

α -WISKUNDE/ MATHEMATICS

24 Oktober 2022/ *October 24, 2022*
Graad/ Grade 12

Tyd/ Time: 3 ure/ *hours*
Totaal/ Total: 200 PUNTE/ *MARKS*

VRAAG/ QUESTION 1 [20 PUNTE/ MARKS]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

VRAAG/ QUESTION 2 [20 PUNTE/ MARKS]

2.1 (a)	$P(39) = 6,9(1,011)^{39}$ $\checkmark = 10,57 \checkmark$	1: 39 1: Vervang reg 1: Antwoord [3]
(b)	$2 = 1,011^t \checkmark$ $t = \log_{1,011} 2 \checkmark$ $= 63,36 \checkmark$ Dus 2074 \checkmark	1: 2 1: log 1: 63,4 1: Antwoord [4]
(c)	$P'(t) = 1,011^t \checkmark \cdot \ln 1,011 \cdot 6,9 \checkmark$ $\therefore P'(11) = 0,0851 \dots \checkmark$ $= 85 \text{ miljoen } \checkmark$	2: Differentieer 1: 11 1: 0,085... 1: Antwoord [5]
2.2 (a)	$\ln(x+1) - 1 = 0 \checkmark ; \ln(x+1) = 1$ $x+1 = e$ $x = e - 1 = 1,72 \checkmark$ $y = \ln 1 \checkmark - 1 = -1 \checkmark$	1: Stel = 0 1: y-afsnit 1: Stel $x = 0$ 1: x-afsnit [4]
(b)		1: y-afsnit 1: x-afsnit 1: asimptoot 1: Grafiek [4]

VRAAG/ QUESTION 3 [23 PUNTE/ MARKS]

3.1	<p>As $4x - 12 \geq 0$ ✓ $4x - 12 = -6x + 6$ ✓ $x = 1,8$ nie geldig ✓</p> <p>As $4x - 12 < 0$ ✓ $-4x + 12 = -6x + 6$ ✓ $x = -3$ ✓</p>	<p>2: Voorwaardes 2: Berekeninge 1: 1,8 nie geldig 1: -3</p> <p style="text-align: right;">[6]</p>
3.2 (a)	<p>$(x^2 - 2)(x^2 + 2)$ ✓✓ $(x - \sqrt{2})(x + \sqrt{2})$ ✓ $(x - i\sqrt{2})(x + i\sqrt{2})$ ✓</p>	<p>1: 1e ontbinding 1: 1e twee hakies 2: 2e twee hakies</p> <p style="text-align: right;">[4]</p>
(b)	<p>1 - i en $2 + \sqrt{2}$ ook wortels $x - 1 = \pm i$ en $x - 2 = \sqrt{2}$ ✓✓ $x^2 - 2x + 1 = -1$ en $x^2 - 4x + 4 = 2$ ✓✓ Dus $x^2 - 2x + 2$ en $x^2 - 4x + 2$ is faktore ✓✓ $2x^5 - 13x^4 + 30x^3 - 36x^2 + 20x - 4 = 0$ $(x^2 - 2x + 2)(x^2 - 4x + 2)(2x - 1) = 0$ ✓ Dus $x = \frac{1}{2}$ ✓</p>	<p>1: Ander 2 wortels 6: Bereken faktore 1: Ander faktor 1: Antwoord</p> <p style="text-align: right;">[8]</p>
3.3	<p>$\sqrt{1 - 3x} = (1 - 3x)^{\frac{1}{3}}$ ✓ $n = \frac{1}{3}$ $1 + \frac{1}{3}(-3x) + \frac{(\frac{1}{3})(-\frac{2}{3})(-3x)^2}{2} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(-3x)^3}{6}$ ✓ $= 1 - x$ ✓ $- x^2$ ✓ $-\frac{5x^3}{3}$ ✓</p>	<p>1: Eksponent 1: Vervang reg 3: Terme reg bereken</p> <p style="text-align: right;">[5]</p>

VRAAG/ QUESTION 4 [20 PUNTE/ MARKS]

