

# **$\alpha$ -MATHEMATICS**

**Alpha Wiskunde Graad 11 / *Alpha Mathematics Grade 11***

**Junie Eksamen Toets 2021 / *June Examination 2021***

## **MEMORANDUM**

**Totaal / *Total*: 150 punte / *marks***

**Eksaminator / *Examiner*: Lanice Liebenberg**

**Moderator: Anna Muller**

**Hierdie memorandum bestaan uit 10 bladsye. /**

***This memorandum consists of 10 pages.***

Vraag / Question 1

[20 punte / marks]

|      |   |   |   |   |
|------|---|---|---|---|
| 1.1  | A | B | C | D |
| 1.2  | A | B | C | D |
| 1.3  | A | B | C | D |
| 1.4  | A | B | C | D |
| 1.5  | A | B | C | D |
| 1.6  | A | B | C | D |
| 1.7  | A | B | C | D |
| 1.8  | A | B | C | D |
| 1.9  | A | B | C | D |
| 1.10 | A | B | C | D |

| NR. NO | ANTWOORD ANSWER | BEREKENINGE (nie vir nasien doeleindes nie) CALCULATIONS (not for marking purpose)                                       | PUNTE MARKS |
|--------|-----------------|--|-------------|
| 1.1    | D               | $(0 ; 5)$  | (2)         |
| 1.2    | B               | $\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n$   | (2)         |
| 1.3    | C               | $y = \arcsin(x - 2) - \frac{\pi}{2}$   | (2)         |
| 1.4    | A               | $\binom{11}{4} = 330$  | (2)         |
| 1.5    | B               | $y = \cos 2x + 3$<br>$x = \cos 2y + 3$<br>$x - 3 = \cos 2y$<br>$2y = \arccos(x - 3)$<br>$y = \frac{1}{2} \arccos(x - 3)$ | (2)         |

|      |   |  |     |
|------|---|--|-----|
| 1.6  | D | $7n - 5 = 79$<br>$n = 12$<br><br>$\frac{12(7(12) - 3)}{2}$<br>$= 486$                          | (2) |
| 1.7  | C | $x = 0$ one solution   | (2) |
| 1.8  | C | $3\left(-\frac{2}{3}\right)^3 + 5\left(-\frac{2}{3}\right)^2 + 2\left(-\frac{2}{3}\right) = 0$ | (2) |
| 1.9  | D | Domain: $x \in (-\infty; \infty)$ Range: $y \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$    | (2) |
| 1.10 | A | $n + 1 = 13 + 1 = 14$  | (2) |

**Vraag / Question 2**

**[37 punte / marks]**

| NR. NO | ANTWOORD / ANSWER   | PUNTE / MARKS |
|--------|---|---------------|
| 2.1.1  | $f(0) =  2(0) - 4  - 1 = 3\checkmark\checkmark$   | (2)           |
| 2.1.2  | x-intercept: $ 2x - 4  = 1$<br>$2x - 4 = -1\checkmark$ or $2x - 4 = 1\checkmark$<br>$2x = 3$ $2x = 5$<br>$x = \frac{3}{2}\checkmark$ $x = \frac{5}{2}\checkmark$<br>y-intercept: $y =  -4  - 1$<br>$= 3\checkmark$                      | (5)           |
| 2.1.3  | $x = \frac{4}{2} = 2$<br>$(2; -1)\checkmark\checkmark$  | (2)           |
| 2.1.4  | $f(x) = g(x)$<br>$ 2x - 4  - 1 = x + 1$<br>$ 2x - 4  = x + 2$<br>$2x - 4 = x + 2\checkmark$ if $x \geq 2\checkmark$ or $2x - 4 = -x - 2\checkmark$ if $x < 2\checkmark$<br>$x = 6\checkmark$ or $3x = 2$<br>$x = \frac{2}{3}\checkmark$ | (6)           |
| 2.1.5  | $ 2x - 4  - 1 \leq 3$<br>$ 2x - 4  \leq 4\checkmark$<br>$-4 \leq 2x - 4 \leq 4\checkmark\checkmark$<br>$0 \leq 2x \leq 8\checkmark$   | (5)           |

