x}$ $\therefore 0{,}1x=\ln(8100)$ ✓ $\therefore x=90$ decibels/*decibles* ✓	1: Vervang/*Substitute* 1: Verander na/ *Change to* $\ln$ 1:Antwoord/*Answer* **[3]**
2.2a	✓ ✓ $p=2\text{cis}\left(-\frac{\pi}{4}\right)$ OF/*OR* $p=2\text{cis}\left(\frac{7\pi}{4}\right)$ ✓ $k=2\text{cis}\left(\frac{\pi}{3}\right)$	1:2 1: $\frac{-\pi}{4}$ OF/*OR* $\frac{7\pi}{4}$ 1: $\frac{\pi}{3}$ **[3]**
2.2b	✓ ✓ $=4\text{cis}\left(\frac{\pi}{12}\right)$ OF/*OR* $4\text{cis}\left(\frac{25\pi}{12}\right)$	1:4 1: $\frac{\pi}{12}$ OF/*OR* $\frac{25\pi}{12}$ **[2]**
2.2c	Stel $z_1=4\text{cis}\left(\frac{\pi}{12}\right)$ en $z_2=4\text{cis}\left(\frac{25\pi}{12}\right)$ OF/*OR* $z_2=4\text{cis}\left(\frac{49\pi}{12}\right)$ ✓ ✓ ✓ $\therefore\sqrt{z}=2\text{cis}\left(\frac{\pi}{24}\right)=2\text{cis}\left(\frac{25\pi}{24}\right)$ OF/*OR* $z_2=4\text{cis}\left(\frac{49\pi}{24}\right)$	1:2 1: $\frac{\pi}{24}$ 1: $\frac{25\pi}{24}$ OF/*OR* $\frac{49\pi}{24}$ **[3]**
2.3	As /If $x\geq\frac{1}{3}$: ✓ $3x-1=\frac{2}{x}$ ✓ $\therefore 3x^2-x-2=0$ $\therefore(3x+2)(x-1)=0$ ✓ ✓ $\therefore x=-\frac{2}{3}$ or / of $x=1$ n.v.t./*n/a* As /If $x<\frac{1}{3}$: ✓ $-3x+1=\frac{2}{x}$ ✓ $\therefore -3x^2+x-2=0$ $\therefore\therefore 3x^2-x+2=0$ Geen oplossing nie/*No solution* ✓	1: Opskrif/*Heading* 1: Vgl/*Eq* 1: $x=-\frac{2}{3}$ n.v.t./*n/a* 1: $x=1$ 1: Opskrif/*Heading* 1: Vgl/*Eq* 1: Geen oplossing nie/*No solution* **[7]**

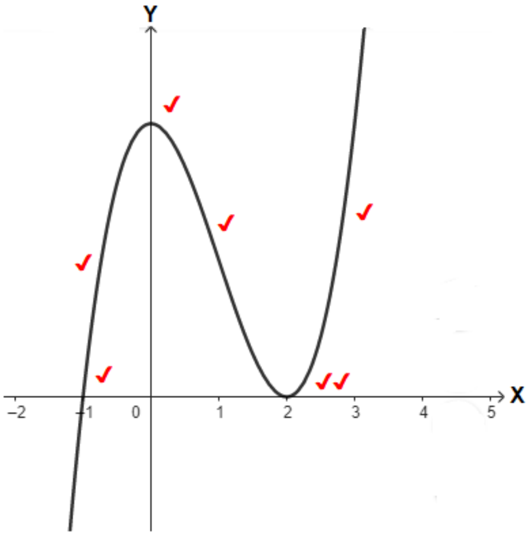
VRAAG/ *QUESTION* 3 [23 PUNTE/*MARKS*]

