α-MATHEMATICS

Alpha Wiskunde Graad 11 / *Alpha Mathematics Grade 11*

Finale Eksamen 2022 / *Final Examination 2022*

MEMORANDUM

Totaal / *Total*: 150 punte / *marks*

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Moderator: Anna Muller

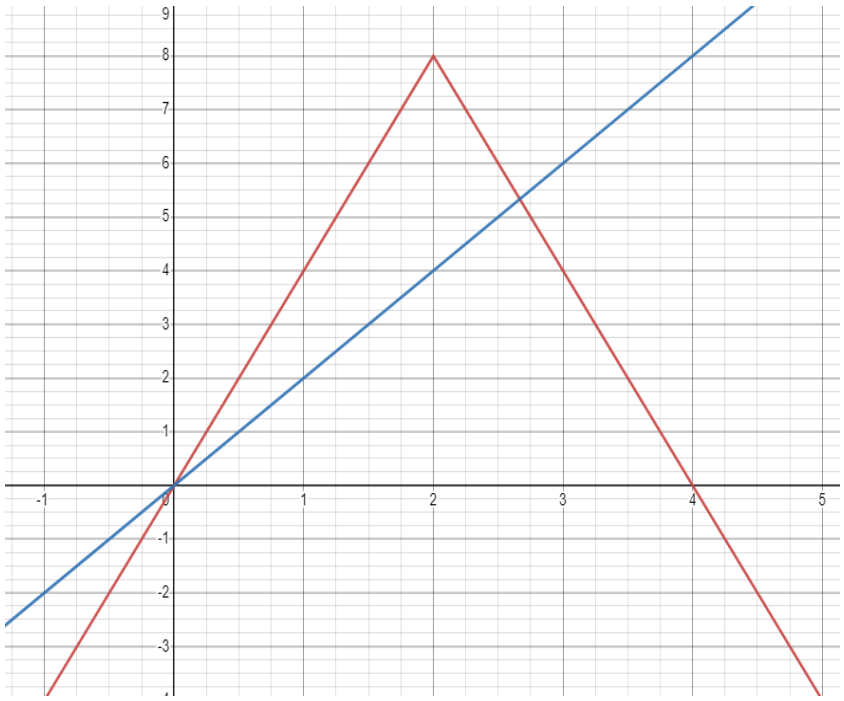
Hierdie memorandum bestaan uit 11 bladsye. /
This memorandum consists of 11 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Vraag / *Question* 2 **[16 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1.1	$4x - 8 = 0$ $x = 2$ Salient point $(2; 8)$✓✓	**(2)**
2.1.2	$-\|4x - 8\| + 8 \geq 4$ $-\|4x - 8\| \geq -4$✓ $\|4x - 8\| \leq 4$✓ $-4 \leq 4x - 8 \leq 4$ $4 \leq 4x \leq 12$✓ $1 \leq x \leq 3$✓	**(4)**
2.1.3	$-\|4x - 8\| + 8 = 2x$ $-\|4x - 8\| = 2x - 8$✓ $\|4x - 8\| = 8 - 2x$✓ For $x \geq 2$✓ $4x - 8 = 8 - 2x$ $6x = 16$ $x = \frac{8}{3}$✓ For $x < 2$✓ $4x - 8 = -8 + 2x$ $2x = 0$ $x = 0$✓	**(6)**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.2		**(4)** ✓$g(x)$ ✓**f(x) intercepts with the axis** ✓**Salient point** ✓**Shape f(x)**

Vraag / *Question* 3 **[27 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1	$2-\sqrt{2}$ is also a zero of p.✓ $p(x)=\left(x-2+\sqrt{2}\right)\left(x-2-\sqrt{2}\right)(x^2-2)$✓ $=(x^2-2)(x^2-2x-2x+4-2)$ ✓ $=x^4-2x^3-2x^3+4x^2-2x^2-2x^2+4x+4x-8+4$ $=x^4-4x^3+8x-4$✓ $a=4$✓ $b=8$✓	**(7)**
3.2	$\frac{4x^3-6x^2+4x-8}{x^2(x^2+4)}\equiv\frac{A}{x}$✓$+\frac{B}{x^2}$✓$+\frac{Cx+D}{x^2+4}$✓ $4x^3-6x^2+4x-8=Ax(x^2+4)+B(x^2+4)+(Cx+D)x^2$✓ $4x^3-6x^2+4x-8=Ax^3+4Ax+Bx^2+4B+Cx^3+Dx^2$✓ $-8=4B$ $B=-2$✓ $4A=4$ $A=1$✓	**(10)**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
	$A + C = 4$ $C = 3$✓ $B + D = -6$ $D = -4$✓ $\frac{4x^3-6x^2+4x-8}{x^2(x^2+4)} = \frac{1}{x} - \frac{2}{x^2} + \frac{3x-4}{x^2+4}$✓	