4.1	<p>Stel $n=1$: LK = 1 RK = 1, dus waar vir $n=1$ ✓ Aanvaar ✓ waar vir $n=k$:</p> $\sum_{p=1}^k (3p - 2)^2 = \frac{1}{2}k(6k^2 - 3k - 1) \checkmark$ <p>Stel $n=k+1$: LK = $\frac{1}{2}k(6k^2 - 3k - 1) \checkmark + (3(k+1) - 2)^2 \checkmark$</p> $= \frac{6k^3 - 3k^2 - k + 2(3k+1)^2}{2} \checkmark$ $= \frac{6k^3 - 3k^2 - k + 18k^2 + 12k + 2}{2}$ $= \frac{6k^3 + 15k^2 + 11k + 2}{2} \checkmark$ <p>RK = $\frac{(k+1)(6(k+1)^2 - 3(k+1) - 1)}{2} \checkmark$</p> $= \frac{(k+1)(6k^2 + 12k + 6 - 3k - 3 - 1)}{2} \checkmark$ $= \frac{(k+1)(6k^2 + 9k + 2)}{2}$ $= \frac{6k^3 + 15k^2 + 11k + 2}{2}$ <p>Volgens die beginsel van wiskundige induksie is die stelling waar vir alle $n \in N$ $\frac{6k^3 + 15k^2 + 11k + 2}{2} \checkmark$</p>	<p>1: $n=1$ 1: $n=k$ 1: Aanvaar 2: Lk $n=k+1$ 2: Vereenvoudig LK 2: Vereenvoudig RK 1: Storie</p> <p style="text-align: right;">[10]</p>
4.2	<p>$r = 2 \checkmark$ en $\theta = \pi - \frac{\pi}{6} \checkmark = \frac{5\pi}{6} \checkmark$</p> $2^8 \left(\text{cis} \frac{5\pi}{6} \right)^8 = 256 \checkmark \text{cis} \frac{20\pi}{3} \checkmark$ $x = 256 \cos \frac{20\pi}{3} = -128 \text{ en } y = 256 \sin \frac{20\pi}{3} = 128\sqrt{3}$ <p>Dus $-128 \checkmark + 128\sqrt{3}i \checkmark$</p>	<p>1: Radius 2: Hoek 1: 256 1: Maal hoek met 8 2: Antwoord</p> <p style="text-align: right;">[7]</p>
4.3	$x = \frac{\begin{vmatrix} 7 & 2 & -1 \\ 9 & 1 & -4 \\ 2 & 0 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & 2 & -1 \\ 0 & 1 & -4 \\ 9 & 0 & 4 \end{vmatrix}} \checkmark$	<p>2: Elke determinant 1: Regte volgorde</p> <p style="text-align: right;">[3]</p>

VRAAG/ QUESTION 5 [18 PUNTE/ MARKS]

5.1	$AC = 3a\checkmark + 2b\checkmark$ $AM = \frac{3}{2}a + b\checkmark$	2: AC 1: AM [3]
5.2 a	$u = DE = (-3; 1; -2) = -3i + j - 2k\checkmark$ $v = EF = (5; 5; -3) = 5i + 5j - 3k\checkmark$	1: u 1: v [2]
b	$u \cdot v = -15 + 5 + 6 = -4\checkmark$ $ u = \sqrt{9 + 1 + 4} = \sqrt{14}\checkmark$ $ v = \sqrt{25 + 25 + 9} = \sqrt{59}\checkmark$ $\cos \theta = -\frac{4}{\sqrt{14 \times 59}}\checkmark\checkmark$ $\theta = 1,71 \text{ rad}\checkmark$	1: u.v 1: Grootte u 1: Grootte v 2: Verander formule en vervang reg 1: Antwoord [6]
c	$u \times v = \begin{vmatrix} i & j & k \\ -3 & 1 & -2 \\ 5 & 5 & -3 \end{vmatrix}\checkmark$ $= i(-3 + 10)\checkmark - j(9 + 10)\checkmark + k(-15 - 5)\checkmark$ $= 7i - 19j - 20k\checkmark$ $\text{Area} = \sqrt{49 + 361 + 400}\checkmark$ $= \sqrt{810} (28,46)\checkmark$	1: Determinant 3: Vereenvoudig 1: Kruisproduk 1: Bereken oppervlakte 1: Antwoord [7]

VRAAG/ QUESTION 6 [23 PUNTE/ MARKS]

