α-MATHEMATICS

Alpha Wiskunde Graad 11 / *Alpha Mathematics Grade 11*

Junie Eksamen 2022 / *June Examination 2022*

MEMORANDUM

Totaal / *Total*: 150 punte / *marks*

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Moderator: Anna Muller

Hierdie memorandum bestaan uit 10 bladsye. /
This memorandum consists of 10 pages.

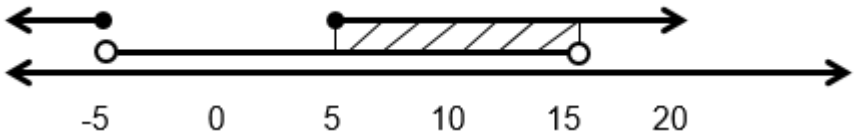
Vraag / *Question* 1 **[20 punte / *marks*]**

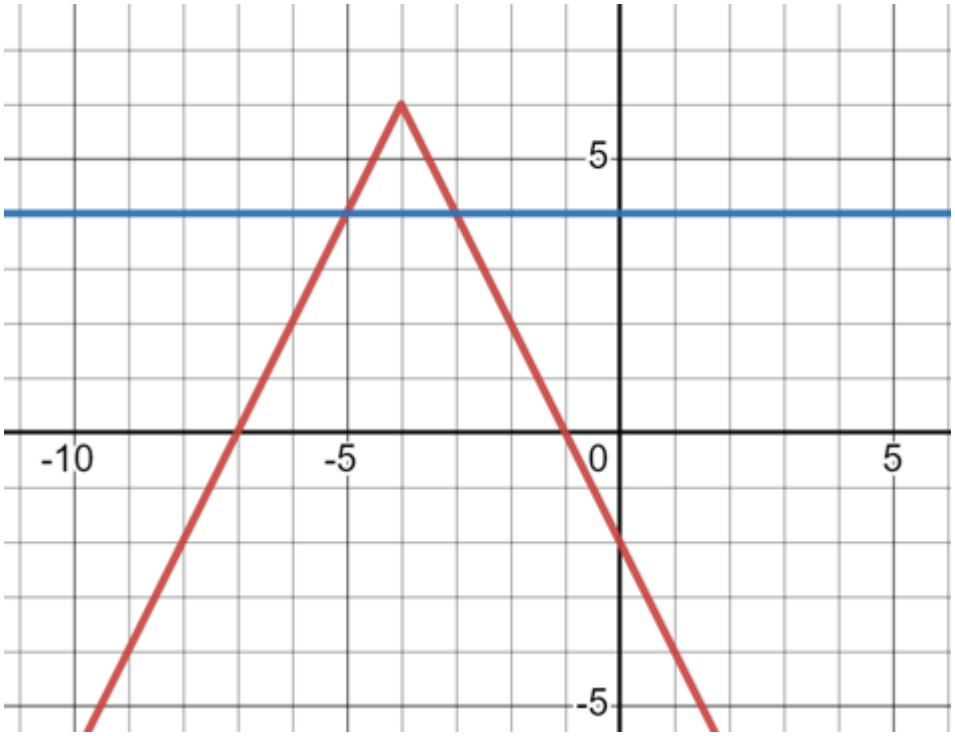
1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

NR. NO	**ANTWOORD *ANSWER***	**BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)***	**PUNTE *MARKS***
1.1	**C**	bestaan, maar die antwoord sal altyd positief wees	**(2)**
1.2	**C**	n kan nie negatief wees nie.	**(2)**
1.3	**B**	$\sum_{k=1}^{3} k^3 = 1 + 8 + 27$	**(2)**
1.4	**C**	$\frac{1}{\sec^2 \pi + 3} = \frac{1}{\frac{1}{\cos^2 \pi} + 3} = 0{,}25$	**(2)**
1.5	**B**	$f^{-1}(x)$ se definisieversameling is dieselfde as die definisieversameling van f.	**(2)**
1.6	**D**	$r + 1 = 5$ $r = 4$	**(2)**
1.7	**A**	$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^4 + 3\left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1 = 0$	**(2)**
1.8	**C**	$-1 - \sqrt{3}$ is ook 'n nulpunt.	**(2)**
1.9	**A**	$x \in \mathbb{R}$	**(2)**

