

α -WISKUNDE

Alpha Wiskunde Graad 12 / *Alpha Mathematics Grade 12*
Rekord eksamen 2021 / *Preliminary exam 2021*

MEMORANDUM

Totaal / *Total*: 200 punte / *marks*

Eksaminator / *Examiner*: Pieter van Onselen

Moderator: Anna Muller

Hierdie memorandum bestaan uit 12 bladsye. /
This memorandum consists of 12 pages.

Vraag / Question 1

[20 punte / marks]

NR. NO	ANTWOORD ANSWER	BEREKENINGE (nie vir nasien doeleindes nie) CALCULATIONS (not for marking purpose)	PUNTE MARKS
1.1	D	$m < 0$ negatiewe helling/negative slope $f'' > 0$ konkaf op/concave up	2
1.2	B	$0 \leq 2x \leq \pi$ $0 \leq x \leq \frac{\pi}{2} \quad x \in \left[0; \frac{\pi}{2}\right]$	2
1.3	B	$ 2x - 1 = -4$ Geen oplossing/No solution	2
1.4	D	$f(x) = e^{x^{\frac{1}{2}}} f(x)$ $= \frac{1}{2} \left(x^{-\frac{1}{2}}\right) e^{x^{\frac{1}{2}}}$ $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$	2
1.5	A	Sprong diskontinuiteit/Jump discontinuity : $\lim_{x \rightarrow a} f(x)$ bestaan nie/does not exist.	2
1.6	C	$\int_0^8 (x+1)^{-\frac{1}{2}} dx$ $= \frac{(x+1)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} \Big _0^8$ $= 2\sqrt{9} - 2\sqrt{1}$ $= 4$	2
1.7	A	$(\text{cis } \frac{\pi}{2}) \times 2(\text{cis } \frac{\pi}{2}) \Rightarrow 2 \text{cis } \pi \Rightarrow -2$	2
1.8	C	$\sqrt{4\left(1 - \frac{x}{4}\right)}$ Konvergeer/converges: $\left -\frac{x}{4}\right < 1$ $ x < 4$	2
1.9	C	$f(x) = 2\log_3 x$ $f'(x) = \frac{2}{x \ln 3}$	2
1.10	B	$\frac{2x^2+x}{3x^3-x^2}$ $= \lim_{x \rightarrow \infty} \frac{x^2 \left(2 + \frac{1}{x}\right)}{x^3 \left(3 - \frac{1}{x^2}\right)}$ $= \lim_{x \rightarrow \infty} \left(\frac{1}{x}\right) \frac{\left(2 + \frac{1}{x}\right)}{\left(3 - \frac{1}{x^2}\right)} \Rightarrow 0$	2

Vraag / Question 2

[28 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
2.1 (a)	$C(t) = 500e^{0,92(0)}$ $= 500$ bakterieë/bacteria	✓ $t = 0$ ✓ 500 [2 punte / marks]
2.1 (b)	$C(6) = 500e^{0,92(6)}$ $= 124817$ bakterieë/bacteria	✓ $t = 6$ ✓ 124817 bakterieë/bacteria [2 punte / marks]
2.1 (c)	$10^5 = 500e^{0,92t}$ $e^{0,92t} = \frac{10^5}{500}$ $t = \frac{\ln(200)}{0,92}$ $t = 5,76$ uur/hours	✓ $10^5 = 500e^{0,92t}$ ✓ $e^{0,92t} = \frac{10^5}{500}$ ✓ $t = \frac{\ln(200)}{0,92}$ ✓ 5,76 uur/hours [4 punte / marks]
2.2 (a)	$x + 1 = 0 \Rightarrow x = -1$ Knakpunt/vertex: $C(-1; -1)$	✓ -1 ✓ -1 [2 punte / marks]
2.2 (b)	$ x + 1 - 1 = \left \frac{2}{x + 1} \right $ Negatiewe snypunt/negative intercept $x < -1$: $-x - 1 - 1 = -\frac{2}{x + 1}$ $x^2 + 3x = 0$ $x = -3$ or $x = 0$ Dus/therefore: $x = -3$	✓ $x < -1$ ✓ $-x - 2 = -\frac{2}{x+1}$ ✓ $x^2 + 3x = 0$ ✓ $x = -3$ [4 punte / marks]
2.2(c)	$ x + 1 - 1 < 2$ $ x + 1 < 3$ $-3 < x + 1 < 3$ $-4 < x < 2$	✓ $ x + 1 < 3$ ✓ $-3 < x$ ✓ $x < 2$ [3 punte / marks]
2.3(a)	$(2x - x^2)^7$ $= \binom{7}{3} (2x)^4 (-x^2)^3$ $= -560x^{10}$	✓ $n = 7$ ✓ Vervang/substitute ✓ x^{10} ✓ -560 [4 punte / marks]
2.3(b)	$\sqrt{16 - x} = 4 \left(1 - \frac{x}{16} \right)^{\frac{1}{2}}$ $= 4 \left(1 + \binom{1}{2} \left(-\frac{x}{16} \right) + \frac{\binom{1}{2} \left(-\frac{1}{2} \right) \left(-\frac{x}{16} \right)^2}{2} + \dots \right)$ $= 4 - \frac{x}{8} - \frac{x^2}{512}$	✓ 4 faktoriseer/factorise ✓ Mag/power ✓ 1 ✓ $\binom{1}{2} \left(-\frac{x}{16} \right)$ ✓ $\frac{\binom{1}{2} \left(-\frac{1}{2} \right) \left(-\frac{x}{16} \right)^2}{2}$ ✓ $-\frac{x}{8}$ ✓ $-\frac{x^2}{512}$ [7 punte / marks]

