

α MATHEMATICS

PRELIMINARY EXAM

September 2022

Grade 12

Time: 3 hours

Total: 200 marks

INSTRUCTIONS AND INFORMATION

Please read the following instructions carefully before answering any questions:

1. Answer all nine questions of this paper.
2. Write your name on the answer sheet.
3. Non-programmable calculators may be used unless otherwise indicated.
4. Unless indicated otherwise, all answers, where applicable, must be given correctly to **two decimal places**.
5. The diagrams in the question paper are not necessarily drawn to scale.
6. **All angles are given in radians.** Answers must be given in radians where applicable.
7. This question paper consists of seven pages, a formula sheet of three pages and an answer sheet of three pages.
8. Question 1 consists of 10 multiple choice questions. Answer this on the answer sheet.
9. All necessary calculations must be shown. The correct answer on its own will not necessarily lead to full marks.
10. Write neatly and legibly.

QUESTION 1**[20 MARKS]**

- The following questions have only ONE correct answer. Only mark the correct letter, A, B, C or D, with a X (cross) on the answer sheet provided. Each question counts 2 marks.

1.1 Solve for x if $2\ln(3x + 1) = 5$

(A) $x = 6,36$

(B) $x = 4,39$

(C) $x = 3,73$

(D) $x = 11,18$

1.2 Solve for x : $|x| = 2x + 1$

(A) $x = -\frac{1}{3}$

(B) $x = -\frac{1}{3}$ of $x = -1$

(C) $x = -1$

(D) No solution

1.3 The second term in the binomial expansion of $\frac{1}{2-x}$ is

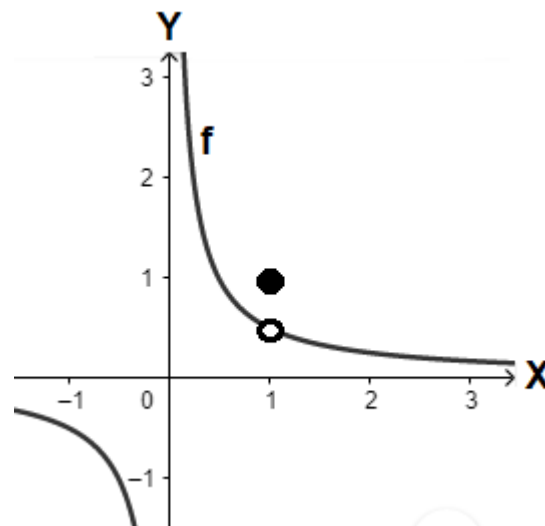
(A) x

(B) $\frac{x}{2}$

(C) $\frac{-x}{2}$

(D) $\frac{x}{4}$

1.4 The graph shows the function of f . What type of discontinuity occurs at $x = 1$?



(A) Asymptotic discontinuity

(B) Removable discontinuity

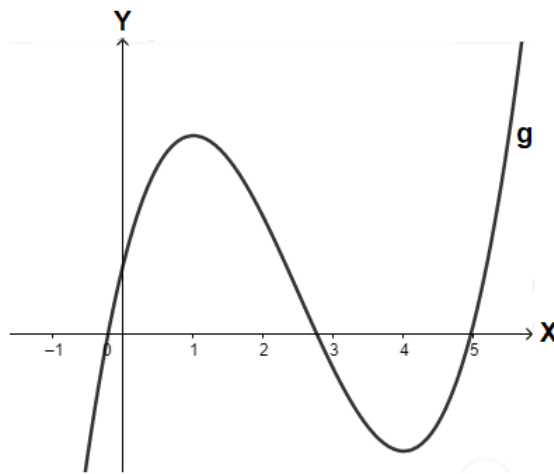
(C) Jump discontinuity

(D) None of the above

1.5 Given: $f(x) = \frac{x^2+x-2}{x-1}$ Which one of the following statements is true?

- (A) f has a vertical asymptote at $x = 1$
- (B) f is undefined at $x = 1$
- (C) f is continuous at $x = 1$
- (D) None of the above

1.6 The following sketch shows the graph of g . The x -intercepts of the function are between $x = 2$ and $x = 3$ and stationary points at $x = 1$ and $x = 4$. If the Newton-Rhapson method is used, which value of x would have been the best initial approximation to calculate the intercept between $x = 2$ and $x = 3$?



