α-MATHEMATICS

Grade 11 Alpha Mathematics

Final Examination 2021

Examiner: L Liebenberg **Time: $2\frac{1}{2}$ hours**

Moderator: A Muller **Total: 150**

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 10 pages and an answer sheet.
2. Answer ALL 11 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 **[20 marks]**

This question must be answered **on the answer sheet**.
Every question has **ONLY** one correct answer. Mark the correct answer with an **X** on the answer sheet.

1.1 $x^4 - 4x^3 + 8x - 4$ has a zero at $x = -2 + \sqrt{2}$, another zero will be:

A $x = -\sqrt{2} + 2$

B $x = -2 + \sqrt{2}$

C $x = -\sqrt{2} - 2$

D None of the above.

1.2 If $f(x) = \sin^3 3x$, then $f'(x) =$

A $3\cos 3x$

B $3\sin 3x$

C $9\sin^2 3x.\cos 3x$

D $3\sin^2 3x\,.\cos 3x$

1.3 The fraction $\frac{-2x-7}{x^2-4x+4}$ must be divided into partial fractions, the correct form will be:

A $\frac{A}{(x-2)} + \frac{B}{(x-2)^2}$

B $\frac{A}{x^2-2x+4} + \frac{B}{x^2-2x+4}$

C $\frac{Ax^2+B}{x^2-2x+4}$

D $\frac{Ax+B}{x^2-2x+4}$

1.4 Given the vectors $a = (1\,;2\,;3)$ and $b = (4\,;5\,;6)$, determine the vector ba.

A $(2\,;4\,;8)$

B $(3\,;3\,;3)$

C $(-3\,;\,-3\,;\,-3)$

D $(5\,;7\,;9)$

1.5 Given that $a.b = 0$, the vectors a and b are

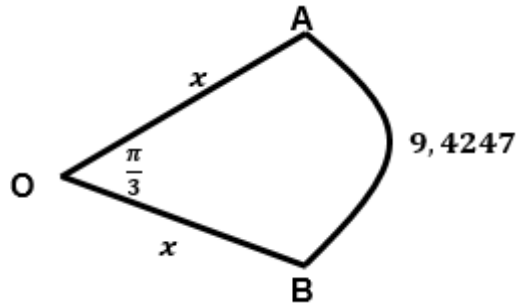
A Equal

B Perpendicular

C Parallel

D Opposite

1.6 In the diagram below $A\hat{O}B = \frac{\pi}{3}$ radians and $AB = 9{,}4247$. Calculate the value of x.



A 9

B $9{,}87$

C $8{,}90$

D $22{,}05$

1.7 If $f(x) = \arccos x$ the domain and range will be as follows:

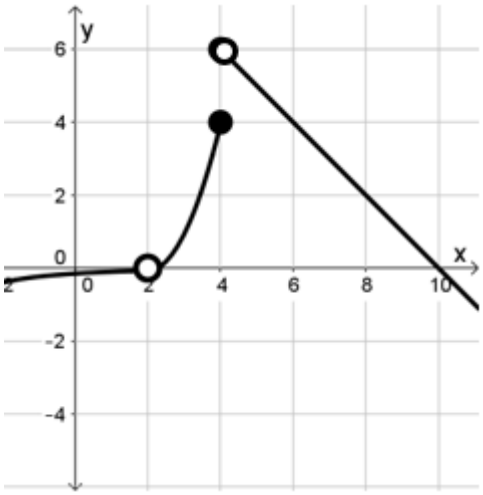
A $x \in (-\infty\,;\,\infty)\ \ \&\ \ y \in (-\frac{\pi}{2}\,;\frac{\pi}{2})$

B $x \in [-1\,;\,1]\ \ \&\ \ y \in [-\frac{\pi}{2}\,;\frac{\pi}{2}]$

C $x \in [-1\,;\,1]\ \ \&\ \ y \in [0\,;\pi]$

D $x \in (-\infty\,;\,\infty)\ \ \&\ \ y \in (0\,;\pi)$

Use the following diagram to answer questions 1.8 – 1.10



1.8 $f'(6) =$

A 2

B 4

C -1

D 1

1.9 $f(2) =$

A 0

B Undefined

C 2

D None of the above

1.10 Which one of the following statements are true?

A There is a jump discontinuity at $x = 2$.

B The limit does not exist at $x = 4$.