Vraag / Question 3

[31 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
3.1	<p>Beskou nou/consider $n = k + 1$:</p> <p>LK/LS: $\left(\frac{k+1}{2k}\right)\left(1 - \frac{1}{(k+1)^2}\right)$ ✓ $= \left(\frac{k+1}{2k}\right)\left(\frac{k^2+2k+1-1}{(k+1)^2}\right)$ ✓✓ $= \left(\frac{k+2}{2(k+1)}\right)$ ✓</p> <p>RK/RS: $\frac{k+2}{2(k+1)}$ ✓</p> <p>Dus/therefore: LK=RK LS=RS</p> <p>Volgens die beginsel van wiskundige induksie is die bewering waar vir alle waardes van $n \in \mathbb{N}$.</p> <p>According the principle of mathematical induction the statement is true for all values of $n \in \mathbb{N}$.</p>	<p>✓ $n = k + 1$</p> <p>✓ LK=RK</p> <p>✓ sin/sentence</p> <p>[8 punte / marks]</p>
3.2	<p>Faktore/factors: $(x - 2)(x^2 + 6x + 13)$</p> <p>As/if $x - 2 - \sqrt{2}$ dan is/then $x - 2 + \sqrt{2}$ ook wortel/also root.</p> <p>$(x - 2 - \sqrt{2})(x - 2 + \sqrt{2})$ $= (x - 2)^2 - 2$ $= x^2 - 4x + 2$</p> $ \begin{array}{r} 1 \quad -4 \quad 2 \quad \overline{) \quad 2 \quad 1} \\ \underline{2 \quad -7 \quad 0 \quad 2} \\ 2 \quad -8 \quad 4 \\ \underline{ \quad 1 \quad -4 \quad 2} \end{array} $ <p>$(x^2 - 4x + 2)(2x + 1)$</p> <p>$(x - 2 - \sqrt{2})(x - 2 + \sqrt{2})(2x + 1)$</p>	<p>✓ $x - 2 + \sqrt{2}$</p> <p>✓ $(x - 2)^2 - 2$</p> <p>✓ $x^2 - 4x + 2$</p> <p>Lang deling/Long division</p> <p>✓ $2 \quad 1$</p> <p>Antwoord/Answer</p> <p>✓ $(x - 2 \pm \sqrt{2})$</p> <p>✓ $(2x + 1)$</p> <p>[6 punte / marks]</p>
3.3 (a)	<p>$\theta = \text{bgtan}\left(\frac{1}{1}\right) = \frac{\pi}{4}$</p> <p>II kwadrant/quadrant: $\theta = \pi - \frac{\pi}{4}$ of/or $\theta = \frac{3\pi}{4}$</p> <p>$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$</p> <p>$-1 + i = \sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$</p>	<p>✓ $\theta = \frac{3\pi}{4}$</p> <p>✓ $r = \sqrt{2}$</p> <p>[2 punte / marks]</p>

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
3.3 (b)	$\left(\sqrt{2}cis\left(\frac{3\pi}{4}\right)\right)^6$ $= \sqrt{2}cis\left(6\frac{3\pi}{4}\right)$ $= 8cis\left(\frac{18\pi}{4}\right)$ $p = 8 \quad \beta = \frac{\pi}{2}$	<p>✓ Vervang/substitute <i>cis</i></p> <p>✓ $(\sqrt{2})^6$</p> <p>✓ $cis\left(\frac{6 \times 3\pi}{4}\right)$</p> <p>✓ $8cis\left(\frac{18\pi}{4}\right)$</p> <p>✓ $p = 8$</p> <p>✓ $\beta = \frac{18\pi}{4} / \frac{9\pi}{2} / \frac{\pi}{2}$</p> <p style="text-align: right;">[6 punte / marks]</p>
3.2 (c)	$8cis\left(\frac{\pi}{2}\right)$ $= 8\cos\frac{\pi}{2} + 8i\sin\frac{\pi}{2}$ $= 8i$	<p>✓ $8\cos\frac{\pi}{2}$</p> <p>✓ $8i\sin\frac{\pi}{2}$</p> <p>✓ 0</p> <p>✓ $8i$</p> <p style="text-align: right;">[4 punte / marks]</p>
3.3	$\frac{8cis\left(\frac{\pi}{2}\right)}{4cis\left(-\frac{\pi}{2}\right)}$ $= 2cis\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= 2cis(\pi)$	<p>✓ $8cis\left(\frac{\pi}{2}\right)$</p> <p>✓ 2</p> <p>✓ $\frac{\pi}{2} + \frac{\pi}{2}$</p> <p>✓ $cis(\pi)$</p> <p style="text-align: right;">[4 punte / marks]</p>

Vraag / Question 4

[20 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
4.1(a)	$\begin{pmatrix} 1 & a \\ 2a & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$	<p>✓ $\begin{pmatrix} 1 & a \\ 2a & 5 \end{pmatrix}$ ✓ $\begin{pmatrix} x \\ y \end{pmatrix}$ ✓ $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$</p> <p style="text-align: right;">[3 punte / marks]</p>
4.1(b)	$\det A = \begin{vmatrix} 1 & a \\ 2a & 5 \end{vmatrix}$ $= 5 - 2a^2$ $\det A_x = \begin{vmatrix} 3 & a \\ 6 & 5 \end{vmatrix}$ $= 15 - 6a$ $x = \frac{\det A_x}{\det A} = -1$ $\frac{15 - 6a}{5 - 2a^2} = -1$ $2a^2 - 6a - 10 = 0$ $a = 5 \text{ of/or } a = -2$	<p>✓ $5 - 2a^2$ ✓ $15 - 6a$ ✓ $x = \frac{\det A_x}{\det A} = -1$</p> <p>✓ $2a^2 - 6a - 10$</p> <p>✓ $a = 5 \text{ of/or}$ ✓ $a = -2$</p> <p style="text-align: right;">[6 punte / marks]</p>
4.2(a)	$AC = (1 - 3; 1 - 1; 3 - 2) = (-2; 0; 1)$ $AB = (2 - 3; 3 - 1; 2 - 2) = (-1; 2; 0)$	<p>✓ ✓</p> <p style="text-align: right;">[2 punte / marks]</p>
4.2(b)	$ AC = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$ $ AB = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$ $AC \cdot AB = 2 + 0 + 0 = 2$ $\theta = \text{bgcos} \left(\frac{2}{\sqrt{5}\sqrt{5}} \right)$ $\theta \approx 1.16$	<p>✓ $AC = \sqrt{5}$ ✓ $AB = \sqrt{5}$ ✓ $AC \cdot AB = 2$ ✓ $\theta \approx 1.16$</p> <p style="text-align: right;">[4 punte / marks]</p>
4.2(c)	$AB \times AC = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ -2 & 0 & 1 \end{vmatrix}$ $= i \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} - j \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} + k \begin{vmatrix} -1 & 2 \\ -2 & 0 \end{vmatrix}$ $= 2j + j + 4k$ $\text{Oppervlakte/area } AB \times AC = \sqrt{2^2 + (1)^2 + (4)^2}$ $= \sqrt{21}$	<p>✓ $\begin{vmatrix} i & j & k \\ -2 & 0 & 1 \\ -1 & 2 & 0 \end{vmatrix}$ ✓ $i \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} - j \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} + k \begin{vmatrix} -1 & 2 \\ -2 & 0 \end{vmatrix}$ ✓ $+k \begin{vmatrix} -1 & 2 \\ -2 & 0 \end{vmatrix}$ ✓ $2j + j + 4k$ ✓ $AB \times AC$ ✓ $\sqrt{21}$</p> <p style="text-align: right;">[5 punte / marks]</p>