- (A) $x = 4$
- (B) $x = 5$
- (C) $x = 3,9$
- (D) $x = 2,5$

1.7 Calculate the horizontal asymptote of $h(x) = \frac{2x^3+3x^2+x-7}{5x^3+x^2+3x-2}$

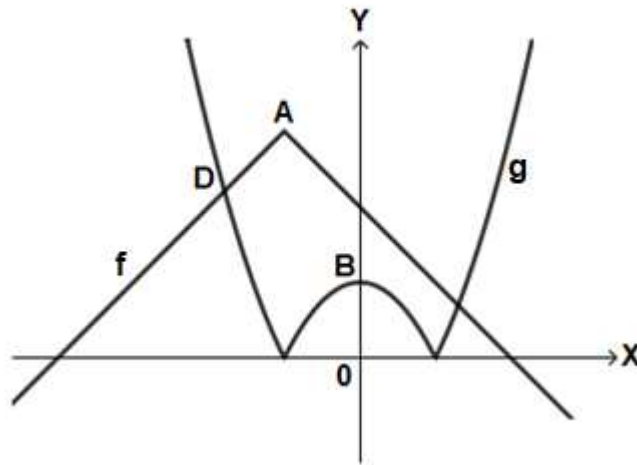
- (A) $y = \frac{7}{2}$
- (B) $y = 0$
- (C) $y = \frac{2}{5}$
- (D) $y = 3$

1.8 Which integration method will be used to calculate $\int \frac{x^2-4x+2}{(2x-1)(-x+1)^2} dx$?

- (A) Integration by partial fraction
- (B) Integration by parts
- (C) Integration by substitution
- (D) No solution

QUESTION 3**[26 MARKS]**

3.1 The sketch shows the graphs of $f(x) = -|x + 1| + 4$ and $g(x) = |x^2 - 1|$



- (a) Determine the coordinates of A, the salient point of f . (2)
- (b) Determine the coordinates of B, the y -intercept of g . (1)
- (c) Determine the coordinates of D, the intersection of f and g . (5)
- (d) Determine algebraically the value(s) of x for which $f(x) < 1$. (5)

3.2 Create a polynomial, g , with the following properties:

- The function has a zero at $x = 1 + i$
- $g(-1) = 0$ (5)

3.3 The constant term of the binomial expansion of $\left(x + \frac{a}{x^3}\right)^8$ is 7.

Determine the value(s) of a . (8)

QUESTION 4**[24 MARKS]**

4.1 Use mathematical induction to prove that the following statement is true for all $n \in \mathbb{N}$:

$$1(2) + 2(5) + 3(8) + \dots + n(3n - 1) = n^2(n + 1) \quad (10)$$

4.2 Given vectors $\mathbf{t} = (a; -1; -2)$ and $\mathbf{p} = (2; -a; 3)$

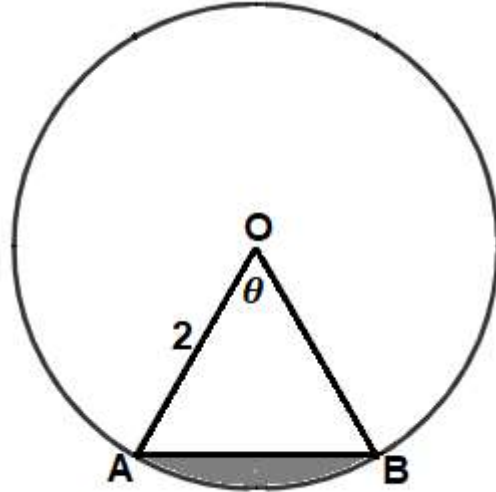
- (a) The vectors \mathbf{t} and \mathbf{p} are perpendicular. Show that $a = 2$. (4)
- (b) Determine two vectors perpendicular to both \mathbf{t} and \mathbf{p} . (4)
- (c) Determine the area of the parallelogram with \mathbf{t} and \mathbf{p} as adjacent sides. (2)
- (d) Determine the size of the angle formed between vector \mathbf{p} and the z -axis. (4)

QUESTION 5**[21 MARKS]**

5.1 Given $f(x) = \arccos\left(\frac{x}{2}\right) - \frac{\pi}{4}$

Sketch f on the answer sheet. Clearly indicate all intercepts with the axes, as well as the starting point and the end point of the function (if applicable). (7)

- 5.2 The diagram shows a circle with centre O and a radius of 2cm. Points A and B lie on the circle.



