

α -MATHEMATICS

Alpha Wiskunde Graad 11 / *Alpha Mathematics Grade 11*

Junie Eksamen 2023 / *June Examination 2023*

MEMORANDUM

Totaal / *Total*: 150 punte / *marks*

Eksaminator / *Examiner*: Lanice Liebenberg

Moderator: Rika Grobler

Hierdie memorandum bestaan uit 9 bladsye. /

This memorandum consists of 9 pages.

Vraag / Question 1

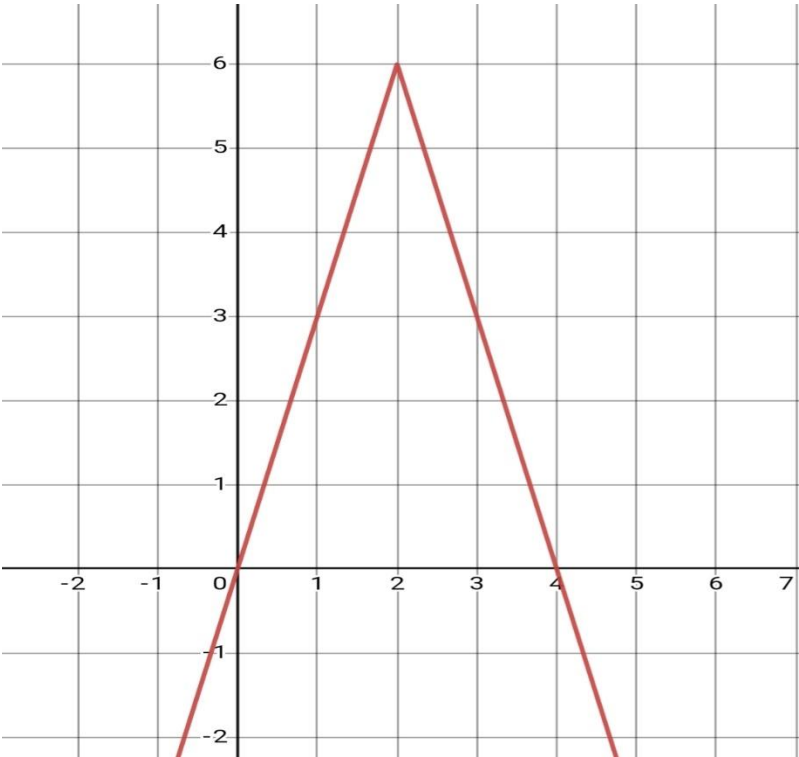
[20 punte / marks]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Vraag / Question 2

[42 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
2.1.1	$p = 3$ ✓ $q = -1$ ✓ $0 = a 2 - 3 - 1$ $a = 1$ ✓	(3)
2.1.2	$x \in \mathbb{R}$ ✓ ; $x \neq 3$ ✓	(2)
2.1.3	$(3; -1)$ ✓✓	(2)
2.2.1	$(0; 0)$ ✓	(1)
2.2.2	$0 = -3 2 - x + 6$ $ x - 2 = 2$ $x = 4$ ✓ ; $x = 0$ ✓	(2)

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
2.2.3		<p style="text-align: right;">(4)</p> <p>✓✓ x-intercepts ✓ salient point ✓ shape</p>
2.2.4	$-3 2 - x + 6 < 0$ $ x - 2 > \sqrt{2}$ ✓ $x \geq 4$ ✓ or $x < 0$ ✓	(2)
2.3.1	No solutions✓✓	(2)
2.3.2	$ x + 4 = x$ $x \geq -4$ or $x < -4$ ✓ $x + 4 = x$ or $x + 4 = -x$ ✓ No solution✓ or $x = -2$ ✓ but $(-2 \notin -4)$ There are no solutions for $x \in \mathbb{R}$. ✓	(5)
2.3.3	$\frac{6}{ x + 1 } > 3$ $3 x + 1 < 6$ ✓ $ x + 1 < 2$ ✓ $-2 < x + 1 < 2$ ✓ $-3 < x < 1$ ✓	(5)

2.4.1	D✓✓	(2)
2.4.2	A✓✓	(2)
2.4.3	F✓✓	(2)
2.4.4	C✓✓	(2)
2.4.5	E✓✓	(2)
2.4.6	B✓✓	(2)

Vraag / Question 3

[15 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
3.1	$\frac{B}{x^2} \checkmark$	(1)
3.2	$Cx + D \checkmark$	(1)
3.3	$Cx^3 + Dx^2 \checkmark$	(1)
3.4	$1 \checkmark$	(1)
3.5	$0 \checkmark$	(1)
3.6	$0 \checkmark$	(1)
3.7	$x \checkmark$	(1)
3.8	$\frac{-x^3 - 4x^2 - 7x - 2}{(x + 2)^4}$ $\equiv \frac{A}{x + 2} + \frac{B}{(x + 2)^2} + \frac{C}{(x + 2)^3} + \frac{D}{(x + 2)^4} \checkmark$ $-x^3 - 4x^2 - 7x - 2 = A(x + 2)^3 + B(x + 2)^2 + C(x + 2) + D$ $-x^3 - 4x^2 - 7x - 2 \checkmark$ $= Ax^3 + 6Ax^2 + 12Ax + 8A + Bx^2 + 4Bx + 4B + Cx + 2C + D \checkmark$ $A = -1 \checkmark$ $6A + B = -4$ $B = 2 \checkmark$ $12A + 4B + C = -7$ $C = -3 \checkmark$ $8A + 4B + 2C + D = -2$ $D = 4 \checkmark$ $\frac{-x^3 - 4x^2 - 7x - 2}{(x + 2)^4}$ $\equiv \frac{-1}{x + 2} + \frac{2}{(x + 2)^2} - \frac{3}{(x + 2)^3} + \frac{4}{(x + 2)^4} \checkmark$	(8)

Vraag / Question 4

[17 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
4.1	$\sum_{i=2}^k 2(i+2)^2 = 2(2+2)^2 + 2(3+2)^2 + 2(4+2)^2 + 2(5+2)^2 \dots$ $= 32\checkmark + 50\checkmark + 72\checkmark + 98\checkmark + \dots$	(4)
4.2	$\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$ <p> $2 + 6 + 12 + \dots + k(k+1) = \frac{n(n+1)(n+2)}{3}$ </p> <p>STEP 1 Consider $n = 1$: \checkmark</p> <p> $LHS = 1(1+1) = 2\checkmark$ $RHS = \frac{1(1+1)(1+2)}{3} = 2\checkmark$ </p> <p>$\therefore LHS = RHS\checkmark$ and the statement is true for $n=1$. \checkmark</p> <p>STEP 2 Accept \checkmark the statement is true for a natural number $n = k$:</p> <p> $2 + 6 + 12 + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3} \checkmark$ </p> <p>STEP 3 Consider $n = k + 1$:</p> <p> $RHS = \frac{(k+1)(k+2)(k+3)}{3} \checkmark$ </p> <p> $LHS = 2 + 6 + 12 + \dots + k(k+1) + (k+1)(k+2) \checkmark$ </p> <p> $LHS = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) \checkmark$ </p> <p> $LHS = (k+1)(k+2) \left(\frac{k}{3} + 1 \right) \checkmark$ </p> <p> $LHS = (k+1)(k+2) \left(\frac{k+3}{3} \right)$ </p> <p> $LHS = \frac{(k+1)(k+2)(k+3)}{3}$ </p> <p> $LHS = RHS\checkmark$ </p> <p>STEP 4 The statement is true for $n = 1$. If it is true for $n = k$, it is also true for $n = k + 1$ and consequently, true for all natural numbers n. \checkmark</p>	(13)

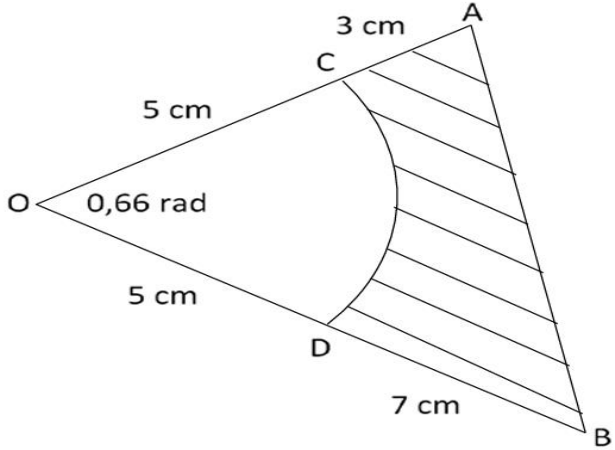
Vraag / Question 5

[20 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
5.1.1	$\left(2x - \frac{y}{4}\right)^3 = \binom{3}{0} (2x)^3 \left(\frac{-y}{4}\right)^0 + \binom{3}{1} (2x)^2 \left(\frac{-y}{4}\right)^1 \checkmark$ $+ \binom{3}{2} (2x)^1 \left(\frac{-y}{4}\right)^2 + \binom{3}{3} (2x)^0 \left(\frac{-y}{4}\right)^3 \checkmark$ $= 8x^3 \checkmark - 3x^2 y \checkmark + \frac{3xy^2}{8} \checkmark - \frac{y^3}{64} \checkmark$	(6)
5.1.2	$(a + b)^n = \binom{n}{r} a^{n-r} \cdot b^r$ $a = x \quad b = -\frac{2}{x} \quad n = 8$ $\left(x - \frac{2}{x}\right)^8 = \binom{8}{r} \cdot (x)^{8-r} \cdot \left(-\frac{2}{x}\right)^r \checkmark$ $= \binom{8}{r} \cdot x^{8-r} \cdot (-2)^r \cdot (x)^{-r} \checkmark$ $8 - r - r = 2 \checkmark$ $r = 3 \checkmark$ $\left(x - \frac{2}{x}\right)^8 = \binom{8}{3} \cdot x^{8-3} \cdot (-2)^3 \cdot (x)^{-3} \checkmark$ $= 448x^2 \checkmark \checkmark$	(7)
5.2	$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots$ $\frac{1}{4 + x^2} = \frac{1}{4 \left(1 + \frac{x^2}{4}\right)} = \frac{1}{4} \left(1 + \frac{x^2}{4}\right)^{-1} \checkmark$ $= \frac{1}{4} \left[1 + (-1) \left(\frac{x^2}{4}\right) + \frac{-1(-1-1)}{2!} \left(\frac{x^2}{4}\right)^2 + \dots \right] \checkmark$ $= \frac{1}{4} \checkmark - \frac{x^2}{4} \checkmark + \frac{x^4}{16} \checkmark + \dots$ $\left \frac{x^2}{4}\right < 1 \checkmark$ $ x < 2 \checkmark$	(7)

