

α -WISKUNDE/ MATHEMATICS

Junie/ June 2022
Graad/ Grade 12

Tyd/ Time: 2,5 uur/ hours
Totaal/ Total: 170 PUNTE/ MARKS

VRAAG/ QUESTION 1 [20 PUNTE/ MARKS]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

VRAAG/ QUESTION 2 [22 PUNTE/ MARKS]

2.1a	$\checkmark \checkmark$ $A\left(\frac{1}{2}; 3\right)$ $B(0; 2) \checkmark$ $C(-1; 0) \checkmark$ $D(2; 0) \checkmark$	A:2 B:1 C:1 D:1 <p style="text-align: right;">[5]</p>
2.1b	$ 2x - 1 = -2x + 3 \checkmark$ As $x \geq \frac{1}{2}$: $2x - 1 = -2x + 3 \checkmark$ $\therefore x = 1 \checkmark$ As $x < \frac{1}{2}$: $2x - 1 = 2x - 3 \checkmark$ $-1 \neq 3$ \therefore geen oplossing \checkmark	1: Vereenvoudig/ <i>Simplify</i> 1: Vereenvoudig/ <i>Simplify</i> 1: Antwoord/ <i>Answer</i> 1: Vereenvoudig/ <i>Simplify</i> 1: Antwoord/ <i>Answer</i> <p style="text-align: right;">[5]</p>
2.2	\checkmark $x = -i$ is ook 'n nulpunt van f $(x + i)(x - i) \checkmark$ is 'n faktor $= x^2 + 1 \checkmark$ $\frac{2x^3 - 3x^2 + 2x - 3}{x^2 + 1} = 2x - 3 \checkmark$ $\therefore f(x) = (x - i)(x + i)(2x - 3) \checkmark$	1: $x = -i$ 1: $(x + i)(x - i)$ 1: Antwoord/ <i>Answer</i> 1: Langdeling/ <i>Long division</i> 1: Antwoord/ <i>Answer</i> <p style="text-align: right;">[5]</p>
2.3	$ A = \begin{vmatrix} k & 2 \\ 2 & k \end{vmatrix} = k^2 - 4 \checkmark$ $ A_y = \begin{vmatrix} k & 3 \\ 2 & 0 \end{vmatrix} \checkmark$ $= -6 \checkmark$ $\therefore -\frac{6}{k^2 - 4} \checkmark$ $= 2 \checkmark$ $k = \pm 1 \checkmark \checkmark$	1: Determinant van/ <i>Determinant of A</i> 1: Matriks/ <i>Matrix</i> 1: Determinant van/ <i>Determinant of A_y</i> 1: Toon/ <i>Show Cramer</i> 1: Antwoord/ <i>Answer</i> 2: Antwoord/ <i>Answer</i> <p style="text-align: right;">[7]</p>

3.1a	$f(60) \checkmark$ $= \frac{184}{(\ln 60)^{0,9} + 1,84} \checkmark$ $= 34,10\% \checkmark$	1: Vervang/ <i>Substitute</i> 1: Vereenvoudig/ <i>Simplify</i> 1: Antwoord/ <i>Answer</i> [3]
3.1b	\checkmark $30 = \frac{184}{(\ln t)^{0,9} + 1,84}$ $\therefore \sqrt[0,9]{\frac{184}{30}} - 1,84 = \ln t \checkmark$ $\therefore \ln t = 5,04 \dots \checkmark$ $\therefore t = e^{5,04 \dots} \checkmark$ $\therefore t = 155,68 \text{ minute/minutes} \checkmark$	1: Vervang/ <i>Substitute</i> 1: Vereenvoudig/ <i>Simplify</i> 1: Vereenvoudig/ <i>Simplify</i> 1: Verander na/ <i>Change to e</i> 1: Antwoord/ <i>Answer</i> [5]
3.1c	$f(t) = 184((\ln t)^{0,9} + 1,84)^{-1}$ $f'(t) = -184((\ln t)^{0,9} + 1,84)^{-2} \left(0,9(\ln t)^{-0,1} \cdot \frac{1}{t} \right)$	4: Differensieer/ <i>Differentiate</i> [4]
3.2a	\checkmark $1 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right) \checkmark$	1:1 1: $\frac{\pi}{6}$ [2]
3.2b	\checkmark $2(\cos(\pi) + i \sin(\pi)) \checkmark$	1:2 1: π [2]
3.3c	\checkmark $\frac{1}{2} \left(\cos\left(\frac{-5\pi}{6}\right) + i \sin\left(\frac{-5\pi}{6}\right) \right) \checkmark$	1: $\frac{1}{2}$ 1: $\frac{-5\pi}{6}$ [2]
3.3d	\checkmark $LK = \frac{1}{64} (\cos(-5\pi) + i \sin(-5\pi)) \checkmark$ $= \frac{1}{64} (-1 + 0) \checkmark$ $= \frac{-1}{64} \checkmark$ $RK = \frac{-1}{64} \checkmark$	1: $\frac{1}{64}$ 1: -5π 1: reghoekige vorm/rectangular form 1: Antwoord LK/ <i>Answer LHS</i> 1: Antwoord RK/ <i>Answer RHS</i> [5]