6.1	Sektor se hoek = $2\pi - 1,4 = 4,88$ ✓ Sektor oppervl = $\frac{1}{2} \times 100 \times (2\pi - 1,4) = 244,16$ ✓ Volume = $20 \times 244,16 = 4883,19$ kub m ✓ Massa van graan = $\frac{80}{100} \times 4883,19 \times 760$ $= 2\,968\,976,67$ kg ✓	1: Hoek 1: Oppervlakte 1: Volume 1: Antwoord	[4]
6.2	$\frac{\pi}{3} = a \text{ b gcos}(1) + p$ $p = \frac{\pi}{3}$ ✓ $0 = a \text{ b gcos}\left(\frac{1}{2} + \frac{\pi}{3}\right)$ ✓ $a \cdot \frac{\pi}{3} = -\frac{\pi}{3}, a = -1$ ✓	1: Vervang 1: p 1: Vervang 1: b gcos $\frac{1}{2}$ 1: a	[4]
6.3 a	$b^2 - 8 = 2b$ ✓ $(b - 4)(b + 2) = 0$ ✓ $b = -2$ ✓ $-4 = -4(0) + c$ ✓ $c = -4$ ✓	1: $x = b$ 1: Faktoreer 1: Net een antwoord 1: $x = 1$ 1: Antwoord	[5]
b	$\lim_{x \rightarrow 1^-} f'(x) = 0$ ✓ $\lim_{x \rightarrow 1^+} f'(x) = -8(1 - 1) = 0$ ✓✓ Ja, differensieerbaar by $x = 1$ ✓	1: Limiet van LK 1: Differensieer 1: =0 1: Ja	[4]
6.4	$m = f'(x) = 2(\ln x) \cdot \frac{1}{x} + 1$ ✓ Maks as $f''(x) = 0$ ✓ $\frac{2}{x^2} - \frac{2 \ln x}{x^2} = 0$ ✓ $\ln x = 1$ ✓ $x = e$ ✓	1: 1e afgeleide 1: $f''(x) = 0$ 2: Differensieer 1: Vereenvoudig 1: Antwoord	[6]

VRAAG/ QUESTION 7 [20 PUNTE/ MARKS]

7.1 a	$\frac{dy}{dx} = e^{\tan 3x} \cdot \sec^2(3x) \cdot 3 - \frac{1}{x \cdot \ln 10} \cdot (\log(3x) = \log 3 + \log x)$	4: Soos getoon [4]
b	$D_x [\arcsin(\sqrt{1-x^2})] = \frac{1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2} (1-x^2)^{-\frac{1}{2}} \cdot (-2x)$ $= \frac{1}{ x } \cdot -\frac{x}{\sqrt{1-x^2}} = -\frac{1}{\sqrt{1-x^2}}$	5: Soos getoon [5]
c	$f'(x) = \sec(\cos x) \times \tan(\cos x) \times (-\sin x)$	3: Soos getoon [3]
7.2	$2x \cdot \ln y + x^2 \cdot \frac{1}{y} \cdot y' + 2 + 5y' = 0$ $y' \left(\frac{x^2}{y} + 5 \right) = -2x \cdot \ln y - 2$ $m = y' = \frac{-2x \cdot \ln y - 2}{\frac{x^2}{y} + 5}$ $m(\text{raaklyn}) = \frac{0-2}{9+5} = -\frac{1}{7}$ $m(\text{normaal}) = 7$ $y - 1 = 7(x - 3); y = 7x - 20$	3: Differensieer 2: y' 1: m raaklyn 1: m normaal 1: Vergelyking [8]

VRAAG/ QUESTION 8 [20 PUNTE/ MARKS]

8.1 a	$\frac{dy}{dx} = \frac{3x^2(2-5x)\checkmark - (x^3+8)(-5)\checkmark}{(2-5x)^2} = 0\checkmark$ $6x^2 - 15x^3 + 5x^3 + 40 = 0\checkmark$ $5x^3 - 3x^2 - 20 = 0$	2: Differensieer 1: =0 1: vereenvoudig [4]
(b)	Stel $f(x) = 5x^3 - 3x^2 - 20$ $f'(x) = 15x^2 - 6x\checkmark$ Newton: $x_1 = x_0 - \frac{5x_0^3 - 3x_0^2 - 20}{15x_0^2 - 6x_0}\checkmark$ $x = 1,815\checkmark\checkmark$	1: Differensieer 1: Newton 1: Antwoord 1: Desimale reg [4]
8.2 a	$x = 3\checkmark$	1: Antwoord [1]
b	$x = 0\checkmark$ buigpunt, \checkmark want $h''(0) = 0\checkmark$ $x = 3\checkmark$ maks draaipunt \checkmark want $h''(3) < 0\checkmark$	2: Elke stasionêre punt 4: Watter soort en redes [6]
8.3	$y = \frac{(x-2)(x^2+x-1)}{(x-2)(x+2)}\checkmark$ $x = -2\checkmark$ Vertikaal \checkmark Skuins \checkmark : $y = x - 1\checkmark$	1: (x-2) 2: x=-2 2: y=x-1 [5]