| NR. NO | ANTWOORD / ANSWER  | PUNTE / MARKS  |
|--------|--|--|
|        | $0 \leq x \leq 4$ ✓  |  |
| 2.1.6  |  | <p style="text-align: right;"><b>(6)</b></p> <p><b>Graph of g:</b><br/> ✓ x-intercept<br/> ✓ y-intercept</p> <p><b>Graph of f:</b><br/> ✓✓ x-intercept<br/> ✓ y-intercept<br/> ✓ Overall Shape</p> |
| 2.2    | $3 4 - x  > 15$ $ 4 - x  > 5$ $4 - x < -5 \quad \text{or} \quad 4 - x > 5$ $-x < -9 \qquad \qquad -x > 1$ $x > 9 \qquad \qquad \qquad x < -1$<br>$x^2 \leq 121$ $x^2 - 121 \leq 0$ $(x + 11)(x - 11) \leq 0$ $-11 \leq x \leq 11$<br><br>$-11 \leq x < -1 \quad \text{or} \quad 9 < x \leq 11$ | <p style="text-align: right;"><b>(11)</b></p>  |

Vraag / Question 3

[23 punte / marks]

| NR. NO | ANTWOORD / ANSWER   | PUNTE / MARKS |             |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
|--------|---|---------------|-------------|------|-------|-------|---------|----------|-------------|-------|---------|----------|--|--|--------|---------|-------------|--|--------|---------|--------|--|--|---------|------------|--|--|---------|------------|--|--|--|-----|------------|
| 3.1.1  | $x = 2 - 5i$ is also a root ✓<br><br>$(x - 2 - 5i)(x - 2 + 5i)$ ✓<br>$= x^2 - 4x + 4 - 25i^2$<br>$= x^2 - 4x + 29$ is a quadratic factor of $k$ . ✓✓  | <b>(4)</b>    |             |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
| 3.1.2  | $x^2 - 4x + 29$ <table style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"><math>x^2</math></td> <td style="padding-right: 10px;"><math>-x</math></td> <td style="padding-right: 10px;"><math>-2</math></td> <td style="padding-right: 10px;"><math>✓✓✓</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"><math>x^4</math></td> <td style="padding-right: 10px;"><math>-5x^3</math></td> <td style="padding-right: 10px;"><math>+31x^2</math></td> <td style="padding-right: 10px;"><math>-21x - 58</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"><math>x^4</math></td> <td style="padding-right: 10px;"><math>-4x^3</math></td> <td style="padding-right: 10px;"><math>-29x^2</math></td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-right: 10px;"><math>-x^3</math></td> <td style="padding-right: 10px;"><math>+2x^2</math></td> <td style="padding-right: 10px;"><math>-21x - 58</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-right: 10px;"><math>-x^3</math></td> <td style="padding-right: 10px;"><math>+4x^2</math></td> <td style="padding-right: 10px;"><math>-29x</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-right: 10px;"></td> <td style="padding-right: 10px;"><math>-2x^2</math></td> <td style="padding-right: 10px;"><math>+8x - 58</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-right: 10px;"></td> <td style="padding-right: 10px;"><math>-2x^2</math></td> <td style="padding-right: 10px;"><math>+8x - 58</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-right: 10px;"></td> <td style="padding-right: 10px;"></td> <td style="padding-right: 10px;"><math>0</math></td> </tr> </table><br>$k(x) = (x^2 - 4x + 29)(x^2 - x - 2)$ ✓✓<br>$= (x^2 - 4x + 29)(x - 2)(x + 1)$ ✓ | $x^2$         | $-x$        | $-2$ | $✓✓✓$ | $x^4$ | $-5x^3$ | $+31x^2$ | $-21x - 58$ | $x^4$ | $-4x^3$ | $-29x^2$ |  |  | $-x^3$ | $+2x^2$ | $-21x - 58$ |  | $-x^3$ | $+4x^2$ | $-29x$ |  |  | $-2x^2$ | $+8x - 58$ |  |  | $-2x^2$ | $+8x - 58$ |  |  |  | $0$ | <b>(6)</b> |
| $x^2$  | $-x$  | $-2$          | $✓✓✓$       |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
| $x^4$  | $-5x^3$   | $+31x^2$      | $-21x - 58$ |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
| $x^4$  | $-4x^3$   | $-29x^2$      |             |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
|        | $-x^3$  | $+2x^2$       | $-21x - 58$ |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
|        | $-x^3$  | $+4x^2$       | $-29x$      |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
|        |   | $-2x^2$       | $+8x - 58$  |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
|        |   | $-2x^2$       | $+8x - 58$  |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
|        |   |               | $0$         |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
| 3.1.3  | $k(x) = (x^2 - 4x + 29)(x - 2)(x + 1)$<br>$= (x - 2 - 5i)✓(x - 2 + 5i)✓(x - 2)(x + 1) ✓$  | <b>(3)</b>    |             |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |
| 3.2.1  | $2x - 1 = 0$<br>$x = \frac{1}{2}$ ✓<br><br>$p\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^3 - 7\left(\frac{1}{2}\right)^2 + 1$ ✓<br>$= 0$ ✓<br>$(2x - 1)$ is a factor of $p(x)$ .   | <b>(3)</b>    |             |      |       |       |         |          |             |       |         |          |  |  |        |         |             |  |        |         |        |  |  |         |            |  |  |         |            |  |  |  |     |            |

3.2.2

$$\begin{array}{r}
 3x^2 \quad -2x \quad -1 \\
 2x - 1 \overline{) 6x^3 \quad -7x^2 \quad +0x \quad +1} \\
 \underline{6x^3 \quad -3x^2} \phantom{+0x \quad +1} \\
 -4x^2 \quad +0x \quad +1 \\
 \underline{-4x^2 \quad +2x} \phantom{+1} \\
 -2x \quad 1 \\
 \underline{-2x \quad +1} \\
 0
 \end{array}$$

$$p(x) = (2x - 1)(3x^2 - 2x - 1) \checkmark \checkmark \checkmark$$

$$0 = (2x - 1)(3x + 1)(x - 1) \checkmark$$

$$x = \frac{1}{2}; -\frac{1}{3}; 1 \checkmark \checkmark \checkmark$$

(7)

| NR.<br>NO | ANTWOORD / ANSWER  | PUNTE / MARKS |
|-----------|--|---------------|
| 4.1       | $2 + 9 + 16 + \dots + (7n - 5) = \frac{n(7n - 3)}{2}$ <p><b>STEP 1</b> Consider <math>n = 1</math>:</p> $LHS = 2 \checkmark \quad RHS = \frac{1(7(1) - 3)}{2} = 2 \checkmark$ <p><math>\therefore</math> LHS = RHS and the statement is true for <math>n=1</math>. <math>\checkmark</math></p> <p><b>STEP 2</b> Accept the statement is true for a natural number <math>n = k</math>: <math>\checkmark</math></p> $2 + 9 + 16 + \dots + (7k - 5) \checkmark = \frac{k(7k - 3)}{2} \checkmark$ <p><b>STEP 3</b> Consider <math>n = k + 1</math>: <math>\checkmark</math></p> $RHS = \frac{(k + 1)(7(k + 1) - 3)}{2} = \frac{(k + 1)(7k + 4)}{2} \checkmark \checkmark$ $LHS = 2 + 9 + 16 + \dots + (7k - 5) + 7(k + 1) - 5 \checkmark$ $= \frac{k(7k - 3)}{2} + \frac{2(7k + 7 - 5)}{2} \checkmark$ $= \frac{7k^2 + 11k + 4}{2}$ $= \frac{(k + 1)(7k + 4)}{2} \checkmark$ <p><b>STEP 4</b> The statement is true for <math>n = 1</math>. If it is true for <math>n = k</math>, it is also true for <math>\checkmark</math></p> <p><math>n = k + 1</math> and consequently, true for all natural numbers <math>n</math>. <math>\checkmark</math></p> | (13)          |