3.1a	$n=6$ ✓ $\binom{6}{r}(px)^{6-r}(x^{-2})^r$ ✓ $\therefore x^{6-r}\times x^{-2r}=x^0$ $\therefore 6-3r=0$ ✓ $\therefore r=2$ ✓ $\therefore \binom{6}{2}(px)^4(x^{-2})^2=240$ ✓ $\therefore 15p^4x^4x^{-4}=240$ ✓ $\therefore 15p^4=240$ $p=\pm 2$ ✓✓	1: n 1: Formule/*Formula* 1: Stel gelyk aan 0/*Set equal to 0* 1: r 1: Stel gelyk aan 240/ *Equals 240* 1: Vereenvoudig/*Simplify* 2:Antwoord/*Answer* **[8]**
3.2	Stel/ *Set* $n=1$: LK/*LHS* $=1$ RK/*RHS* $=\frac{1}{2}(3^1-1)=1$ ✓ $\therefore$ Die bewering is waar as/ The statement is true for $n=1$ Aanvaar die bewering is waar vir/*Accept the statement is true for* $n=k$: $3^0+3^1+3^2+\cdots+3^{k-1}=\frac{1}{2}(3^k-1)$ ✓ Beskou nou/ *Consider* $n=k+1$: ✓ ✓ LK/*LHS* $=\frac{1}{2}(3^k-1)+3^k$ $=\frac{3^k-1+2\times 3^k}{2}$ ✓ $=\frac{1}{2}(3^{k+1}-1)$ ✓ RK/R*HS* $=\frac{1}{2}(3^{k+1}-1)$ ✓ *LK* $=$ *RK*/*LHS* $=$ *RHS* en die bewering is dus waar vir $n=k+1$/ *and the statement is therefore true for* $n=k+1$. Volgens die beginsel van wiskundige induksie is die bewering dus waar vir alle $n\in\mathbb{N}$ /*By the principle of mathematical induction the statement is true for all* $n\in\mathbb{N}$ ✓✓	1: Bewys waar vir $n=1$/ *Prove true for* $n=1$ 1: Aanvaar waar vir/ Accept true for $n=k$ 1: Vervang/*Substitute* 1: $(k+1)$de term/ $(k+1)$*th term* 2: Vereenoudig LK/ *Simplify LHS* 1: Rk Vervang/*Substitute* 2: Afleiding/*Deduction* **[9]**
3.3a	✓ ✓ $\pi r^2=8\left(\frac{1}{2}r^2\alpha\right)$ $\frac{\pi}{4}=\alpha$ ✓	2: Formule/*Formula* 1:Antwoord/*Answer* **[3]**
3.3b	✓ ✓ Boog/*Arc* AB $=3\left(\frac{\pi}{4}\right)=\frac{3\pi}{4}$ $\therefore$ Omtrek $=3+3+\frac{3\pi}{4}$ $=6+\frac{3\pi}{4}$ ✓	1: Formule/*Formula* 1: $\frac{3\pi}{4}$ 1:Antwoord/*Answer* **[3]**

VRAAG/ *QUESTION* 4 [24 PUNTE/*MARKS*]

