α-MATHEMATICS

Grade 11 Alpha Mathematics

June Examination 2023

Examiner: Lanice Liebenberg **Time: $2\frac{1}{2}$ hours**

Moderator: Rika Grobler **Total: 150**

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 9 pages and an answer sheet.
2. Answer ALL 7 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 **[20 marks]**

This question must be answered **on the answer sheet**.
Every question has **ONLY** one correct answer. Mark the correct answer with an **X** on the answer sheet.

1.1 The x-intercepts of $y = |x + 2| + 3$ are: (2)

A $x = 1$ and $x = -5$

B $x = -1$ and $x = 5$

C $x = -2$ and $x = 3$

D There are no x-intercepts.

1.2 Given that $f(x) = x^3 - 2x^2 - 5x + 6 = 0$ which of the following statements are false? (2)

A -2 is a zero of f.

B $x + 2$ is a factor of f.

C $x^2 - x - 6$ is a quadratic factor of f.

D -1 is a zero of f.

1.3 In Pascal's triangle the number in the 4th position in the 11th row is: (2)

A 330

B 165

C 210

D 120

1.4 The inverse of a function is a reflection over: (2)

A $y = x$

B The x-axis.

C $y = -x$

D The y-axis.

1.5 The power series of $\sqrt[3]{1-6x}$ will converge if: (2)

A $|x| < -\frac{1}{6}$

B $|x| > \frac{1}{6}$

C $|x| < \frac{1}{6}$

D $|x| > -\frac{1}{6}$

1.6 If $3|2x+1| \leq 2|-1-2x| + 7$ then: (2)

A $x \in [-4; 4]$

B $x \in (-\infty; -4]$ or $x \in [4; \infty)$

C $x = 4$ or $x = -4$

D There are no solutions for x.

1.7 Given (2)

$$\frac{x+7}{(x-2)(x+1)} \equiv \frac{A}{x+1} + \frac{B}{x-2}$$

then:

A $A = 3$

B $A = -2$

C $B = -2$

D $B = 7$

1.8 Given $y = \frac{1}{2}\arccos(x-3)$. (2)

If y is enlarged by a factor of 4, and translated 4 units to the left the new equation will be:

A $y = 4\arccos(x-4)$

B $y = \frac{1}{8}\arccos(x+1)$

C $y = 2\arccos(x+1)$

D $y = 2\arccos(x-7)$

1.9 Given $p(x) = x^3 + ax^2 + 4x + 20$ and (x^2+4) is a factor of $p(x)$, then (2)

A $a = 5$

B $a = 4$

C $a = -4$

D $a = -5$

1.10 If the graph of $y = |x-2| - 3$ is translated 2 units to the left and 3 units up, the new equation will be: (2)

A $y = |x-4|$

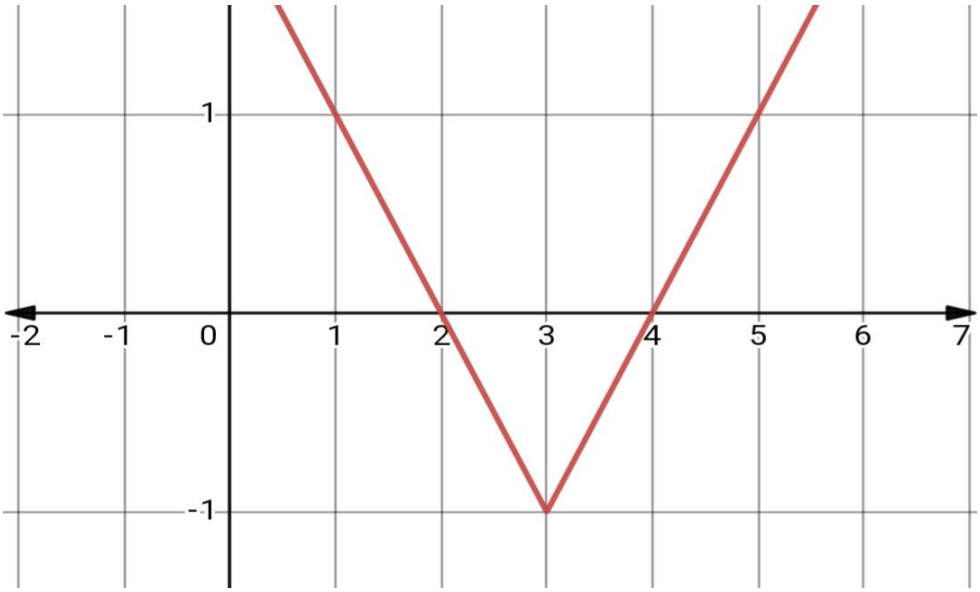
B $y = |x-4| - 6$

C $y = |x|$

D $y = |x| - 6$

Question 2 **[42 marks]**

2.1 Study the graph below with equation $y = a|x - p| + q$ and answer the questions that follow.



2.1.1 Determine the values of a, p and q. (3)

2.1.2 For which value(s) of x is the graph increasing? (2)

2.1.3 Write down the coordinates of the salient point. (2)

2.2 Given the equation of an absolute value function $y = -3|2 - x| + 6$.

2.2.1 Write down the coordinates of the y-intercept. (1)

2.2.2 Calculate the x-intercepts. (2)

2.2.3 Sketch the graph on the **answer sheet** provided. (4)

2.2.4 Determine algebraically for which value(s) of x, $y < 0$. (4)

2.3 Solve for x:

2.3.1 $|x + 2| = -4$ (2)

2.3.2 $|x + 4| = x$ (5)

2.3.3 $\dfrac{6}{|x + 1|} > 3$ (5)

2.4 This question must be answered **on the answer sheet**. (12)
Match **COLUMN B** to **COLUMN A**. The options (**A-F**) from **COLUMN B** can be used more than once or not at all as solutions to **COLUMN A**.

COLUMN A		**COLUMN B**	
2.4.1	$\|x+2\| \geq 0$	**A**	$x \in \mathbb{R}\,; x \neq -2$
2.4.2	$0 < \|x+2\|$	**B**	$x = 6$ or $x = -10$
2.4.3	$\|x+2\| = 0$	**C**	No solutions for $x \in \mathbb{R}$
2.4.4	$0 \geq \|x+2\|$	**D**	$x \in \mathbb{R}$
2.4.5	$\|x+2\| \geq 8$	**E**	$x \geq 6$ or $x < -10$
2.4.6	$\|x+2\| = 8$	**F**	$x = -2$

Question 3 **[15 marks]**

3.1 The fraction (7)

$$\frac{x^3+2x^2+8}{x^4+4x^2}$$

was decomposed into partial fractions, but some of the steps are incomplete. Write down the steps in question **3.1 – 3.7**.

$$\frac{x^3+2x^2+8}{x^4+4x^2} \equiv \frac{A}{x} + \mathbf{3.1} + \frac{\mathbf{3.2}}{x^2+4}$$

$$x^3+2x^2+8 = Ax^3+4Ax+Bx^2+4B+\mathbf{3.3}$$

$$A+C = \mathbf{3.4} \qquad B+D = 2 \qquad 4A = \mathbf{3.5} \qquad 4B = 8$$

$$\frac{x^3+2x^2+8}{x^4+4x^2} = \frac{\mathbf{3.6}}{x} + \frac{2}{x^2} + \frac{\mathbf{3.7}}{x^2+4}$$

3.8 Decompose (8)

$$\frac{-x^3-4x^2-7x-2}{(x+2)^4}$$

into partial fractions.