|       |  |       |      |       |      |       |      |     |
|-------|--|-------|------|-------|------|-------|------|-----|
| 4.2   | $\frac{4x^3 + 6x^2 + x + 2}{x^4 + x^2} = \frac{4x^3 + 6x^2 + x + 2}{x^2(x^2 + 1)} \checkmark \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{(x^2 + 1)} \checkmark$ $4x^3 + 6x^2 + x + 2 \equiv Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)x^2 \checkmark$ <p>Let <math>x = 0</math><br/> <math>2 = B \checkmark</math></p> $4x^3 + 6x^2 + x + 2 \equiv Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2 \checkmark$ $A = 1 \checkmark$ $B + D = 6$ $D = 4 \checkmark$ $A + C = 4$ $C = 3 \checkmark$ $\frac{4x^3 + 6x^2 + x + 2}{x^4 + x^2} = \frac{1}{x} + \frac{2}{x^2} + \frac{3x + 4}{(x^2 + 1)} \checkmark$ | (9)   |      |       |      |       |      |     |
| 4.3   | <table border="1"> <tr> <td data-bbox="225 1106 384 1189">4.3.1</td> <td data-bbox="384 1106 539 1189">B ✓✓</td> </tr> <tr> <td data-bbox="225 1189 384 1272">4.3.2</td> <td data-bbox="384 1189 539 1272">C ✓✓</td> </tr> <tr> <td data-bbox="225 1272 384 1355">4.3.3</td> <td data-bbox="384 1272 539 1355">A ✓✓</td> </tr> </table>  | 4.3.1 | B ✓✓ | 4.3.2 | C ✓✓ | 4.3.3 | A ✓✓ | (6) |
| 4.3.1 | B ✓✓   |       |      |       |      |       |      |     |
| 4.3.2 | C ✓✓   |       |      |       |      |       |      |     |
| 4.3.3 | A ✓✓   |       |      |       |      |       |      |     |

**Vraag / Question 5**

**[21 punte / marks]**

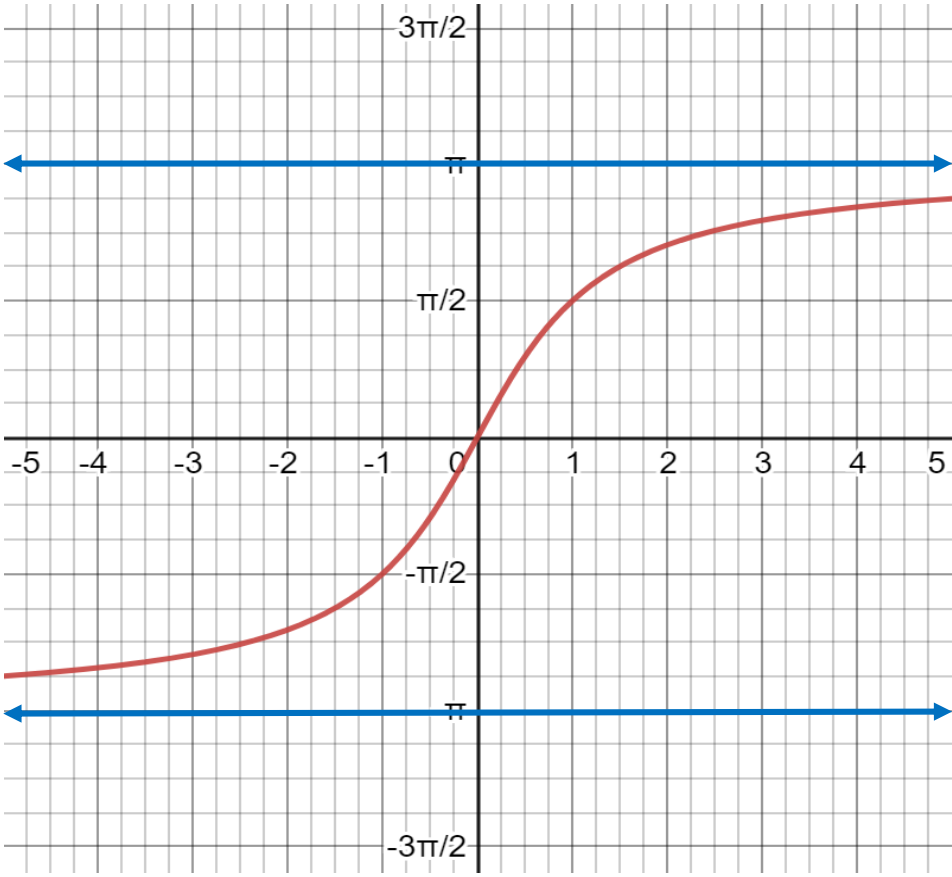
| NR. NO | ANTWOORD / ANSWER  | PUNTE / MARKS |
|--------|--|---------------|
| 5.1.1  | $(a + b)^n = a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}ab^{n-1} + b^n$ $a = ax \quad b = \frac{1}{2y} \quad n = 10$ $\left(ax + \frac{1}{2y}\right)^{10} = (ax)^{10} + 10(ax)^{10-1}\left(\frac{1}{2y}\right) + \binom{10}{2}(ax)^{10-2}\left(\frac{1}{2y}\right)^2$ $= a^{10}x^{10} \checkmark \checkmark + \frac{5a^9x^9}{y} \checkmark \checkmark + \frac{45a^8x^8}{4y^2} \checkmark \checkmark$ | (6)           |