C The function is differentiable at $x = 4$.

D None of the above.

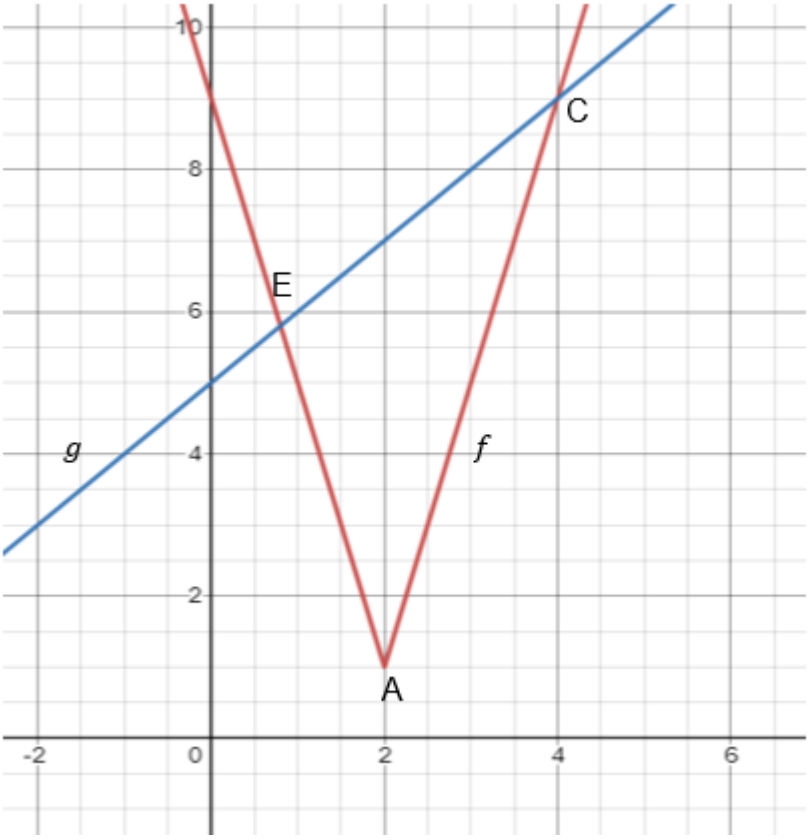
Question 2 **[12 marks]**

2.1 Determine and simplify the first three terms of $\sqrt[3]{27-x}$. (8)

2.2 Determine the constant term in the expansion of $\left(x+\frac{1}{x}\right)^6$. (4)

Question 3 **[16 marks]**

3.1 The diagram shows the graphs of $f(x)=2|4-2x|+1$ and $g(x)=x+5$



3.1.1 Write down the coordinates of the salient point A. (2)

3.1.2 Determine the coordinates of E and C, the intercepts of f and g. (8)

3.1.3 Use the diagram to determine for which values of x, $f(x)<g(x)$. (2)

3.2 Solve for x if: (4)

$$2|3-x|-3|x-3|-1\geq -10$$

Question 4 **[28 marks]**

4.1 Decompose $\frac{7x^2+6x+5}{(x+1)(x^2+x+1)}$ into partial fractions. (9)

4.2 Given that $P(x) = x^4 - 5x^3 + 31x^2 - 21x - 58 = 0$ has a root at $x = 2 + 5i$, solve for all $x \in \mathbb{R}$. (11)

4.3 The following statement has been partially proven by Mathematical induction. Complete the proof: (8)

$$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$$

is true for all $n \geq 2, n \in \mathbb{N}$.

STEP 1: Consider $n = 2$:

$$LHS = \left(1-\frac{1}{4}\right) \qquad RHS = \left(\frac{2+1}{2(2)}\right)$$
$$= \frac{3}{4} \qquad = \frac{3}{4}$$

$$\therefore LHS = RHS$$

The statement is true for $n = 2$.

STEP 2: Accept that the staement is true for $n = k$:

$$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{k^2}\right) = \frac{k+1}{2k}$$

STEP 3: ...