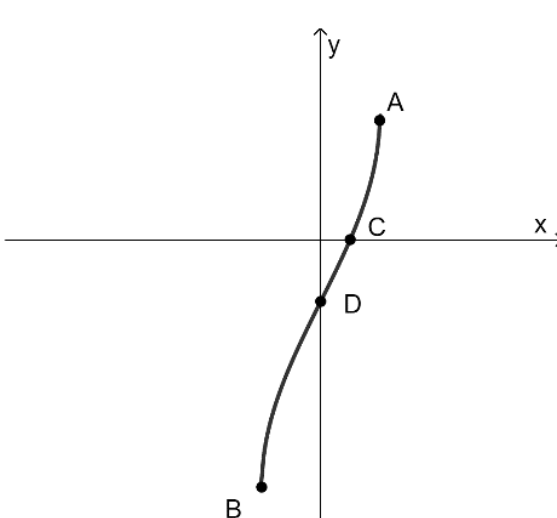
Vraag / Question 5

[20 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
5.1	$\lim_{x \rightarrow c^-} px + 1 = f(c)$ $p^2 + 1 = 2p$ $p^2 - 2p + 1 = 0$ $(p - 1)(p - 1) = 0$ $p = 1$ $\lim_{x \rightarrow q^+} \sqrt{x + 1} = 2p$ $\sqrt{q + 1} = 2(1)$ $q + 1 = 4$ $q = 3$	<p>✓ $\lim_{x \rightarrow c^-} px + 1 = f(c)$</p> <p>✓ $p^2 + 1 = 2p$</p> <p>✓ $p = 1$</p> <p>✓ $\lim_{x \rightarrow q^+} \sqrt{x + 1} = 2p$</p> <p>✓ $\sqrt{q + 1} = 2(1)$</p> <p>✓ $q = 3$</p> <p style="text-align: right;">[6 punte / marks]</p>
5.2	$Dx[e^y \ln(x + y)] = Dx[y]$ $\checkmark \quad \checkmark \quad \frac{\checkmark e^y}{x + y} \quad \checkmark \quad \checkmark \quad \checkmark$ $e^y y' \ln(x + y) + \frac{e^y}{x + y} (1 + y') = y'$ $\frac{e^y}{x + y} = y' \left(1 - e^y \ln(x + y) - \frac{e^y}{x + y} \right)$ $y' = \frac{e^y}{(x + y) \left(1 - e^y \ln(x + y) - \frac{e^y}{x + y} \right)} \quad \checkmark$	<p style="text-align: right;">[9 punte / marks]</p>
5.3	$h(x) = \log_5(3x + 2) - e^{-x}$ $h'(x) = \frac{3}{\ln 5(3x + 2)} + e^{-x}$ $x_{n+1} = x_n - \frac{\log_5(3x_n + 2) - e^{-x_n}}{\frac{3}{\ln 5(3x_n + 2)} + e^{-x_n}}$ $x_0 = 0,2$ $x_1 = 0,34654$ $x_2 = 0,358939$ <p>...</p> $x \approx 0,3590$	$h'(x):$ <p>✓ 3</p> <p>✓ $\ln 5(3x_n + 2)$</p> <p>✓ $+ e^{-x_n}$</p> <p>✓ Formule/formula</p> <p>✓ $x \approx 0,3590$</p> <p style="text-align: right;">[5 punte / marks]</p>