- (a) Calculate the area of the **small** sector AOB , in terms of θ . (2)
- (b) Calculate the area of the **large** sector AOB , in terms of θ . (2)
- (c) Show that $\theta = \frac{\pi}{3}$, if the area of the **large** sector AOB is **five** times the area of the **small** sector AOB . (2)
- (d) Calculate the **perimeter** of the coloured segment. (Give your answer in terms of π). (3)

- 5.3 Use Newton-Rhaphson's method to calculate the point of intersection of $f(x) = \tan x$ and $g(x) = \arccos x$. Use $x = 0,5$ as initial approximation and give your answer correct to four decimal places. Clearly show all the methods used to determine your answer. (5)

QUESTION 6**[19 MARKS]**

6.1 Consider the function $g(x) = \begin{cases} 2 - x^2 & \text{if } x \leq a \\ x - 4 & \text{if } x > a \end{cases}$

- (a) Find the value(s) of a which will make g continuous at $x = a$. Show all calculations. (4)
- (b) If $a = 2$, is g differentiable at $x = 2$? Motivate your answer thoroughly. (4)

6.2 Differentiate the following functions with respect to the variable:

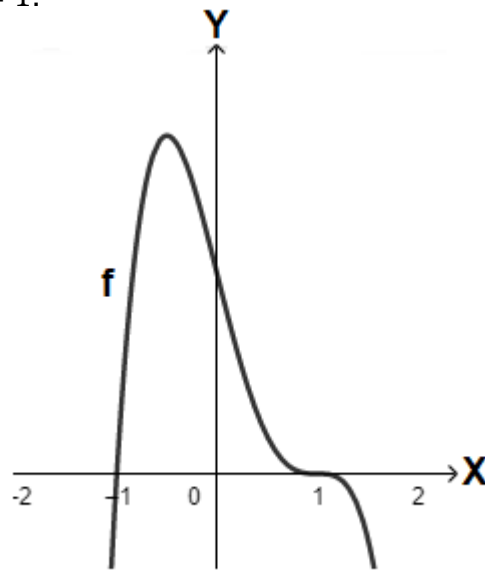
(a) $p(x) = 4^{3x} + 4x^3 + \log_3(4x)$ (7)

(b) $t(x) = [\arctan(2x)]^5$ (4)

QUESTION 7

[21 MARKS]

7.1 The sketch shows the graph of $f(x)$. The function has x -intercepts at $x = -1$ and $x = 1$, stationary points at $x = -\frac{1}{2}$ and $x = 1$ and inflection points at $x = 0$ en $x = 1$.



At which value(s) of x is

(a) $f'(x) > 0$? Motivate your answer. (2)

(b) $f'(x) = 0$? Motivate your answer. (3)

(c) Use the answer sheet and draw a possible sketch of f'' . (5)

7.2 (a) Use implicit differentiation to determine $\frac{dy}{dx}$ if $e^{xy} = \ln y + e$ (7)

(b) Calculate the gradient of the normal at the point $(1; 1)$. (4)

QUESTION 8**[19 MARKS]**

8.1 Use the answer sheet to draw a graph of the rational function, f , with the following properties. Indicate all intercepts, stationary points, and asymptotes on your sketch.

- $f(-2) = 0$
- $f(2) = 4$
- $f'(2) = 0$
- $f''(2) > 0$
- $\lim_{x \rightarrow 0^-} f(x) = \infty$ en $\lim_{x \rightarrow 0^+} f(x) = \infty$
- $\lim_{x \rightarrow \pm\infty} (f(x) - x) = 0$ (8)

8.2 Use a Riemann sum to calculate the value of $\int_1^2 (-x^2) dx$. (11)

QUESTION 9**[26 MARKS]**

9.1 Determine the following integrals:

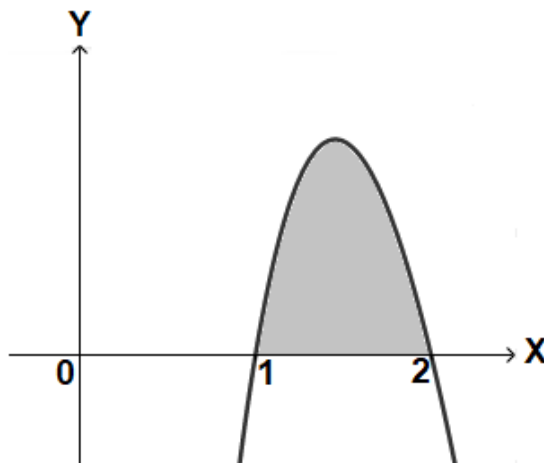
(a) $\int 5^{3x} dx$ (3)

(b) $\int \frac{e^{2x}}{1+e^{2x}} dx$ (5)

(c) $\int \frac{e^x}{1+e^{2x}} dx$ (2)

(d) $\int (\sec x \cdot \tan x + \sin 2x \cdot \cos x) dx$ (5)

9.2 The sketch shows part of the graph of $y = (3 \ln x)(-2x + 4)$. Use integration by parts to calculate the area between $x = 1$ en $x = 2$. Show how you have used integration by parts. (11)

**END OF PAPER**