3.3	$\sum_{r=1}^{n} r(r-1) = \frac{n(n^2-1)}{3}$ $\therefore 0 + 2 + 6 + 12 + \cdots + n(n-1) = \frac{n(n^2-1)}{3}$ **Test if $n = 1$** $LHS = 0$ $\quad RHS = \frac{1(1^2-1)}{3} = 0$ $\therefore LHS = RHS$ The statement is true for $n = 1$. ✓ Accept that the statement is true for $n = k, k \in \mathbb{N}$. ✓ $\sum_{r=1}^{n} r(r-1) = \frac{k(k^2-1)}{3}$ ✓ Consider $n = k + 1$: $LHS = \frac{k(k^2-1)}{3}$ ✓ $+ (k+1)(k+1-1)$ ✓ $= \frac{k^3 - k + 3k^2 + 3k}{3}$ $= \frac{k^3 + 3k^2 + 2k}{3}$ ✓ $RHS = \frac{(k+1)((k+1)^2-1)}{3}$ ✓ $= \frac{(k+1)(k^2+2k)}{3}$ $= \frac{k^3+2k^2+k^2+2k}{3}$ ✓ $= \frac{k^3+3k^2+2k}{3}$ ✓ $LHS = RHS$ **According to mathematical induction the statement is true for all $n \in \mathbb{N}$.** ✓	**(10)**

Vraag / *Question* 4 **[20 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1	$\binom{6}{r}(4x)^{6-r}(a)^r(x^{-2})^r$✓ $6-r-2r=0$✓ $r=2$✓ $\binom{6}{2}(4x)^{6-2}(a)^2(x^{-2})^2$✓ $=34560$✓ $3840a^2=34560$ $a^2=9$ $a=\pm 3$✓	**(6)**
4.2.1	$\frac{1}{(2+x^2)^6}=(2+x^2)^{-6}$✓ $=2^{-6}$✓$\left(1+\frac{x^2}{2}\right)^{-6}$ ✓ $=2^{-6}\left[1-6\left(\frac{x^2}{2}\right)\right.$✓$-\frac{6(-6-1)}{2!}\left(\frac{x^2}{2}\right)^2$ ✓$\left.-\frac{6(-6-1)(-6-2)}{3!}\left(\frac{x^2}{2}\right)^3\right]$✓ $=\frac{1}{64}$✓$-\frac{3x^2}{64}$✓$+\frac{21x^4}{256}$✓$-\frac{7x^6}{64}$✓	**(10)**
4.2.2	$\left\lvert\frac{x^2}{2}\right\rvert<1$✓ $\lvert x^2\rvert<2$✓ $\lvert x\rvert<\sqrt{2}$✓ $-\sqrt{2}<x<\sqrt{2}$✓	**(4)**

Vraag / *Question* 5 **[19 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1.1	$0\checkmark = \arcsin(-1)\checkmark + q$ $\frac{\pi}{2} = q\checkmark$ OR $\pi\checkmark = \arcsin(1)\checkmark + q$ $\frac{\pi}{2} = q\checkmark$	**(3)**
5.1.2	$x = \arcsin(-y) + \frac{\pi}{2}\checkmark$ $x - \frac{\pi}{2}\checkmark = \arcsin(-y)$ $\sin\left(x - \frac{\pi}{2}\right) = -y\checkmark$ $f^{-1}(x) = -\sin\left(x - \frac{\pi}{2}\right)\checkmark$	**(4)**
5.1.3	Range: $y \in [-1; 1]$ ✓✓	**(2)**
5.2.1	Sector ACB $= \frac{1}{2}(10^2)\left(\frac{\pi}{3}\right)\checkmark = \frac{50\pi}{3} = 52{,}36\checkmark$ ΔACB $= \frac{1}{2}(10^2)\sin\frac{\pi}{3}\checkmark = 25\sqrt{3} = 43{,}30\checkmark$ ΔOAB $= \frac{1}{2}(6^2)\sin(1{,}97)\checkmark = 16{,}58\checkmark$ $\therefore$ Area $= 52{,}36 - 43{,}30 + 16{,}58 = 25{,}64\checkmark$	**(7)**
5.2.2	Sector OAXB $= \frac{1}{2}(6^2)\checkmark(1{,}97)\checkmark = 35{,}46$ Area of shaded part $= 35{,}46 - 25{,}64 = 9{,}82\checkmark$	**(3)**