4.1a	$y = 2\text{bgsin}\left(\frac{3}{2}\right)$ ✓ Geen oplossing want $\frac{3}{2} \notin D_f$ ✓ OF/*OR* $\frac{3}{2} > 1$	1: Vervang/*Substitute* $x = 0$ 1: Afleiding/*Deduction* **[2]**
4.1b	$f\left(-\frac{5}{2}\right) = 2\text{bgsin}\left(-\frac{5}{2} + \frac{3}{2}\right)$ ✓ $= -\pi$ ✓ $f\left(-\frac{1}{2}\right) = 2\text{bgsin}\left(-\frac{1}{2} + \frac{3}{2}\right)$ ✓ $= \pi$ ✓ $\therefore y \in [-\pi; \pi]$ ✓	1: Vervang/*Substitute* $x = -\frac{5}{2}$ 1: $-\pi$ 1: Vervang/*Substitute* $x = -\frac{1}{2}$ 1:π 1: $y \in [-\pi; \pi]$ **[5]**
4.2	$x: 2\cos\frac{\pi}{6} = \sqrt{3}$ ✓ $y: 2\sin\frac{\pi}{6} = 1$ ✓ $\therefore \boldsymbol{u} = (\sqrt{3}; 1)$	1:$x = \sqrt{3}$ 1:$y = 1$ **[2]**
4.3	$\boldsymbol{a} \cdot \boldsymbol{b} = \vert\boldsymbol{a}\vert(4)\cos\frac{\pi}{3}$ ✓ ✓ $\therefore 10 = \vert\boldsymbol{a}\vert \times 4 \times \frac{1}{2}$ $\therefore \vert\boldsymbol{a}\vert = 5$ ✓	1: Formule/*Formula* 1: Vervang/*Substitute* $\boldsymbol{a} \bullet \boldsymbol{b} = 10$ 1:Antwoord/*Answer* **[3]**
4.4a	$\boldsymbol{AB}(1; 2; 2)$ ✓ $\boldsymbol{AD}(-3; 2; 0)$ ✓	1:$\boldsymbol{AB}$ 1:$\boldsymbol{AD}$ **[2]**
4.4b	$\vert\boldsymbol{AB}\vert = \sqrt{1^2 + 2^2 + 2^2} = 3$ ✓ $\therefore \left(\frac{1}{3}; \frac{2}{3}; \frac{2}{3}\right)$ ✓	1:$\vert\boldsymbol{AB}\vert = 3$ 1:Antwoord/*Answer* **[2]**
4.4c	✓ ✓ $\beta = \text{bgcos/arccos}\left(\frac{2}{3}\right) = 0{,}84$ ✓	1: bgcos/arccos 1:$\left(\frac{2}{3}\right)$ 1:$0{,}84$ **[3]**
4.4d	$\begin{vmatrix} i & j & \mathrm{k} \\ 1 & 2 & 2 \\ -3 & 2 & 0 \end{vmatrix}$ ✓ $= i(2(0) - 2(2)) - j\big(1(0) - 2(-3)\big) + k(1(2) - 2(-3))$ ✓ $= -4i - 6j + 8k$ ✓ ✓ Nee, want/*No, because* $\boldsymbol{AB}$ x $\boldsymbol{AD} \neq 0$ ✓	1:Determinant 1:Brei uit/*Expand* 1:Antwoord/*Answer* 1:Nee 1: $\boldsymbol{AB}$ x $\boldsymbol{AD} \neq 0$ **[5]**

VRAAG/ *QUESTION* 5 [25 PUNTE/ *MARKS*]

5.1a	$\lim_{x\to3^-} f(x) = 2k+1$ ✓ en/*and* $\lim_{x\to3^+} f(x) = \frac{3}{k}$ ✓ $\therefore 2k+1 = \frac{3}{k}$ ✓ $\therefore 2k^2 + k - 3 = 0$ $\therefore (2k+3)(k-1) = 0$ $\therefore k = -\frac{3}{2}$ ✓ of/or $k = 1$ ✓	1: $\lim_{x\to3^-} f(x)$ 1:$\lim_{x\to3} f(x)$ 1:$2k+1 = \frac{3}{k}$ 1: $k = -\frac{3}{2}$ 1: $k = 1$ **[5]**
5.1b	$\lim_{x\to3^-} f'(x) = 0$ ✓ en/*and* $\lim_{x\to3^-} f'^{(x)} = -\frac{2}{3}$ ✓ $\lim_{x\to3^-} f'(x) \neq \lim_{x\to3^-} f'^{(x)}$ $\therefore f$ is nie differensieerbaar by/*is not differentiable at* $x = 3$ ✓	1: $\lim_{x\to3^-} f'(x) = 0$ 1: $\lim_{x\to3^-} f'^{(x)} = -\frac{2}{3}$ 1:Afleiding/*Deduction* **[3]**
5.2		2:x-afsnitte by/ *intercepts at* $x = -1$ en/*and* $x = 2$ 2:draaipunte by/ *stationary points at* $x = 0$ en/*and* $x = 2$ 2:stygend as/ *increasing if* $x < 0$ en/*and* $x > 2$ 1:dalend as/*decreasing if* $0 < x < 2$ **[7]**
5.3a	$y = -\ln x - \ln y$ ✓ $\therefore \frac{dy}{dx} = -\frac{1}{x} - \frac{1}{y} \times \frac{dy}{dx}$ ✓ ✓ ✓ ✓ $\therefore \frac{dy}{dx} = \frac{-\frac{1}{x}}{1+\frac{1}{y}}$ ✓ ✓	1: Brei uit/*Expand* 1: $\frac{dy}{dx}$ 1: $-\frac{1}{x}$ 1: $-\frac{1}{y}$ 1: $\frac{dy}{dx}$ 1: $-\frac{1}{x}$ 1: $1+\frac{1}{y}$ **[7]**
5.3b	$\left(\frac{1}{e};1\right)$: $\frac{dy}{dx} = \frac{-e}{1+1} = -\frac{e}{2}$ ✓ ✓ ✓	1:Vervang/*Substitute* $x = \frac{1}{e}$ 1: Vervang/*Substitute* $y = 1$ 1:Antwoord/*Answer* **[3]**