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.10	**D**	die definisieversameling van $f^{-1}(x)$ $x \in [-1\,;1]$	**(2)**

Vraag / *Question* 2 **[30 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1.1	$\|10-x\| = -4$ No solutions / Geen oplossings✓✓	**(2)**
2.1.2	$\|3x-6\| = \begin{cases} 3x-6 & ;x \geq 2✓ \\ -3x+6 & ;x < 2✓ \end{cases}$ $3x-6 = x+2$ OR/OF $-3x+6 = x+2$ $2x = 8$ $-4x = -4$ $x = 4$✓ $x = 1$✓ N/A✓	**(5)**
2.1.3	$\|2x-10| < 20$ AND/EN $x^2-25 \geq 0$✓ $-20 < 2x-10 < 20$✓ $(x+5)(x-5) \geq 0$✓ $-10 < 2x < 30$ $x \leq -5$ OR/OF $x \geq 5$✓ $-5 < x < 15$✓  $\rightarrow 5 \leq$ ✓$x < 15$✓	**(7)**
2.1.4	$\frac{-8}{\|x-4\|} < 2$ $-8 < 2\|x-4\|$✓ $\|x-4| > -4$✓ $x \in \mathbb{R}$✓ ; $x \neq 4$✓	**(4)**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.2.1		**(8)** ✓✓$g(x)$ $f(x)$ ✓✓$(-4;6)$ ✓✓x-int/afsnitte ✓y-int/afsnit ✓shape/vorm
2.2.2	$x \in [-5\checkmark\ ;\ -3\checkmark]$ OR/OF $-5 \leq \checkmark x \leq -3\checkmark$	**(2)**
2.2.3	$(4\ ;6)$ ✓✓	**(2)**

Vraag / *Question* 3 **[10 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1.1	$A\hat{B}C = \frac{\pi}{3}$ ✓✓	**(2)**
3.1.2	radius= 3 cm✓✓	**(2)**
3.1.3	$Area\ \Delta = \frac{1}{2}.6.6.\sin\frac{\pi}{3}$✓ $Area\ \Delta = 9\sqrt{3}$ $\approx 15{,}59$✓ $Area\ sector = \frac{1}{2}.3^2.\frac{\pi}{3}$✓ $Area\ sector = \frac{3\pi}{2}$ $\approx 4{,}71$✓ $Area\ shaded = 9\sqrt{3} - \frac{3\pi}{2}$✓ $Area\ shaded \approx 10{,}88$ cm²✓	**(6)**

Vraag / *Question* 4 **[24 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1	$\sum_{r=1}^{n}(3r^2-3r)=n^3-n$ $0+6+18+\cdots+(3n^2-3n)=n^3-n$ **STEP 1** Consider $n=1$: ✓ $LHS=3(1)^2-3(1)=0$✓ $RHS=(1)^3-1=0$✓ $\therefore$ LHS = RHS✓ and the statement is true for n=1. ✓ **STEP 2** Accept✓ the statement is true for a natural number $n=k$: $0+6+18\ldots+(3k^2-3k)=k^3-k$✓ **STEP 3** Consider $n=k+1$: $RHS=(k+1)^3-(k+1)$✓ $RHS=(k+1)((k+1)^2-1)$ $RHS=(k+1)(k^2+2k)$ $RHS=k^3+3k^2+2k$✓ $LHS=\big(3(1)^2-3(1)\big)+\big(3(2)^2-3(2)\big)+\cdots+(3k^2-3k)+\big(3(k+1)^2-3(k+1)\big)$✓ $LHS=(k^3-k)$✓$+\big(3(k+1)^2-3(k+1)\big)$ $LHS=k^3-k+3(k+1)(k+1-1)$ $LHS=k^3-k+3k^2+3k$ $LHS=k^3+3k^2+2k$✓ **STEP 4** The statement is true for $n=1$. If it is true for $n=k$, it is also true for $n=k+1$ and consequently, true for all natural numbers n. ✓	**(13)**

