α-MATHEMATICS

Grade 10 Alpha Mathematics

Final Examination 2021

Examiner: L Liebenberg **Time: 2 hours**

Moderator: A Muller **Total: 120**

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 8 pages and an answer sheet.
2. Answer ALL 9 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 **[20 marks]**

This question must be answered **on the answer sheet**.
Every question has **ONLY** one correct answer for TWO marks each. Mark the correct answer with an **X** on the answer sheet.

1.1 $i^5 =$

A 1

B $-i$

C i

D -1

1.2 To decompose $\frac{3}{x^3}$ into partial fractions the correct form will be:

A $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

B $\frac{A}{x^3}$

C $\frac{A+B+C}{x^3}$

D $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3}$

1.3 Given that $6x^2 + x - 10 = Ax^2 + 5A + 2Bx^2 + Cx$, use comparison of the coefficients to determine the values of A, B and C.

A $A = 6 \;\; ; \;\; B = 0 \;\; ; \;\; C = 1$

B $A = -2 \;\; ; \;\; B = 4 \;\; ; \;\; C = 1$

C $A = 6 \;\; ; \;\; B = 3 \;\; ; \;\; C = 1$

D Cannot be solved.

1.4 Given the matrices $A = \begin{pmatrix} 1 & 3 \\ -2 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & y \\ x & b_{22} \end{pmatrix}$, if $2A = B$ then:

A $x = -2 \;\; ; \;\; y = 3$

B $x = -4 \;\; ; \;\; y = 6$

C $x = 3 \;\; ; \;\; y = -2$

D $x = -1 \;\; ; \;\; y = \frac{3}{2}$

1.5 Given that $C = \begin{bmatrix} 1 & 5 & -3 \\ 2 & -6 & 4 \end{bmatrix}$ then $C^T =$

A $\begin{pmatrix} 2 & -6 & 4 \\ 1 & 5 & -3 \end{pmatrix}$

B $\begin{bmatrix} 2 & 1 \\ -6 & 5 \\ 4 & -3 \end{bmatrix}$

C $\begin{bmatrix} 1 & 2 \\ 5 & -6 \\ -3 & 4 \end{bmatrix}$

D $\begin{pmatrix} -3 & 5 & 1 \\ 4 & -6 & 2 \end{pmatrix}$

1.6 If $f(x) = 5$ then $f'(x) =$

A 1

B 5

C $\frac{1}{5}$

D 0

1.7 If $f(x) = \sqrt{x+1}$ and $(g \circ f)(x) = x + 1$, then $g(x) =$

A x

B $\sqrt{x}$

C $\sqrt{x+1}$

D x^2

1.8 $60°$ converted to radians is:

A $\frac{\pi}{3}$

B $\frac{\pi}{6}$

C 6π

D 3π

1.9 Given the vector $w = (1\,;1)$ the direction of the vector will be:

A North West

B $\frac{\pi}{6}$

C South East

D $\frac{\pi}{4}$

1.10 Given the vectors $u = (2\,;3)$ and $v = -2u$, the scalar of v will influence u as follows:

(i) $|v| = -2|u|$

(ii) The direction of v is opposite to the direction of u.

(iii) $|v| = 2|u|$

(iv) The direction of v is the same as the direction of u.

Which of the statements above are true?

A (i) and (ii)

B (ii) and (iii)

C (i) and (iv)

D (iii) and (iv)

Question 2 **[22 marks]**

Given that $a = -2 + i$ and $b = 1 - 2i$, answer the questions that follow:

2.1 Represent $a + b$ graphically. Make use of the DIAGRAM SHEET provided. (4)

2.2 $b - a$ (3)

2.3 $a.a^*$ (4)
Where a^* is the conjugate of a.

2.4 $Re\left(\frac{b}{a}\right)$ (7)

2.5 $b.i^{127}$ (4)

Question 3 **[12 marks]**

Decompose
$$\frac{3x^2 - 1}{(x^2 - x)(x - 1)}$$
into partial fractions.

Question 4 **[9 marks]**

Given $P(x) = x^3 + x^2 + 9x + 9 = 0$, answer the questions that follow.

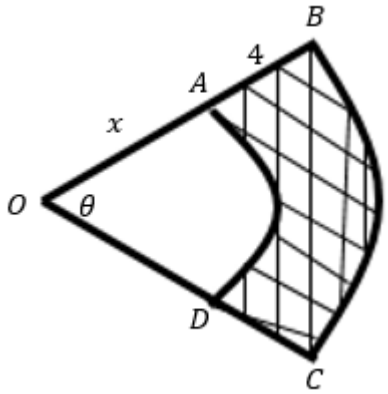
4.1 Make use of the factor theorem to find a real root of the equation. Show all calculations. (2)

4.2 Determine a quadratic factor of P. (HINT: Make use of long division or synthetic division). (3)

4.3 Solve $P(x) = 0$ in $C[x]$. (4)

Question 5 **[14 marks]**

In the diagram below, AD and AB are the arcs of a circle with centre O such that $OA = OD = x$ cm, $AB = DC = 4$ cm and $A\hat{O}D = \theta$ radians.