Vraag / *Question* 6 **[20 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1.1	$\|a\| = \sqrt{1^2 + 3^2 + 5^2} = \sqrt{35}$✓ $\gamma = \arccos\left(\frac{5}{\sqrt{35}}\right)$ ✓ $= 0{,}56\ rad$✓	**(3)**
6.1.2	$b \times c = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ -3 & 0 & 1 \end{vmatrix}$ ✓ $= i(-1) - j(2) + k(-3)$✓ $= -i - 2j - 3k$✓ Area=$\sqrt{1^2 + 2^2 + 3^2}$✓ $= \sqrt{14}$✓	**(5)**
6.1.3	$cos\theta = \frac{a.b}{\|a\|\|b\|}$ $= \frac{2 - 3}{\sqrt{35}.\sqrt{5}}$ ✓✓ $\theta = 1{,}65\ rad$✓✓	**(4)**
6.2	$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}$ ✓ $\|A\| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{vmatrix}$ $= \begin{vmatrix} 5 & 3 \\ 0 & 8 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 1 & 8 \end{vmatrix} + 3\begin{vmatrix} 2 & 5 \\ 1 & 0 \end{vmatrix}$ ✓ $= 40 - 26 - 15$ $= -1$✓ $\|A_a\| = \begin{vmatrix} 1 & 2 & 3 \\ 6 & 5 & 3 \\ -6 & 0 & 8 \end{vmatrix}$ ✓ $= \begin{vmatrix} 5 & 3 \\ 0 & 8 \end{vmatrix} - 2\begin{vmatrix} 6 & 3 \\ -6 & 8 \end{vmatrix} + 3\begin{vmatrix} 6 & 5 \\ -6 & 0 \end{vmatrix}$ ✓ $= 40 - 132 + 90$ $= -2$✓ $\frac{\|A_a\|}{\|A\|} = \frac{-2}{-1} = 2$✓✓	**(8)**

Vraag / Question 7 **[8 punte / marks]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	Gradient of straight line $= f'(8) = \frac{6-0}{4-10} = -1$✓✓	**(2)**
7.2	$x = 2$✓✓	**(2)**
7.3	$x = 2$✓ Removable discontinuity✓ $x = 4$✓ Jump discontinuity✓	**(4)**

Vraag / Question 8 **[20 punte / marks]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1.1	$\frac{d}{dx}\left[(\arctan 4x)\left(\sqrt{x^2+1}\right)\right]$ $= \frac{3}{1+16x^2}.$✓$\sqrt{x^2+1}$✓ $+ \arctan 4x\,.$✓$\frac{1}{2}(x^2+1)^{-\frac{1}{2}}$✓$.2x$✓	**(5)**
8.1.2	$g(x) = \frac{2x^{10}-9x^5}{4x^2-7}$ $g'(x) = \frac{(20x^9-45x^4)(4x^2-7)$✓$-(2x^{10}-9x^5)(8x)$✓$}{(4x^2-7)^2$✓}	**(3)**
8.2	$\int_1^k \frac{1}{\sqrt{-x^2+2x}}dx = \frac{\pi}{6}$ $-x^2+2x = -[x^2-2x]$ $= -[x^2-2x+1^2-1^2]$ $= -[(x-1)^2-1]$ $= 1-(x-1)^2$✓ $\int_1^k \frac{1}{\sqrt{1-(x-1)^2}}dx = \frac{\pi}{6}$✓ $\arcsin(x-1)$ ✓$\|_1^k = \frac{\pi}{6}$ $\arcsin(k-1) - \arcsin 0 = \frac{\pi}{6}$✓ $\arcsin(k-1) = \frac{\pi}{6}$ $k = \sin\frac{\pi}{6} + 1$ $k = \frac{3}{2}$✓	**(5)**

8.3.1	$Area = \int_0^{\frac{\pi}{6}} \sin x \ dx$ ✓ $+ \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x \ dx$ ✓ $= [-\cos x]_0^{\frac{\pi}{6}}$ ✓ $+ \left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ ✓ $= \left(-\cos\frac{\pi}{6} + \cos 0\right)$ ✓ $+ \left(\frac{1}{2}\sin\left(2.\frac{\pi}{4}\right) - \frac{1}{2}\sin\left(2.\frac{\pi}{6}\right)\right)$ ✓ $= 0{,}20$ ✓	**(7)**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -