

α -MATHEMATICS

Grade 11 Alpha Mathematics Final Examination 2022

Examiner: Lanice Liebenberg

Time: $2\frac{1}{2}$ hours

Moderator: Anna Muller

Total: 150

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 10 pages, an answer sheet and a diagram sheet.
2. Answer ALL 8 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Clearly show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1**[20 marks]**

This question must be answered **on the answer sheet**.

Every question has **ONLY** one correct answer. Mark the correct answer with an **X** on the answer sheet.

1.1 $|2x - 3| < -5$ (2)

- A $-1 < x < 4$
- B $x < -1$ or $x > 4$
- C $x < -1$
- D No solution

1.2 Consider Pascal's triangle, the number in the 6th position in row 12 is: (2)

- A 792
- B 924
- C 462
- D 1716

1.3 Given $f(x) = 2x^7 + 8x^6 - 17x^3 + x^2 + 6$. Which of the following numbers is, according to the **rational root theorem**, **NOT** a **POSSIBLE** zero of f : (2)

- A 1
- B 3
- C -4
- D -2

1.4 The third term of the expansion of the power series of $\sqrt[3]{1-2x}$ is: (2)

A $\frac{-1}{9}x^2$

B $\frac{1}{9}x^2$

C $\frac{-4}{9}x^2$

D $\frac{4}{9}x^2$

1.5 If $f(x) = g(x) + 8$ for $x \in [0; 2]$, then $\int_0^2 (f(x) + g(x))dx =$ (2)

A $2 \int_0^2 g(x)dx + 8$

B $2 \int_0^2 g(x)dx + 16$

C $2 \int_0^2 g(x)dx + 4$

D $\int_0^2 g(x)dx + 16$

1.6 Given $F(x) = (f \circ g)(x)$, then $F'(x) =$ (2)

A $f(g(x)) \cdot g'(x)$

B $f'(g'(x)) \cdot g(x)$

C $f'(g(x)) \cdot g'(x)$

D $f(g'(x)) \cdot g'(x)$

- 1.7 The area enclosed by the graph of $y = \sin x$ and the x -axis between $x = 0$ and $x = \pi$ is equal to (2)
- A** 2
- B** 0
- C** 1
- D** π
- 1.8 Given $g(x) = x^3 - 2x^2 + 5$. The gradient at the point (2; 5) is equal to (2)
- A** 0 or $\frac{4}{3}$
- B** 5
- C** 4
- D** 2
- 1.9 Two vectors a and b are perpendicular when: (2)
- A** $a \times b = 0$
- B** $a \cdot b = 0$
- C** $a \times b = -1$
- D** $a \cdot b = -1$
- 1.10 How many terms will the expansion of $\left(5x - \frac{1}{x}\right)^{14}$ contain? (2)
- A** 12
- B** 13
- C** 14
- D** 15

Question 2**[16 marks]**

- 2.1 Given $f(x) = -|4x - 8| + 8$ and $g(x) = 2x$.
Determine:
- 2.1.1 The salient point of f . (2)
- 2.1.2 The value(s) of x for which $f \geq 4$. (4)
- 2.1.3 The value(s) of x for which $f(x) = g(x)$ (6)
- 2.2 Sketch the graphs of f and g on the DIAGRAM SHEET provided. (4)

Question 3**[27 marks]**

- 3.1 Given $p(x) = x^4 - ax^3 + bx - a$ where $2a = b$. (7)
 p has a factor $x^2 - 2$ and a zero $2 + \sqrt{2}$.
Determine the equation of p and hence the values of a and b .

- 3.2 Decompose the expression below in partial fractions: (10)

$$\frac{4x^3 - 6x^2 + 4x - 8}{x^2(x^2 + 4)}$$

- 3.3 Use mathematical induction and prove that: (10)

$$\sum_{r=1}^n r(r-1) = \frac{n(n^2-1)}{3}$$

Question 4**[20 marks]**

4.1 Given the expression: (6)

$$\left(4x + \frac{a}{x^2}\right)^6$$

Determine the value of a , if it is given that a is a constant and that the term that is independent of x is 34560.