VRAAG/ *QUESTION* 6 [24 PUNTE/ *MARKS*]

6.1	✓ ✓ $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}=0$✓ $\therefore \frac{1}{2}x^{-\frac{1}{2}}(1-x^{-1})=0$ ✓ $\therefore x\neq 0$ and $1-\frac{1}{x}=0 \therefore x=1$✓ ✓ $f(1)=\sqrt{1}+\frac{1}{\sqrt{1}}=2$✓ ✓ ✓ $f''(x)=\frac{-1}{4}x^{-\frac{3}{2}}+\frac{3}{4}x^{-\frac{5}{2}}$ ✓ ✓ $f''(1)=\frac{-1}{4}+\frac{3}{4}=\frac{1}{2}>0$ $\therefore f$ het 'n minimum draaipunt by/*minimum point at* $(1;2)$	1:$f'(x)$ 1:$\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}$ 1: $=0$ 1: $x\neq 0$ 1: $x=1$ 1: $f(1)$ 1: 2 1: $f''(x)$ 1: $\frac{-1}{4}x^{-\frac{3}{2}}+\frac{3}{4}x^{-\frac{5}{2}}$ 1: $f''(1)$ 1: >0 **[11]**
6.2a	✓ ✓ ✓ $h'(x)=e^{\mathrm{bgtan}x}.\frac{1}{1+x^2}$ ✓ ✓ ✓ ✓ ✓ $h''(x)=e^{\mathrm{bgtan}x}.\frac{1}{1+x^2}.\frac{1}{1+x^2}+e^{\mathrm{bgtan}x}\left(-(1+x^2)^{-2}(2x)\right)=0$✓ $\therefore e^{\mathrm{bgtan}x}.\frac{1}{(1+x^2)^2}(1-2x)=0$ $\therefore x=\frac{1}{2}$✓	1:$h'(x)$ 1:$e^{\mathrm{bgtan}x}$ 1:$\frac{1}{1+x^2}$ 1: $h''(x)$ 1: $e^{\mathrm{bgtan}x}.\frac{1}{1+x^2}$ 1:produkreël/*product rule* 1: $-(1+x^2)^{-2}$ 1: $-(2x)$ 1: $=0$ 1: $\frac{1}{2}$ **[10]**
6.2b	Kies 'n waarde (x_1) nét kleiner en 'n waarde (x_2) nét groter as x en bereken $h''(x_1)$✓ en $h''(x_2)$. ✓ Die tekens van $h''(x_1)$ en $h''(x_2)$ moet verskil, d.w.s. $h''(x_1)>0$ en $h''(x_2)<0$ of andersom.✓ /*Choose a value* (x_1) *just smaller and a value* (x_2) *just bigger than* x *and calculate* $h''(x_1)$✓ *and* $h''(x_2)$✓. *The signs of* $h''(x_1)$ *and* $h''^{(x_2)}$*must differ, i.e.* $h''(x_1)>0$ and $h''(x_2)<0$ *or vice versa.* ✓	1: $h''(x_1)$ 1: $h''(x_2)$ 1: $h''(x_1)>0$ en/*and* $h''(x_2)<0$ of andersom/ *or vice versa* **[3]**

VRAAG/ *QUESTION* 7 [22 PUNTE/ *MARKS*]