4.2	$\frac{1}{x^4+x^2}=\frac{1}{x^2(x^2+1)}\equiv\frac{A}{x}\checkmark+\frac{B}{x^2}\checkmark+\frac{Cx+D}{(x^2+1)}\checkmark$ $1=Ax(x^2+1)\checkmark+B(x^2+1)\checkmark+x^2(Cx+D)\checkmark$ $1=Ax^3+Ax+Bx^2+B+Cx^3+Dx^2$ $B=1\checkmark$ $A=0\checkmark$ $B+D=0$ $D=-1\checkmark$ $A+C=0$ $C=0\checkmark$ $\frac{1}{x^4+x^2}=\frac{1}{x^2(x^2+1)}\equiv\frac{0}{x}+\frac{1}{x^2}+\frac{(0)x-1}{(x^2+1)}=\frac{1}{x^2}-\frac{1}{x^2+1}\checkmark$	**(11)**

Vraag / *Question* 5 **[19 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$(a+b)^n=\binom{n}{r}a^{n-r}.b^r$ $a=6a \quad b=-\frac{2}{3a} \quad n=8$ $\left(6a-\frac{2}{3a}\right)^8=\binom{8}{r}.(6a)^{8-r}.\left(-\frac{2}{3a}\right)^r\checkmark$ $=\binom{8}{r}.6^{8-r}.a^{8-r}.\left(-\frac{2}{3}\right)^r.(a)^{-r}\checkmark$ $8-r-r=0\checkmark$ $r=4\checkmark$ $\left(6a-\frac{2}{3a}\right)^8=\binom{8}{4}.6^{8-4}.a^{8-4}.\left(-\frac{2}{3}\right)^4.(a)^{-4}$ $=17920\checkmark\checkmark$	**(6)**

5.2.1	$(\mathbf{1}+\boldsymbol{x})^{\boldsymbol{n}}=\mathbf{1}+\boldsymbol{nx}+\frac{\boldsymbol{n(n-1)}}{\mathbf{2!}}\boldsymbol{x^2}+\frac{\boldsymbol{n(n-1)(n-2)}}{\mathbf{3!}}\boldsymbol{x^3}+\cdots$ $(1-2x)^{-2}=1+(-2)(-2x)\checkmark+\frac{-2(-2-1)}{2!}(-2x)^2\checkmark$ $+\frac{-2(-2-1)(-2-2)}{3!}(-2x)^3\checkmark$ $(1-2x)^{-2}=1\checkmark+4x\checkmark+12x^2\checkmark+32x^3\checkmark$	**(7)**
5.2.2	$\lvert-2x\rvert<1\checkmark$ $\lvert2x\rvert<1$ $\lvert x\rvert<\frac{1}{2}\checkmark\checkmark$	**(3)**
5.3	$r+1=5$ AND/ EN $n=10$ $\binom{10}{4}\checkmark$ $=210\checkmark\checkmark$	**(3)**