4.2 The expression below is given:

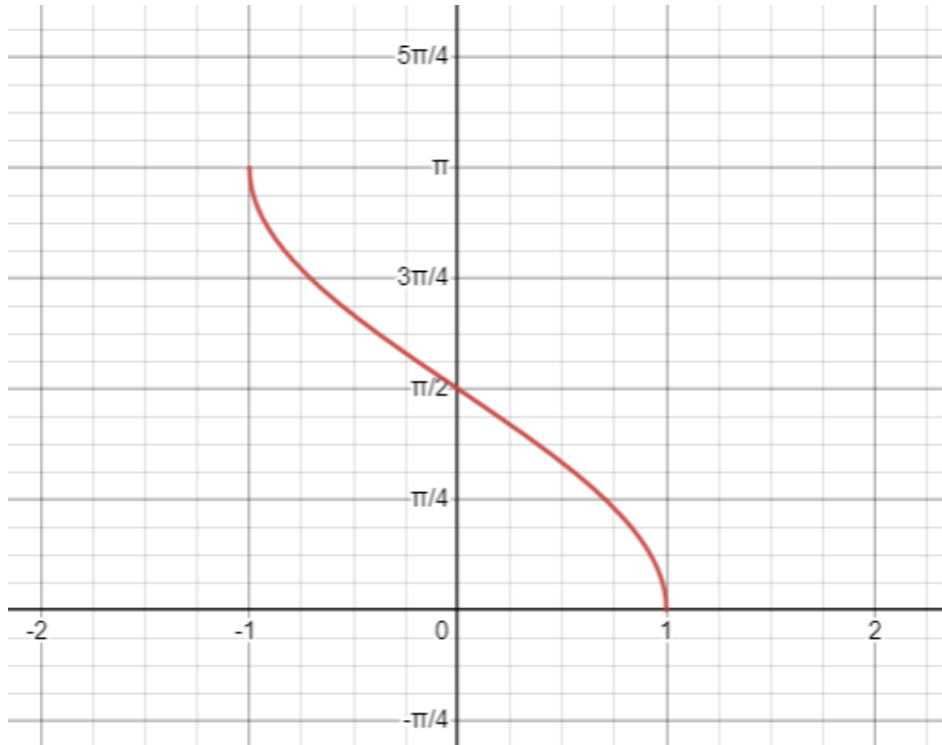
$$\frac{1}{(2 + x^2)^6}$$

4.2.1 Determine the first 4 terms of the expansion of the expression. (10)

4.2.2 For which value(s) of x will the expansion above be valid? (4)

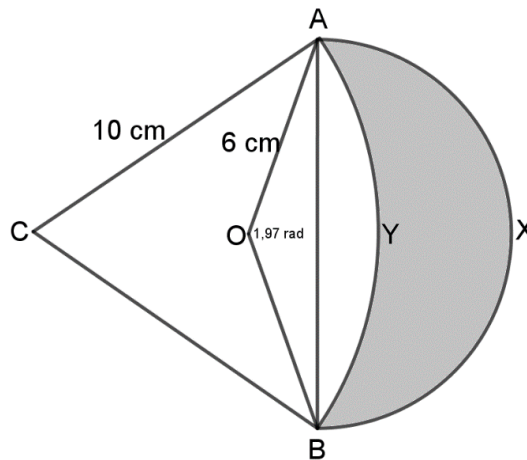
Question 5**[19 marks]**

Given the function $f(x) = \arcsin(-x) + q$. The function has end points at $(-1; \pi)$ and $(1; 0)$.



- 5.1.1 Determine the value of q . (3)
- 5.1.2 Determine the inverse function of f in the form $f^{-1}(x) = \dots$ (4)
- 5.1.3 Determine the range of the inverse function of f . (2)

- 5.2 In the diagram $OAXB$ is a sector of a circle with midpoint O and radius 6 cm.
 $CAYB$ is a sector of another circle with midpoint C and radius 10 cm.
 $\triangle ABC$ is an equilateral triangle and $\widehat{AOB} = 1,97$ radians.



- 5.2.1 Show that the area of $OAYB$ is $25,64 \text{ cm}^2$. (7)
- 5.2.2 Hence, determine the area of the shaded region $AXBY$. (3)

Question 6

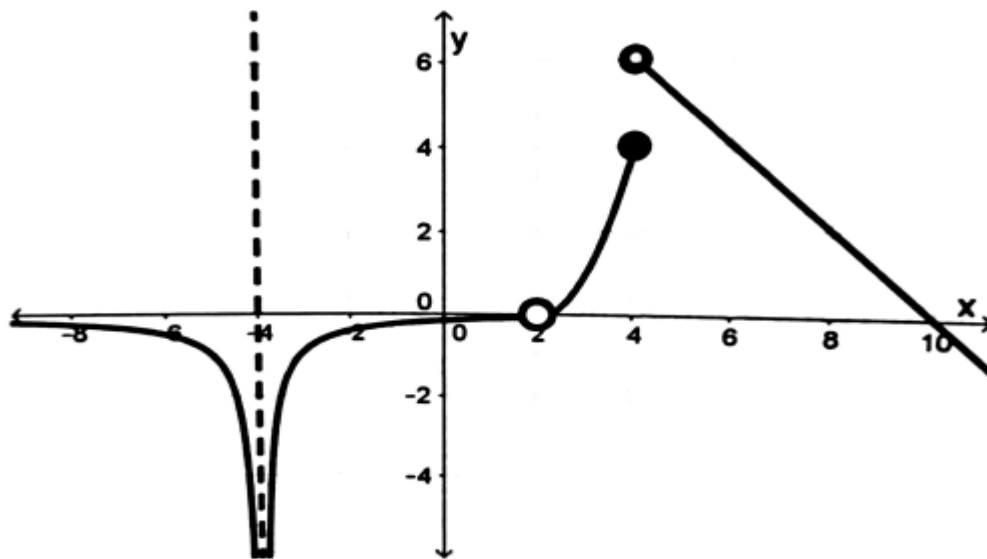
[20 marks]

- 6.1 Given the vectors $a = (1; 3; 5)$, $b = (2; -1; 0)$ and $c = (-3; 0; 1)$. Determine:
- 6.1.1 The angle between a and the z -axis. (3)
- 6.1.2 The area of the parallelogram formed by b and c . (5)
- 6.1.3 Determine the angle between a and b . (4)
- 6.2 Given the system of equations: (8)
- $$\begin{aligned} a + 2b + 3c &= 1 \\ 2a + 5b + 3c &= 6 \\ a + 8c &= -6 \end{aligned}$$

Make use of Cramer's rule to determine the value of a .

Question 7**[8 marks]**

Use the given diagram to answer the questions that follow:



- 7.1 Determine the value of $f'(8)$. (2)
- 7.2 For which value(s) of x is the function undefined? (2)
- 7.3 State two values of x where the function is not continuous, name the type of discontinuity. (4)

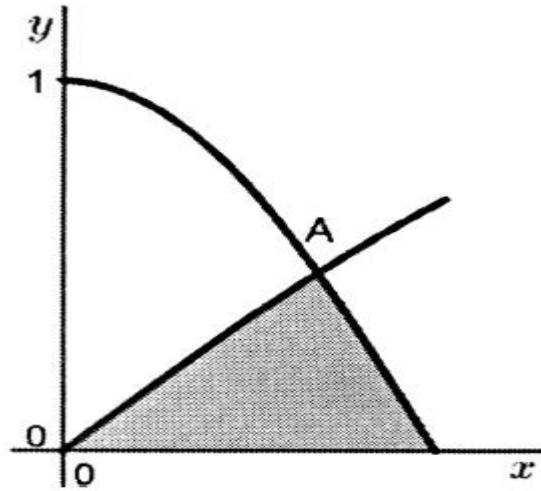
Question 8**[20 marks]**

8.1 Differentiate each of the following:

8.1.1 $\frac{d}{dx}[(\arctan 4x)(\sqrt{x^2 + 1})]$ (5)

8.1.2 $g(x) = \frac{2x^{10} - 9x^5}{4x^2 - 7}$ (3)

8.2 If $\int_1^k \frac{1}{\sqrt{-x^2 + 2x}} dx = \frac{\pi}{6}$, determine the value of k . (5)

8.3 The diagram shows the graphs of $y = \cos 2x$ and $y = \sin x$ on the interval $[0; \frac{\pi}{4}]$. The graphs intersect at A which has an x -value of $\frac{\pi}{6}$.

8.3.1 Determine the area of the shaded part. (7)

- END OF QUESTION PAPER -

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Grade 11 Alpha Mathematics Final Examination 2022 Answer sheet

Name and Surname: _____

Question Total	1 [20]	2 [16]	3 [27]	4 [20]	5 [19]	6 [20]	7 [8]	8 [20]	TOTAL 150
Learner mark									

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Question 2

2.2