Question 5 **[15 marks]**

5.1 Given $h(x) = 2\arctan x$

5.1.1 Sketch the graph of h on the DIAGRAM SHEET provided. (5)

5.1.2 Give the equation of $k(x)$ if k was formed by translating h as follows: (4)

- 2 units down
- $\frac{\pi}{2}$ units to the left
- Reflected over the x-axis

5.1.3 Determine the inverse function of h in the form $y = \cdots$ (3)

5.1.4 Determine the gradient of h at $x = 1$. (3)

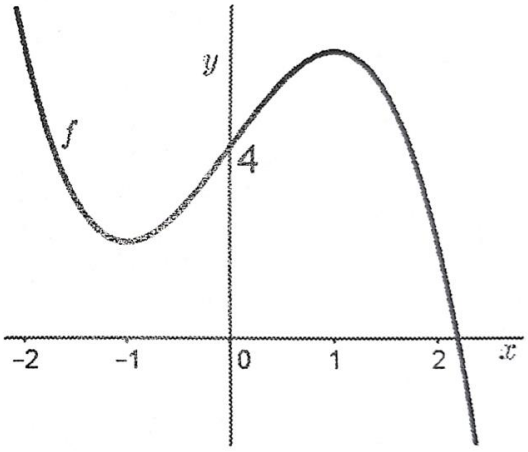
Question 6 **[12 marks]**

6.1 Given $u = i + 2j - k$, and $w = 5i - j + 3k$. (7)
Determine the area of the parallelogram formed by the vectors.

6.2 Given $P = 3i + 5j - 4k$ and $Q = 2i + 4j + 3k$. (5)
Calculate the magnitude of the angle between P and Q.

Question 7 **[11 marks]**

Given the diagram of $f(x)$.



7.1 Determine $f(x)$ if $f'(x) = -3x^2 + 3$. (5)

7.2 Determine the x-intercept with the use of Newton's method correct to 5 decimal places. Use $x = 2{,}2$ as the first approximate value. (6)
Accept that $f(x) = -x^3 + 3x + 4$.

Question 8 **[10 marks]**

Determine each of the following derivatives. (Do not simplify.)

8.1 $D_x\left[\frac{2x^4-8x^2}{x^5-6x}\right]$ (5)

8.2 $f(x) = \cos(x.\sin x)$ (5)

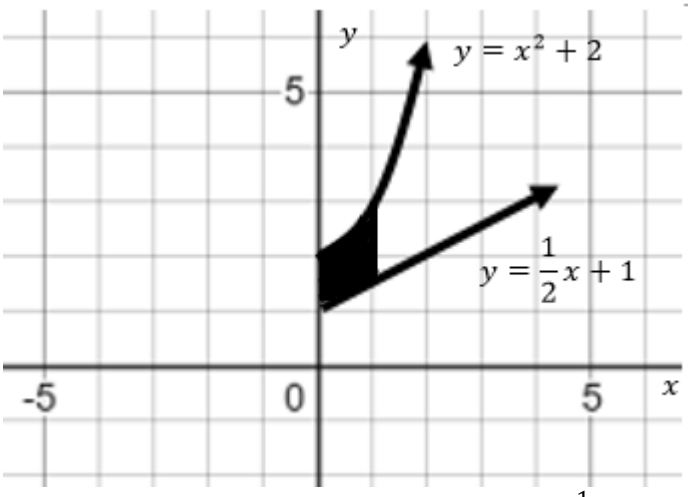
Question 9 **[9 marks]**

Determine each of the following integrals. (Do not simplify.)

9.1 $\int \frac{2}{\sqrt{9-x^2}}\,dx$ (5)

9.2 $\int (6\,\text{cosec}\,4x\,.\cot 4x + 7\sec^2(3x+2))dx$ (4)

Question 10 **[9 marks]**



The diagram shows the graphs of $y = x^2 + 2$ and $y = \frac{1}{2}x + 1$. Determine the volume of the rotating body that is formed when the shaded area (that is between $x = 0$ and $x = 1$) revolves about the x −axis.

Question 11 **[8 marks]**

$$g(x) = \begin{cases} 2k & as \quad x \geq 1 \\ 9 - x^2 & as \quad -1 < x < 1 \\ 2x + 3c & as \quad x \leq -1 \end{cases}$$

Determine the values of k and c such that $g(x)$ is continuous for all $x \in \mathbb{R}$..
Show all calculations.

END OF QUESTION PAPER

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Final Examination 2021 Answer sheet

Name and Surname: _____

Question Total	1 [20]	2 [12]	3 [16]	4 [28]	5 [15]	6 [12]	7 [11]	8 [10]	9 [9]	10 [9]	11 [8]
Learner mark											

TOTAL 150

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D