<p>4.1a</p>	$\binom{14}{5} x^9 \left(\frac{3}{x}\right)^5$ $= 486486x^4$	<p>1: $\binom{14}{5}$ 1: x^9 1: $\left(\frac{3}{x}\right)^5$ 1: 486486 1: x^4</p> <p style="text-align: right;">[5]</p>
<p>4.2a</p>	<p>$4x < 1$ ✓ $\therefore x < \frac{1}{4}$ ✓</p>	<p>1: $4x < 1$ 1: Antwoord/ Answer</p> <p style="text-align: right;">[2]</p>
<p>4.2b</p>	$\frac{\frac{1}{2} \left(\frac{1}{2} - 1\right) \left(\frac{1}{2} - 2\right)}{3!} (4x)^3$ $= 4x^3$	<p>1: $\frac{1}{2} \left(\frac{1}{2} - 1\right) \left(\frac{1}{2} - 2\right)$ 1: $(4x)^3$ 1: 3!</p> <p>1: Antwoord/ Answer</p> <p style="text-align: right;">[4]</p>
<p>4.3</p>	<p>Stel/ Set $n = 1$: LK/ LHS = $\frac{1}{2}$ RK/ RHS = $\frac{1}{2}$ \therefore Die bewering is waar as/ The statement is true for $n = 1$ ✓</p> <p>Aanvaar die bewering is waar vir/ Accept the statement is true for $n = k$:</p> $\frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$ <p>Beskou nou/ Consider $n = k + 1$:</p> $\text{LK/ LHS} = \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$ $= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$ $= \frac{2^{k+1} - 2 + 1}{2^{k+1}}$ $= \frac{2^{k+1} - 1}{2^{k+1}}$ <p>RK/ RHS = $1 - \frac{1}{2^{k+1}}$</p> $= \frac{2^{k+1} - 1}{2^{k+1}}$ <p>Volgens die beginsel van wiskundige induksie is die bewering dus waar vir alle $n \in \mathbb{N}$ /By the principle of mathematical induction the statement is true for all $n \in \mathbb{N}$ ✓✓</p>	<p>1: Bewys $n = 1$ / Prove $n = 1$</p> <p>1: Aanvaar/ Accept $n = k$</p> <p>1: Vervang/ Substitute 1: $(k + 1)$de term/ $(k + 1)$th term 2: Vereenvoudig LK / Simplify LHS</p> <p>1: Rk Vervang/ Substitute 2: Vereenvoudig RK/ Simplify RHS</p> <p>2: Afleiding/ Deduction</p> <p style="text-align: right;">[10]</p>

VRAAG/ QUESTION 5 [22 PUNTE/ MARKS]