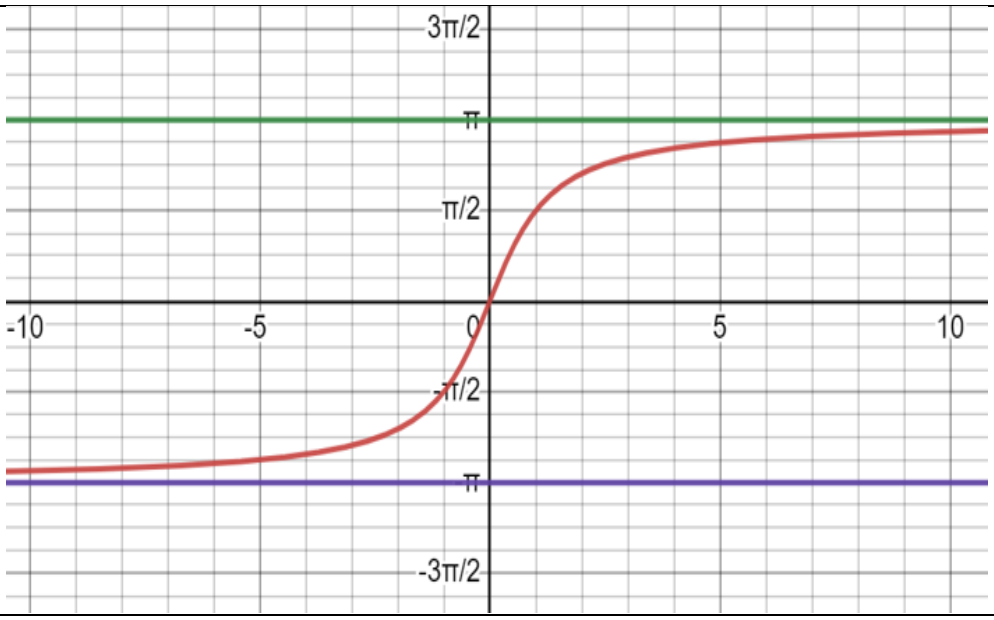
Vraag / *Question* 6 **[17 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1.1	$x=3-2i\checkmark$ is also a zero of f/ is ook 'n nulpunt van f. $(x-3+2i)(x-3-2i)\checkmark$ OR/OF $x-3=\pm2i\checkmark$ $=x^2-6x+13\checkmark$ is a factor of f/ is 'n factor van f. $x^2-6x+13$) $x^4-6x^3+22x^2-54x+117$; $x^2\checkmark$; $+9\checkmark$ $x^4-6x^3+13x^2$ $9x^2-54x+117$ $9x^2-54x+117$ 0 $0=x^4-6x^3+22x^2-54x+117$ $0=(x^2-6x+13)(x^2+9)\checkmark$ $0=(x^2-6x+13)(x+3i)(x-3i)\checkmark$ $x=3\pm2i\checkmark;\ \pm3i\checkmark$	**(9)**

6.2	$x = 2$; $x = -2 + \sqrt{3}$ and $x = -2 - \sqrt{3}$ ✓are zero's /is nulpunte $(x-2)\left(x+2-\sqrt{3}\right)\left(x+2+\sqrt{3}\right)$✓ $= (x-2)(x^2+4x+1)$✓ $= x^3 + 2x^2 - 7x - 2$✓✓ $a = -5$✓ ; $b = 9$✓ ; $c = -2$✓	**(8)**

Vraag / Question 7 **[30 punte / marks]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	$-\frac{\pi}{2} \leq 2x - \frac{\pi}{4} \leq \frac{\pi}{2}$✓ $-\frac{\pi}{4} \leq 2x \leq \frac{3\pi}{4}$ $-\frac{\pi}{8} \leq x \leq \frac{3\pi}{8}$✓✓	**(3)**
7.2	$y = sin\left(2x - \frac{\pi}{4}\right)$ $x = sin\left(2y - \frac{\pi}{4}\right)$✓ $2y - \frac{\pi}{4} = \arcsin x$✓ $2y = \arcsin x + \frac{\pi}{4}$✓ $g^{-1}(x) = \frac{\arcsin x}{2} + \frac{\pi}{8}$✓✓	**(5)**
7.3	$g\left(\sqrt{2}\right) = sin\left(2\sqrt{2} - \frac{\pi}{4}\right)$ $g\left(\sqrt{2}\right) \approx 0{,}89$✓✓	**(2)**
7.4	$sin\left(2x - \frac{\pi}{4}\right) = 0{,}5$ $2x - \frac{\pi}{4} = \arcsin 0{,}5$ ✓ $2x = \arcsin 0{,}5 + \frac{\pi}{4}$✓ $x = \frac{\arcsin 0{,}5 + \frac{\pi}{4}}{2}$✓ $x \approx 0{,}65$✓	**(4)**
7.5	$g^{-1}(1) = \frac{\arcsin(1)}{2}$ ✓ $+ \frac{\pi}{8}$ $g^{-1}(2) \approx 1{,}18$✓	**(2)**
7.6.1	$y = \pm\pi$✓✓	**(2)**

7.6.2		**(5)** ✓✓Asymptotes /Asimptote ✓Intercept /afsnit ✓✓Shape **/Vorm**
7.6.3	$x \in (-\infty; 1)$ ✓✓ OR/OF $x < 1$	**(2)**
7.6.4	$x \in (-\pi; \pi)$ ✓✓	**(2)**
7.6.5	$k(x) = -$✓$2\arctan(x - 3$✓$) + 4$✓	**(3)**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -