α-MATHEMATICS

Grade 10 Alpha Mathematics

Final Examination 2022

Examiner: Lanice Liebenberg **Time: 2 hours**

Moderator: Anna Muller **Total: 120**

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 8 pages, an answer sheet and a diagram sheet.
2. Answer ALL 9 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 **[20 marks]**

This question must be answered **on the answer sheet**.
Every question has **ONLY** one correct answer. Mark the correct answer with an **X** on the answer sheet.

1.1 $.\sqrt{-1}.\sqrt{-1}.\sqrt{-1}.\sqrt{-1}.\sqrt{-1} =$ (2)

A 1

B $-i$

C i

D -1

1.2 The derivative of $(f \circ g)(x) =$ +f

A $f'(x).g'(x)$

B $f'\big(g(x)\big).g'(x)$

C $f(x).g'(f(x))$

D $f(x).g'(x)$

1.3 $60° =$ _____ radians (2)

A $\frac{3}{\pi}$

B $\frac{\pi}{6}$

C $\frac{\pi}{3}$

D $\frac{6}{\pi}$

1.4 For which value(s) of x will the following expression be purely imaginary? (2)

$$\sqrt{x+2}$$

A $x < -2$

B $x = 2$

C $x \geq -2$

D $-2 < x < 2$

1.5 Given $y = tan\theta$ where $\theta = \frac{\pi}{2}$. Which statement is false? (2)

A $\tan\frac{\pi}{2}$ is undefined

B There is a vertical asymptote at $x = \frac{\pi}{2}$

C $\tan\frac{\pi}{2} = \tan 90°$

D There is a horizontal asymptote at $y = \frac{\pi}{2}$

1.6 $x^2 + 9$ is factorised as: (2)

A $(x+3)(x-3)$

B $(x+3i)(x+3i)$

C $(x-3i)(x-3i)$

D $(x+3i)(x-3i)$

1.7 Given $f(x) = \begin{cases} x-3 & if \quad x > -1 \\ x^2 - 2 & if \quad -1 \leq x < 3 \end{cases}$ (2)

Then $f(-1) =$

A -4

B -2

C -1

D None of the above mentioned.

1.8 $\int (qx^2 + p)^n dx =$ (2)

A $2qs + c$

B $\frac{2qx}{n+1} + c$

C $\frac{(qx^2+p)^{n+1}}{(n+1).2qx} + c$

D $n(qx^2 + p)(2qx)$

1.9 The following is a factor of $f(x) = 2x^3 + 13x^2 + 13x - 10$ (2)

A $x = \frac{1}{2}$

B $2x + 1$

C $2x - 1$

D $x + \frac{1}{2}$

1.10 Given the vector $(5; -4)$, the magnitude is given by: (2)

A $\sqrt{5^2 + 4^2}$

B $\tan\theta = \frac{-4}{5}$

C $\sqrt{5^2 - 4^2}$

D $\tan\theta = -\frac{5}{4}$

Question 2 **[14 marks]**

2.1 Simplify each expression fully:

2.1.1 i^{75} (2)

2.1.2 $\sqrt{-12}$ (2)

2.2 Prove that (6)

$$\frac{3i^{13}+i^{14}}{4}=\frac{-3i^2+7i-2}{8-4i}$$

2.3 Do the following calculation graphically, make use of the DIAGRAM SHEET provided. (4)

$$(2-i)+(6+4i)$$

Question 3 **[12 marks]**

3.1 Decompose the following fraction into partial fractions: (7)

$$\frac{x^2-7x+4}{(x+1)(x^2-2x+1)}$$

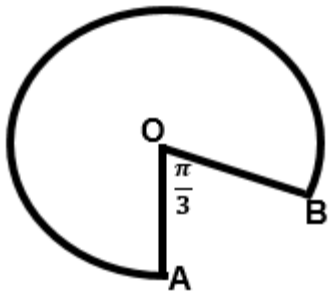
3.2 Given (5)

$$x^4-30x^2+125=0$$

Solve the equation if x^2-5 is a factor.