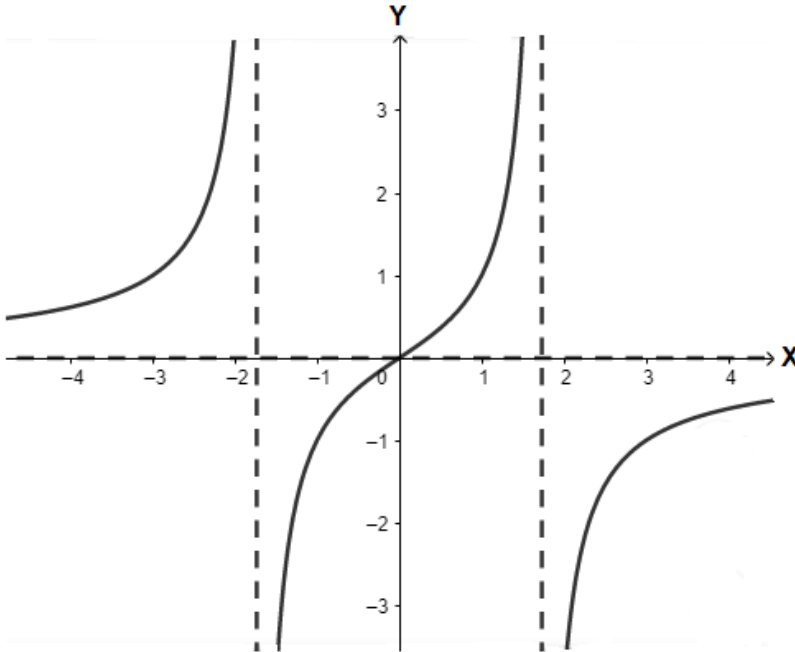
5.1a	$\frac{OC}{12} = \sin \frac{\pi}{3} \quad / \quad \frac{OC}{6} = \tan \frac{\pi}{3} \quad \checkmark$ $\therefore OC = 6\sqrt{3} \quad \checkmark$	1: Trig 1: Antwoord/ Answer [2]
5.1b	$\text{Opp/ Area} = \frac{1}{2} (6)(6\sqrt{3}) \quad \checkmark$ $= 18\sqrt{3} \quad \checkmark$	1: Vervang/ Substitute 1: Antwoord/ Answer [2]
5.1c	$\text{Opp/ Area} = \frac{1}{2} 12^2 \left(\frac{\pi}{3}\right) \quad \checkmark$ $= 24\pi \quad \checkmark$	1: Vervang/ Substitute 1: Antwoord/ Answer [2]
5.1d	$\text{Opp/ Area} = 72\sqrt{3} - (18\sqrt{3} + 24\pi) \quad \checkmark$ $= 18,13 \quad \checkmark$	1: Vereenvoudig/ Simplify 1: Antwoord/ Answer [2]
5.2a	$a = \pi \quad \checkmark$ $0 = 2b \tan(x-1) / 2 \arctan(x-1) \quad \checkmark$ $\therefore x = 1$ $b = 1 \quad \checkmark$ $y = 2b \tan(-1) / 2 \arctan(-1) \quad \checkmark$ $\therefore y = -\frac{\pi}{2}$ $c = -\frac{\pi}{2} \quad \checkmark$	1: a 1: 1: Vervang/ Substitute 1: Antwoord/ Answer 1: 1: Vervang/ Substitute 1: Antwoord/ Answer [5]
5.2b	$x = 2b \tan(y-1) / 2 \arctan(y-1) \quad \checkmark$ $\frac{x}{2} = b \tan(y-1) / \arctan(y-1) \quad \checkmark$ $\therefore y = \tan \frac{x}{2} + 1 \quad \checkmark$	1: Ruil x en y/ Swop x and y 1: Vereenvoudig/ Simplify 1: Antwoord/ Answer [3]
5.3a	$p \cdot q = (2)(-1) - 1(2) + 3(0) \quad \checkmark$ $= -4 \quad \checkmark$	1: Vereenvoudig/ Simplify 1: Antwoord/ Answer [2]
5.3b	$ p = \sqrt{2^2 + (-1)^2 + 3^2} \quad \checkmark$ $= \sqrt{14} \quad \checkmark$	1: Vereenvoudig/ Simplify 1: Antwoord/ Answer [2]
5.3c	$\theta = b \cos \left(-\frac{4}{\sqrt{14}\sqrt{5}} \right) / \arccos \left(-\frac{4}{\sqrt{14}\sqrt{5}} \right) \quad \checkmark$ $= 2,07 \quad \checkmark$	1: Vervang/ Substitute 1: Antwoord/ Answer [2]

VRAAG/ QUESTION 6 [22 PUNTE/ MARKS]

6.1	$g'(x) = \cos e^{5x} \cdot e^{5x} \cdot 5 + \frac{2^{3x} \cdot \ln 2 \cdot 3}{\sqrt{1 - 2^{6x}}}$	1: $\cos e^{5x}$ 1: e^{5x} 1: 5 1: 2^{3x} 1: $\ln 2$ 1: 3 1: $\sqrt{1 - 2^{6x}}$ [7]
6.2a	$f(x) = -\frac{3x^3}{3} + 3x + k$ $(0; 3): 3 = 0 + 0 + k$ $f(x) = -x^3 + 3x + 3$	1: Integreer/ Integrate 1: Vervang/ Substitute 1: Antwoord/ Answer [3]
6.2b	$x_{n+1} = x_n - \frac{-(x_n)^3 + 3x_n + 3}{-3(x_n)^2 + 3}$ $x \approx 2,10380$	1: Newton 2: Antwoord korrek/ Answer correct 5 des [3]
6.3	$\frac{\frac{1}{y \ln 2} \frac{dy}{dx} \cdot \text{bgtan/arctany} - \log_2 y \cdot \frac{1}{1+y^2} \frac{dy}{dx}}{(\text{bgtan/arctany})^2} = 3e$ $\frac{dy}{dx} = \frac{3e(\text{bgtan/arctan } y)^2}{\frac{1}{y \ln 2} \cdot \text{bgtan/arctany} - \log_2 y \cdot \frac{1}{1+y^2}}$	1: $\frac{1}{y}$ 1: $\frac{1}{\ln 2}$ 1: $\frac{dy}{dx}$ 1: 1:produkteël/ product rule 1: $\frac{1}{1+y^2}$ 1: $\frac{dy}{dx}$ 1: 3e 2: Antwoord/ Answer [9]