| NR. NO | ANTWOORD / ANSWER   | PUNTE / MARKS |
|--------|---|---------------|
| 5.1.2  | $\binom{n}{r} a^{n-r} b^r = \binom{10}{4} \checkmark (ax)^{10-4} \checkmark \left(\frac{1}{2y}\right)^4 \checkmark$ $\frac{76545x^6}{8y^4} \checkmark = \frac{210a^6x^6}{16y^4} \checkmark$ $\frac{76545}{8} = \frac{105a^6}{8}$ $a^6 = 729$ $a^6 = 3^6$ $a = \pm 3 \checkmark \checkmark$  | (7)           |
| 5.2    | $\sqrt[4]{16-x}$ $= (16-x)^{\frac{1}{4}} \checkmark$ $= \left[16\left(1-\frac{x}{16}\right)\right]^{\frac{1}{4}}$ $= 2\left(1-\frac{x}{16}\right)^{\frac{1}{4}} \checkmark$ $= 2\left[1 + \frac{1}{4}\left(-\frac{x}{16}\right) + \frac{\frac{1}{4}(\frac{1}{4}-1)}{2!}\left(-\frac{x}{16}\right)^2 + \frac{\frac{1}{4}(\frac{1}{4}-1)(\frac{1}{4}-2)}{3!}\left(-\frac{x}{16}\right)^3\right]$ $= 2\left[1 - \frac{x}{64} - \frac{3}{8192}x^2 - \frac{7}{524288}x^3\right] \checkmark \checkmark$ $= 2\checkmark - \frac{x}{32} \checkmark - \frac{3x^2}{4096} \checkmark - \frac{7x^3}{262144} \checkmark$ | (8)           |

**Vraag / Question 6**

**[21 punte / marks]**

| NR. NO | ANTWOORD / ANSWER   | PUNTE / MARKS |
|--------|---|---------------|
| 6.1.1  | $y = 3 \arcsin(5x + 1)$ $x = 3 \arcsin(5y + 1) \checkmark$ $\frac{x}{3} = \arcsin(5y + 1) \checkmark$ $5y + 1 = \sin\left(\frac{x}{3}\right) \checkmark$ $5y = \sin\left(\frac{x}{3}\right) - 1 \checkmark$ $f^{-1}(x) = \frac{1}{5} \sin\left(\frac{x}{3}\right) - \frac{1}{5} \checkmark$ | (5)           |

| NR. NO | ANTWOORD / ANSWER   | PUNTE / MARKS  |
|--------|---|--|
| 6.1.2  | $\frac{\pi}{2} = 3 \arcsin(5x + 1) \checkmark$ $\frac{\pi}{6} = \arcsin(5x + 1) \checkmark$ $\sin\left(\frac{\pi}{6}\right) = 5x + 1 \checkmark$ $5x = \sin\left(\frac{\pi}{6}\right) - 1 \checkmark$ $x = -0,09 \checkmark \checkmark$ | <b>(6)</b>   |
| 6.2.1  |    | <b>(6)</b><br>$\checkmark \checkmark$ Asymptotes<br>$\checkmark \checkmark$ Intercept<br>$\checkmark \checkmark$ Shape |
| 6.2.2  | Reduced $\checkmark$ by $\frac{1}{2} \checkmark$ <b>OR</b> Asymptotes changed $\frac{\pi}{2} \checkmark$ and $-\frac{\pi}{2} \checkmark$<br>and moved 1 unit $\checkmark$ to the right. $\checkmark$                                    | <b>(4)</b>   |

**- EINDE VAN DIE MEMORANDUM / END OF THE MEMORANDUM -**