VRAAG/ QUESTION 9 [20 PUNTE/ MARKS]

9.1	$\Delta x_i = \frac{3}{n} \checkmark ; x_i = \frac{3}{n} \cdot i \checkmark$ $f(x_i) = -\left(\frac{3}{n} \cdot i\right)^2 + 1 = -\frac{9}{n^2} i^2 + 1 \checkmark$ $\sum_{i=1}^n \left(\frac{9}{n^2} i^2 + 1\right) = \frac{9}{n^2} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) \checkmark + n \checkmark$ $= -2n - \frac{9}{2} - \frac{3}{2n} \checkmark$ $\int_0^3 (-x^2 + 1) dx = \text{basis x hoogte}$ $= \frac{3}{n} \times \left(-2n \checkmark - \frac{9}{2} - \frac{3}{2n}\right) = -6 - \frac{27}{2n} - \frac{9}{2n^2} \checkmark$ $\int_0^3 (-x^2 + 1) dx = \lim_{n \rightarrow \infty} \left(-6 - \frac{27}{2n} - \frac{9}{2n^2}\right) = -6 \checkmark$ <p>Dus $\int_0^3 (-x^2 + 1) dx = -6$</p>	1: Δx_i 1: x_i 1: $f(x_i)$ 1: Sigma 1: Vervang/ Substitute i^2 1: Vervang/ Substitute 1 1: $\Delta x_i \times f(x_i)$ 1: Antwoord/ Answer [9]
9.2	$\int (\sin(5x) \times \cos(2x)) dx = \frac{1}{2} \int (\sin 7x + \sin 3x) \checkmark dx$ $= \frac{1}{2} \left(-\frac{\cos 7x}{7} \checkmark + -\frac{\cos 3x}{3} \checkmark\right) + k \checkmark$ $= -\frac{\cos 7x}{14} + \frac{\cos 3x}{6} + k$	1: Formule 2: Elke integraal 1: +k [4]
9.3	<p>Stel $\frac{x+1}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \checkmark$</p> $x + 1 = A(x - 2) + B \checkmark$ <p>Stel $x = 2$: $3 = B \checkmark$</p> <p>$x = 1 = A, \checkmark$</p> $\int \left(\frac{1}{x-2} + \frac{3}{(x-2)^2}\right) dx = \ln(x-2) \checkmark - 3(x-2)^{-1} \checkmark \checkmark + k$	1: C 1: A 1: B 1: Maak integraal reg/ 1: $\ln(x-2)$ 1: $(x-2)^{-2}$ 1: $-3(x-2)^{-1}$ [7]

VRAAG/ QUESTION 10 [16 PUNTE/ MARKS]

10.1	$V = \pi \int_0^2 x \cdot e^{2x} dx \checkmark$ $\text{Stel } f(x) = x \text{ en } g'(x) = e^{2x} \checkmark$ $\text{Dus } f'(x) = 1 \text{ en } g(x) = \frac{e^{2x}}{2} \checkmark$ $V = \pi \left(\frac{x \cdot e^{2x}}{2} - \int \left(\frac{e^{2x}}{2} \right) dx \right) \checkmark$ $V = \pi \left(\frac{x e^{2x}}{2} - \frac{e^{2x}}{4} \right) \Big _0^2 \checkmark$ $V = \pi \left(\left(e^4 - \frac{e^4}{4} \right) \checkmark - \left(-\frac{1}{4} \right) \checkmark \right)$ $V = \pi \left(\frac{3e^4}{4} + \frac{1}{4} \right) \checkmark$	1: Kwadreer 2: Stel reg 1: 1e stap integreer 1: 2e stap 2: Vervang 1: Antwoord [8]
10.2	$\int \frac{x^2}{1+x^3} dx = 2 \checkmark$ $\frac{1}{3} \ln 1+x^3 \Big _1^p = 2$ $\ln(1+x^3) \Big _1^p = 6 \checkmark$ $\ln(1+p^3) - \ln 2 \checkmark = 6$ $\ln \frac{1+p^3}{2} = 6 \checkmark$ $1+p^3 = 2e^6 \checkmark$ $p = \sqrt[3]{2e^6 - 1} \checkmark = \sqrt[3]{a \cdot e^c + b}$ $\text{Dus } a = 2, c = 6 \text{ en } b = -1 \checkmark$	1: Vergelyking 1: Integreer reg 1: Skei 1: Vervang 1: $2 = \ln e^2$ 1: Bring saam 1: p alleen 1: Antwoorde [8]