5.1 Determine the area of sector OAD in terms of x and θ. (2)

5.2 Determine the area of sector OBC in terms of x and θ. (2)

5.3 If the area of the shaded part is 64 cm^3, show that $\theta = \frac{16}{x+2}$ radians. (5)

5.4 Given that $x = 2$ cm, determine the value of θ. (1)

5.5 Hence, calculate the perimeter of the shaded part. (4)

Question 6 [10 marks]

6.1 Sketch the following piecewise function on the DIAGRAM SHEET provided. (6)

$$f(x) = \begin{cases} x^2 & if \quad x < 0 \\ 2x & if \quad 0 < x < 2 \\ 4 - x & if \quad x \geq 2 \end{cases}$$

6.2 Determine each of the following, all angles are in radians.

6.2.1 $\tan\frac{\pi}{6}$ (2)

6.2.2 $\arcsin\frac{1}{2}$ (2)

Question 7 [14 marks]

7.1 Determine the following derivative: (4)

$f(x) = 5.\sqrt[3]{x} - \frac{4}{x^2} + 9$

7.2 Determine the following integral: (4)

$\int 6x^5 + 3\sqrt{x} + 2\ dx$

7.3 Determine the volume of the rotating body formed when the graph of $f(x) = \sqrt{3x^2 - 8x + 7}$ rotates about the x-axis between the points $x = 1$ and $x = 2$. (6)

Question 8 [7 marks]

Given the system of equations:

$x + y = 8$
$3x + 2y = 21$

8.1 Write the system of equations in matrix form. (2)

8.2 Solve x and y by making use of Cramer's rule. (5)

Question 9 **[12 marks]**

Given the vectors $a = (-8\,;6)$ and $b = (\sqrt{7}\,;3)$.

9.1 Determine the magnitude of b. (2)

9.2 Determine $|a|$. (2)

9.3 Determine $a.b$. (3)

9.4 Are the vectors perpendicular? (1)

9.5 Determine the angle between vectors a and b. (4)

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Final Examination 2021 Answer sheet

Name and Surname: _____

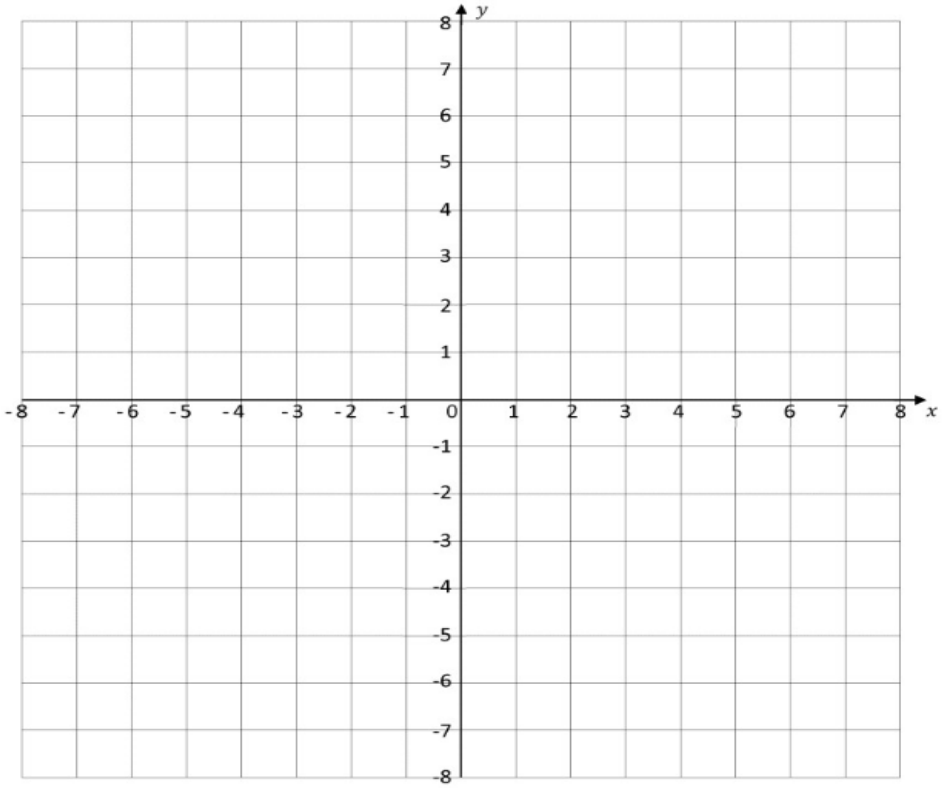
Question Total	1 [20]	2 [22]	3 [12]	4 [9]	5 [14]	6 [10]	7 [14]	8 [7]	9 [12]
Learner mark									

TOTAL 120

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

2.1



6.1