Question 4 **[17 marks]**

4.1 Write down the first four terms of (4)

$$\sum_{i=2}^{k} 2(i+2)^2$$

4.2 Use Mathematical Induction to show that for all natural numbers n: (13)

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

Question 5 **[20 marks]**

5.1 Use the Binomial Theorem to answer this question.

5.1.1 Write down the expansion of $\left(2x - \frac{y}{4}\right)^3$. (6)

5.1.2 Determine the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^8$. (7)

5.2 Make use of the formula for a power series to write down the first 3 terms of the expansion of (7)

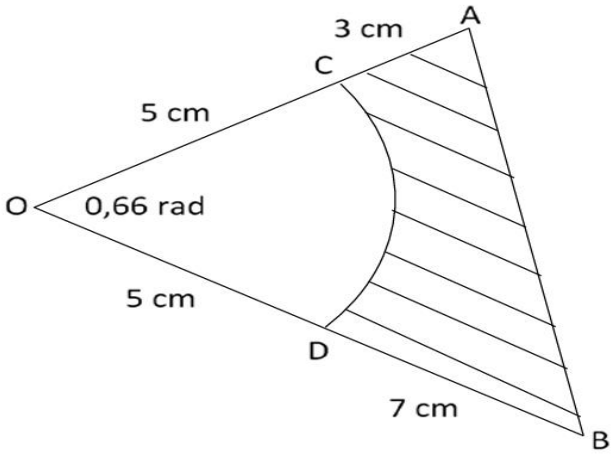
$$\frac{1}{4+x^2}$$

and give the values of x for which the expansion is valid.

Question 6 **[20 marks]**

6.1 $x^3 - 7x^2 + 15x - 25$ has 2 complex roots and one real root. (8)
If $x = 1 - 2i$ is one of the complex roots, determine the real root.

6.2 In the diagram C is a point on OA and D is a point on OB. OCD is a sector with midpoint O inside of triangle AOB.



6.2.1 Determine the area of the shaded part. (6)

6.2.2 Determine the perimeter of the shaded area. (6)

Question 7 **[16 marks]**

7.1 Given $y = \tan\left(x + \frac{\pi}{4}\right)$

7.1.1 Determine the inverse of y. (4)

7.1.2 Draw the graph of the inverse function of y on the **answer sheet**. (4)

7.1.3 Write down the domain of the graph above. (2)

7.1.4 Write down the range of the graph above. (2)

7.2 Solve for x if: (4)

$$3\arcsin(2x + 1) = \pi$$

- END OF QUESTION PAPER -

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June Examination 2023 Answer sheet

Name and Surname: _____

Question Total	1 [20]	2 [42]	3 [15]	4 [17]	5 [20]	6 [20]	7 [16]
Learner mark							

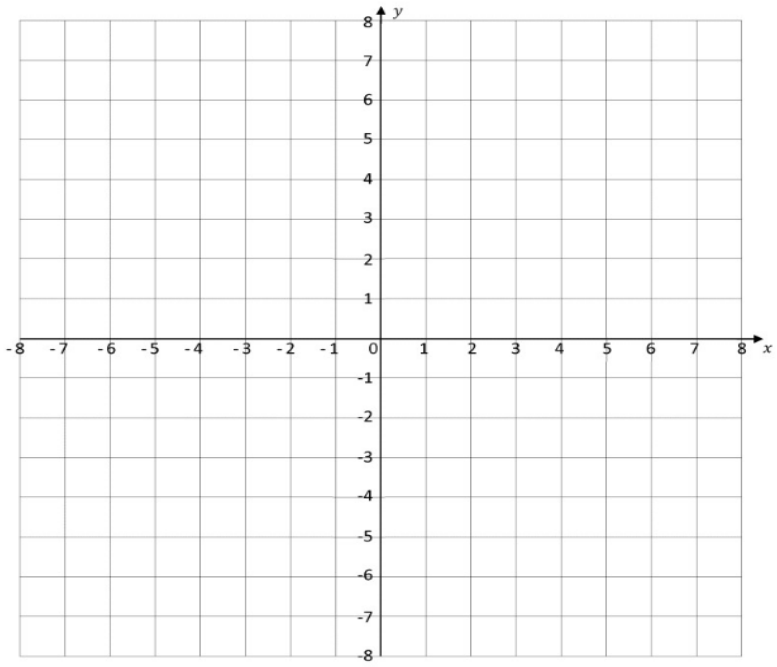
TOTAL 150

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Question 2

2.2.3



2.2.4

2.4.1	
2.4.2	
2.4.3	
2.4.4	
2.4.5	
2.4.6	

Question 7

7.1.2