Question 4 **[7 marks]**

4.1 Given the figure below with radius $6\ cm$ and centre O. Answer the questions that follow:



4.1.1 Determine the perimeter of the figure in terms of π. (3)

4.1.2 Determine the area of the figure in terms of π. (4)

Question 5 **[13 marks]**

5.1 Sketch the piecewise function on the DIAGRAM SHEET. (6)

$$g(x) = \begin{cases} \frac{2}{x} & if \quad x \leq -1 \\ 2x^2 & if \quad -1 < x < 1 \\ -x + 2 & if \quad x \geq 1 \end{cases}$$

5.2 Given that $f(x) = x^2 + 1$ and $g(x) = \frac{1}{\sqrt{x}}$, determine and simplify:

5.2.1 $(f \circ g)(x)$ (2)

5.2.2 $(g \circ f)(3)$ (3)

5.3 If $F(x) = (f \circ g)(x) = \frac{1}{\sqrt{x+5}} + 2(x+5)^2$ determine $f(x)$ and $g(x)$. (2)

Question 6 [13 marks]

6.1 Determine the value of p such that: (7)

$$-19 = \begin{vmatrix} 1 & 4 & p \\ 3 & 1 & -4 \\ p & 5 & 3 \end{vmatrix}$$

6.2 Determine:

6.2.1 $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ (1)

6.2.2 $2\begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix} - 3\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$ (2)

6.2.3 $\begin{bmatrix} 5 & 0 \\ -3 & 4 \\ 7 & 1 \end{bmatrix}^T$ (1)

6.2.4 $\begin{bmatrix} 1 & 2 \\ 9 & 0 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} -4 & 3 \\ 2 & 8 \\ 6 & -5 \end{bmatrix}$ (2)

Question 7 [15 marks]

7.1 Vectors $a = (-2; 3)$ and $b = (4; 2)$ are given, determine:

7.1.1 The magnitude of a. (2)

7.1.2 $a.b$ (2)

7.1.3 The magnitude of the angle between a and b. (4)

7.1.4 Determine the direction of b. (2)

7.1.5 $2a - 3b$ (2)

7.2 If vector $u = (6; 2)$ is perpendicular to vector $v = (-3; a)$ determine the value of a. (3)

Question 8 **[13 marks]**

Determine each of the following derivatives:

8.1 $f(x) = 10x^3 - \frac{1}{2}x^2 + 6x + 3$ (3)

8.2 $\frac{d}{dx}\left(\frac{2x^2+3x-4}{x}\right)$ (3)

8.3 $y = \frac{6}{\sqrt[3]{x^2}} + \frac{2}{x^4} - x^0$ (3)

8.4 $g(x) = \frac{2}{\sqrt[5]{2x^4-3x^3+7x}}$ (4)

Question 9 **[13 marks]**

9.1 Determine the equation of $f(x)$ which has derivative $f'(x) = 2x^3 - 4x^2 + 1$ if $f(x)$ passes through $(0; 2)$. (5)

9.2 Calculate the area between $h(x) = x^3 + x^2 - 3x - 5$ and $k(x) = x^3 - 1$ (8)

-END OF QUESTION PAPER-

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Final Examination 2022 Answer sheet

Name and Surname: _____

Question Total	1 [20]	2 [14]	3 [12]	4 [7]	5 [13]	6 [13]	7 [15]	8 [13]	9 [13]
Learner mark									

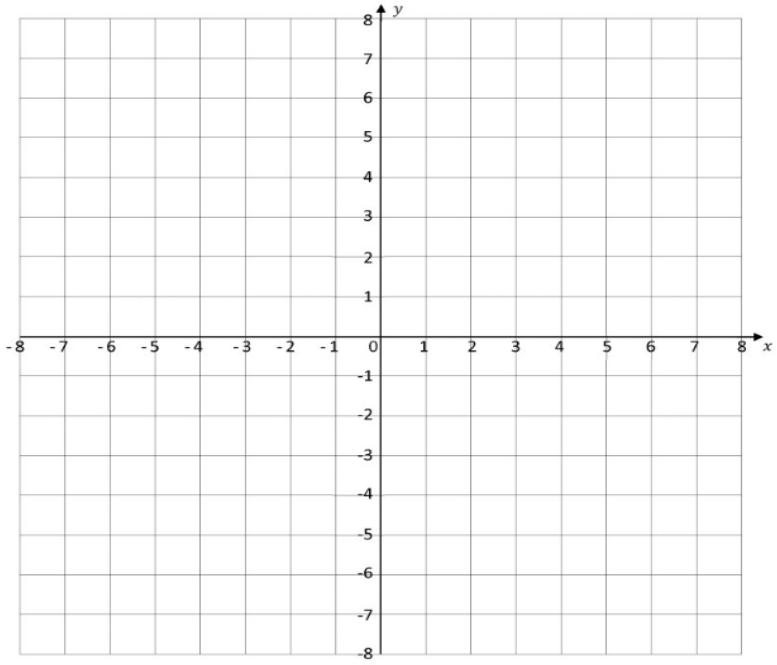
TOTAL 120

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Question 2

2.3



Question 5

5.1