VRAAG/ QUESTION 7 [18 PUNTE/ MARKS]

7.1a	$x = -2$ ✓ Minimum ✓ As $x < -2$, sal $f'(x) < 0$ / If $x < -2$, then $f'(x) < 0$ $\therefore f$ daal / f will decrease As $x > -2$, sal $f'(x) > 0$ / If $x > -2$, then $f'(x) > 0 \therefore f$ styg / f will increase ✓✓✓✓	1: $x = -2$ 1: Minimum 4: Motivering/ <i>Motivation</i> [6]
7.1b	$x = -1$ ✓ f het 'n buigpunt as $f''(x) = 0$, dit is waar f' 'n stasionêre punt het en $f'(-1) \neq 0$ / f has a point of inflection at $f''(x) = 0$, that is the stationary point of f' and $f'(-1) \neq 0$ ✓✓	1: $x = -1$ 2: Motivering/ <i>Motivation</i> [3]
7.1c	$x = 1$ ✓ $f'(1) = 0$ (x -afsnit/ <i>intercept</i>) en/ and $f''(1) = 0$ (f' se stasionêre punt/ <i>the stationary point of f'</i>) ✓✓	1: $x = -1$ 2: Motivering/ <i>Motivation</i> [3]
7.1d	$x < -1$ en/ and $x > 1$ ✓ ✓ f buig konkaaf op as $f''(x) > 0$, dit is as f' styg/ f bends concave <i>upwards if $f''(x) > 0$, that is if f' increases</i> ✓	1: $x < -1$ 1: $x > 1$ 1: Motivering/ <i>Motivation</i> [3]
7.1e	$-1 < x < 1$ ✓ ✓	1: $-1 < x$ 1: $x < 1$ [2]
7.2	$f(0) > -2$ (enige waarde > -2 / <i>any value > -2</i>) ✓	1: Antwoord/ <i>Answer</i> [1]

<p>8.1</p>	$h'(x) = \frac{-2(x^2 - 3) - (-2x)(2x)}{(x^2 - 3)^2} = \frac{2x^2 + 6}{(x^2 - 3)^2} \checkmark\checkmark$ $h''(x) = \frac{4x(x^2 - 3)^2 - 2(x^2 - 3)(2x)(2x^2 + 6)}{(x^2 - 3)^4} \checkmark\checkmark$ <p>Stel/ Set $x = -\frac{1}{2}$: $h''\left(-\frac{1}{2}\right) < 0$ Stel/ Set $x = \frac{1}{2}$: $h''\left(-\frac{1}{2}\right) > 0 \checkmark\checkmark$ \therefore buigpunt by/ inflection point at $x = 0 \checkmark$</p>	<p>2: $h'(x)$ 2: $h''(x)$ 2: Motivering/ Motivation 1: Afleiding/ Deduction [9]</p>
<p>8.2</p>	<p>Vertikaal/ Vertical: $x = \pm\sqrt{3} \checkmark\checkmark$ Horisontaal/ Horizontal: $y = 0 \checkmark\checkmark$</p>	<p>1: Vertikaal/ Vertical 2: $x = \pm\sqrt{3}$ 1: Horisontaal/ Horizontal 1: $y = 0$ [5]</p>
<p>8.3</p>	<p>$h'(x) = \frac{2x^2 + 6}{(x^2 - 3)^2} > 0 \forall x \in \mathbb{R}$ $\therefore h$ is stygend/ increasing $\forall x \in \mathbb{R} \checkmark$</p>	<p>1: $h'(x)$ 1: > 0 1: Afleiding/ Deduction [3]</p>
<p>8.4</p>		<p>1: $x = \sqrt{3}$ 1: $x = -\sqrt{3}$ 1: $y = 0$ (Horisontale asymptoot/ Horizontal asymptote) 1: y-afsnit/ intercept 1: Vorm/ Form [5]</p>