7.1a	$f'(x) = \overset{\checkmark}{2^{\cos^2(3x)}}.\overset{\checkmark}{\ln 2}.\overset{\checkmark}{2\cos(3x)}\overset{\checkmark}{\left(-\sin(3x)\right)}.3\checkmark$	1: $2^{\cos^2(3x)}$ 1: $\ln 2$ 1: $2\cos(3x)$ 1:$\left(-\sin(3x)\right)$ 1:3 **[5]**
7.1b	$\frac{2\checkmark}{(2x-1)\checkmark\ln 5\checkmark} - \frac{e^{-\frac{x}{3}}\checkmark.\frac{-1}{3}\checkmark}{\sqrt{1-\left(e^{-\frac{x}{3}}\right)^2}\ \checkmark}$	1: 2 1: $(2x-1)$ 1: $\ln 5$ 1: $e^{-\frac{x}{3}}$ 1: $\frac{-1}{3}$ 1: $\sqrt{1-\left(e^{-\frac{x}{3}}\right)^2}$ **[6]**

7.2	$\Delta x_i = \frac{3-1}{n} = \frac{2}{n}$ ✓ $\quad x_i = 1 + \frac{2i}{n}$ ✓	1: Δx_i 1: x_i
	$f(x_i) = -2\left(1+\frac{2i}{n}\right)^2$ ✓ $= -2 - \frac{8i}{n} - \frac{8i^2}{n^2}$ ✓	2: $f(x_i)$
	$f(x_i).\Delta x_i = -\frac{4}{n} - \frac{16i}{n^2} - \frac{16i^2}{n^3}$ ✓	1: $f(x_i).\Delta x_i$
	$\therefore \sum_{i=1}^{n} f(x_i).\Delta x_i = -\frac{4}{n}\sum_{i=1}^{n} 1 - \frac{16}{n^2}\sum_{i=1}^{n} i - \frac{16}{n^3}\sum_{i=1}^{n} i^2$ ✓	1:Sigma
	$= -\frac{4}{n}(n)$ ✓ $- \frac{16}{n^2}\left(\frac{n^2}{2}+\frac{n}{2}\right)$ ✓ $- \frac{16}{n^3}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)$ ✓	1:Vervang/*Substitute* i^2 1:Vervang/*Substitute* i 1: Vervang/*Substitute* 1
	$= -4 - 8 - \frac{8}{n} - \frac{16}{3} - \frac{8}{n} - \frac{8}{3n^2}$ ✓	1:Vereenvoudig/*Simplify*
	$\therefore \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i).\Delta x_i) = -4 - 8 - \frac{16}{3} = -\frac{52}{3}$ ✓	1:Antwoord/*Answer*
	$\therefore \int_1^3 (-2x^2)dx = \frac{-52}{3}$	
	OF/ *OR*	
	$\Delta x_i = \frac{2}{n}$ ✓ ; $x_i = 1 + \frac{2i}{n}$ ✓ $f(x_i) = -2\left(1+\frac{2i}{n}\right)^2$ ✓ $= -2 - \frac{8i}{n} - \frac{8i^2}{n^2}$ ✓ $\sum_{i=1}^{n}\left(-2 - \frac{8i}{n} - \frac{8i^2}{n^2}\right) = -2\sum_{i=1}^{n} 1$ ✓ $- \frac{8}{n}\sum_{i=1}^{n} i - \frac{8}{n^2}\sum_{i=1}^{n} i^2$	
	$= -2n$ ✓ $- \frac{8}{n}\left(\frac{n^2}{2}+\frac{n}{2}\right)$ ✓ $- \frac{8}{n^2}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)$ ✓	
	$= -2n - 4n - 4 - \frac{8n}{3} - 4 - \frac{4}{3\text{n}}$ $= -6n - 8 - \frac{8n}{3} - \frac{4}{3n}$ ✓	
	$\int_1^3 -2x^2dx$ = basis x hoogte $= \frac{2}{n} \times \left(-6n - 8 - \frac{8n}{3} - \frac{4}{3n}\right) = -12 - \frac{4}{n} - \frac{16}{3} - \frac{8}{3n^2}$ ✓	
	$\int_1^3 -2x^2dx = \lim_{n\to\infty}\left(-12 - \frac{4}{n} - \frac{16}{3} - \frac{8}{3n^2}\right) = \frac{-52}{3}$ ✓	
	$\therefore \int_1^3 (-2x^2)dx = \frac{-52}{3}$	**[11]**