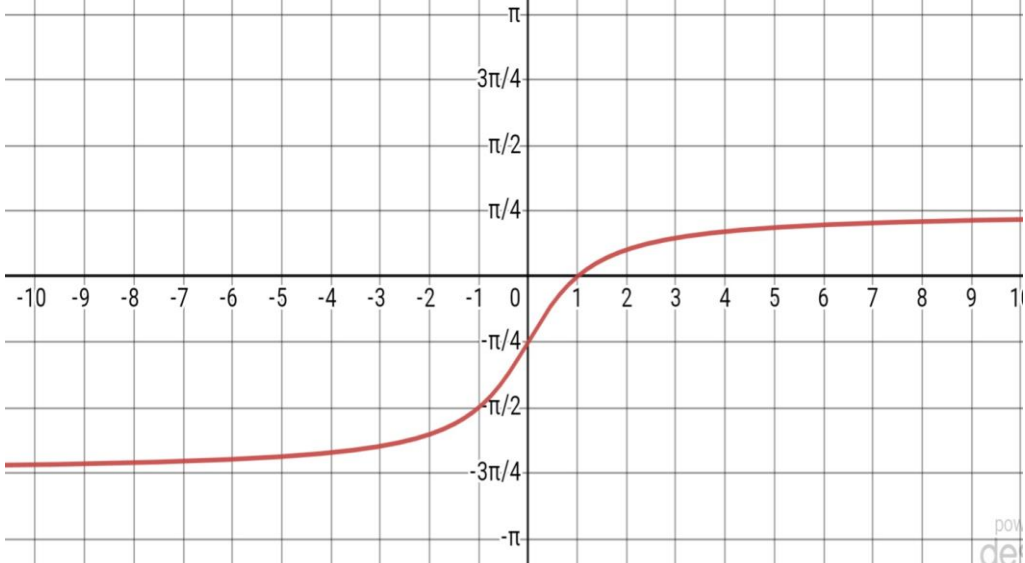
Vraag / Question 6

[20 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
6.1	<p>$x = 1 + 2i$ ✓ is also a zero of f / is ook 'n nulpunt van f. $(x - 1 - 2i)(x - 1 + 2i)$ ✓ OR/OF $x - 1 = \pm 2i$ ✓ $= x^2 - 2x + 5$ ✓ is a factor of f / is 'n factor van f.</p> $ \begin{array}{r} x^2 \quad -2x \quad +5 \quad \left \begin{array}{r} x^3 \quad -7x^2 \quad +15x \quad -25 \\ -x^3 \quad +2x^2 \quad -5x \\ \hline -5x^2 \quad +10x \quad -25 \\ +5x^2 \quad -10x \quad +25 \\ \hline 0 \end{array} \right. \\ \hline x = 5 \checkmark \checkmark \end{array} $	(8)
6.2.1	 <p>Area $\Delta ABO = \frac{1}{2} \cdot 8 \cdot 12 \cdot \sin 0,66$ ✓ = 29,429 ... cm^2 ✓ Area Sector $OCD = \frac{1}{2} \cdot 5^2 \cdot 0,66$ ✓ = 8,25 cm^2 ✓ Area shaded part = 29,429 ... - 8,25 ✓ \approx 21,18 cm^2 ✓</p>	(6)
6.2.2	<p>$CD = 5 \cdot 0,66$ ✓ = 3,3 cm ✓ $AB = \sqrt{OC^2 + OB^2 - 2(OC)(OB) \cdot \cos \widehat{BOC}}$ $AB = \sqrt{8^2 + 12^2 - 2(8)(12) \cdot \cos 0,66}$ ✓ = 7,504 ... cm ✓ Perimeter $\approx 3 + 7 + 3,3 + 7,50$ ✓ = 20,80 cm ✓</p>	(6)

Vraag / Question 7

[16 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
7.1.1	$y = \tan\left(x + \frac{\pi}{4}\right)$ $x = \tan\left(y + \frac{\pi}{4}\right) \checkmark$ $\arctan x = y + \frac{\pi}{4} \checkmark$ $y = \arctan x - \frac{\pi}{4} \checkmark \checkmark$	(4)
7.1.2		(4) \checkmark x-intercept \checkmark y-intercept \checkmark Asymptotes \checkmark Shape
7.1.3	$x \in \mathbb{R} \checkmark \checkmark$	(2)
7.1.4	$y \in \left(-\frac{3\pi}{4}; \frac{\pi}{4}\right) \checkmark \checkmark$	(2)
7.2	$3 \arcsin(2x + 1) = \pi$ $\arcsin(2x + 1) = \frac{\pi}{3} \checkmark$ $2x + 1 = \sin\left(\frac{\pi}{3}\right) \checkmark$ $2x = \sin\left(\frac{\pi}{3}\right) - 1$ $x = \frac{1}{2} \sin\left(\frac{\pi}{3}\right) - \frac{1}{2} \checkmark$ $x = -0,07 \text{ rad} \checkmark$	(4)

- EINDE VAN DIE MEMORANDUM / END OF THE MEMORANDUM -