VRAAG/ *QUESTION* 8 [22 PUNTE/ *MARKS*]

8.1a	$\frac{5^{3x}\checkmark}{3\ln5\,\checkmark\checkmark}-\frac{x^4\checkmark}{20\checkmark}+k$	1: 5^{3x} 1: 3 1: $\ln5$ 1:$-x^4$ 1:$\frac{1}{20}$ OF/*OR* $+\frac{1}{4.5}$ **[5]**
8.1b	$\int\frac{1}{x}(\ln x)^{-3}\checkmark dx$ $=\overset{\checkmark}{-}\frac{(\ln x)^{-2}\checkmark}{2\checkmark}+k$ OF/*OR* Stel/*Let* $u=\ln x$ $\therefore\frac{du}{dx}=\frac{1}{x}$ $\therefore du=\frac{1}{x}dx$ $\therefore\int\frac{1}{u^3}du=-\frac{u^{-2}}{2}+k=-\frac{1}{2(\ln x)^2}+k$	1:-3 1:$-$ 1:$\frac{1}{2}$ 1:$(\ln x)^{-2}$ **[4]**
8.1c	$\int\left(\operatorname{cosec}^2x-1\checkmark+\sec^2\left(\frac{x}{3}\right)\right)dx$ $=-\cot x\checkmark-x\checkmark+3\checkmark\tan\left(\frac{x}{3}\right)\checkmark+k$	1: $\operatorname{cosec}^2x-1$ 1: $-\cot x$ 1: $-x$ 1: 3 1: $\tan\left(\frac{x}{3}\right)$ **[5]**
8.2	$2x^2-2x+5=A(4+x^2)+(x-1)^2\checkmark$ Stel/*Let* $x=1$: $5=5A$ $\therefore A=1\checkmark$ $\therefore\int\left(\frac{A}{(x-1)^2}+\frac{1}{4+x^2}\right)dx$ $=\int\frac{dx}{(x-1)^2}+\int\frac{dx}{4+x^2}$ $=\frac{-1\checkmark}{(x-1)\checkmark}+\int\frac{1}{4\checkmark\left(1+\left(\frac{x}{2}\right)^2\right)}dx$ $=\frac{-1}{(x-1)}+\frac{1}{2}\checkmark\text{bgtan}\checkmark\left(\frac{x}{2}\right)\checkmark+k$	1:Vereenvoudig/*Simplify* 1: $A=1$ 1: -1 1:$\frac{1}{x-1}$ 1:$\frac{1}{4}$ 1: $\frac{1}{2}$ 1: bgtan 1: $\left(\frac{x}{2}\right)$ **[8]**

VRAAG/ *QUESTION* 9 [10 PUNTE/ *MARKS*]

9.	Stel/*Let* $f(x)=\overset{\checkmark}{x+1}$ en/*and* $g'(x)=\overset{\checkmark}{e^x}$	2: $f,\ g'$
	$\therefore f'(x)=\overset{\checkmark}{1}$ en/*and* $g(x)=\overset{\checkmark}{e^x}$	2: $f',\ g$
	$\therefore \int (x+1)e^x dx = (x+1)e^x \checkmark - \int e^x dx \checkmark$	2:1e stap/*1st step*
	$= (x+1)e^x - e^x \checkmark$	1: Integreer/*Integrate*
	$\therefore$ Oppervlakte/*Area* $= [(x+1)e^x - e^x]\|_0^p$	
	$= [\overset{\checkmark}{(p+1)e^p - e^p}] - [\overset{\checkmark}{1-1}]$ $= e^p(p+1-1)$	2: Korrek invervang/ *Correct substitution*
	$= pe^p \checkmark$	1: Antwoord/*Answer*
		[10]