Vraag / Question 6

[19 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
6.1(a)	$\alpha = \frac{s}{r}$ $\alpha = \frac{DE}{AE} = \frac{50\pi}{9} \div 10 = \frac{5\pi}{9}$	<p>✓ Vervang/substute</p> <p>✓ $\frac{5\pi}{9}$</p> <p style="text-align: right;">[2 punte / marks]</p>
6.1(b)	$\widehat{ABC} = \pi - \beta - \alpha$ $\theta = \pi - \frac{\pi}{6} - \frac{5\pi}{9}$ $\theta = \frac{5\pi}{18}$ $\frac{AB}{\sin\beta} = \frac{AC}{\sin\theta}$ $AB = AC(\sin\beta)(\sin\theta)$ $= 20(\sin\frac{\pi}{6})(\sin\frac{5\pi}{18})$ $= 7,66 \text{ cm}$	<p>✓ $\pi - \beta - \alpha$</p> <p>✓ $\theta = \frac{5\pi}{18}$</p> <p>✓ $\frac{AB}{\sin\beta} = \frac{AC}{\sin\theta}$</p> <p>✓ $AC(\sin\beta)(\sin\theta)$</p> <p>✓ $AB = 7,66 \text{ cm}$</p> <p style="text-align: right;">[5 punte / marks]</p>
6.1(c)	$\text{Opp/area } ABC = \frac{1}{2} AB \cdot AC \cdot \sin\alpha$ $= \frac{1}{2} (7,66)(20) \sin\left(\frac{5\pi}{9}\right)$ $= 75,44 \text{ cm}^2$	<p>✓ Formule/formula</p> <p>✓ Vervang/substute</p> <p>✓ $75,44 \text{ cm}^2$</p> <p style="text-align: right;">[3 punte / marks]</p>
6.1(d)	$\text{Opp/area } ADE = \frac{1}{2} r^2 \alpha$ $= \frac{1}{2} (10)^2 \sin\left(\frac{5\pi}{9}\right)$ $= 49,24$ $\text{Opp/area } DECF = \text{Opp/area } ABC - \text{Opp/area } ADE$ $= 75,44 - 49,24$ $= 26,20 \text{ cm}^2$	<p>✓ $\frac{1}{2} (10)^2 \sin\left(\frac{5\pi}{9}\right)$</p> <p>✓ 49,24</p> <p>✓ Opp/area ABC - Opp/area ADE</p> <p>✓ $26,20 \text{ cm}^2$</p> <p style="text-align: right;">[4 punte / marks]</p>
6.2(a)		<p>✓</p> <p>✓ $C = \left(\frac{\sqrt{3}}{4}; 0\right)$</p> <p>✓</p> <p>✓ $D = \left(0; -\frac{\pi}{3}\right)$</p> <p>✓ Vorm/form</p> <p style="text-align: right;">[5 punte / marks]</p>

Vraag / Question 7

[13 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
7.1	$f'(x) = 3^{\tan(\frac{x}{2})} (\ln 3) \left(\sec^2 \left(\frac{x}{2} \right) \frac{1}{2} \right)$	<p>✓ $3^{\tan(\frac{x}{2})}$ ✓ $(\ln 3)$ ✓ $\left(\sec^2 \left(\frac{x}{2} \right) \right)$ ✓ $\left(\frac{1}{2} \right)$</p> <p>[4 punte / marks]</p>
7.2	$y = 2 \sec(e^{2x}) \ln 5x \, dx$ $\frac{dy}{dx} = 2 \sec(e^{2x}) \tan(e^{2x}) e^{2x} (2) (\ln 5x) + \frac{2 \sec(e^{2x}) (5)}{5x}$ <p>OF/OR</p> $y = \sec(e^{2x}) \ln(5x)^2 \, dx$ $\frac{dy}{dx} = \sec(e^{2x}) \tan(e^{2x}) e^{2x} (2) (\ln(5x)^2) + \frac{2 \sec(e^{2x}) (5x)}{(5x)^2}$	<p>✓ $2 \sec(e^{2x}) \tan(e^{2x})$ ✓ $e^{2x} (2)$ ✓ $2 \sec(e^{2x})$ ✓ $\frac{(5)}{5x}$</p> <p>[4 punte / marks]</p>
7.3	$\frac{\frac{1}{\sqrt{1 - (\cos^2 x)^2}} (2 \cos x) (-\sin x) \sqrt[3]{2x} - b \sin(\cos^2 x) \frac{1}{3} \sqrt[3]{2} x^{-\frac{2}{3}}}{(\sqrt[3]{2x})^2}$	<p>✓ $\frac{1}{\sqrt{1 - (\cos^2 x)^2}} (2 \cos x)$ ✓ $(-\sin x) \sqrt[3]{2x}$ ✓ $- b \sin(\cos^2 x)$ ✓ $\frac{1}{3} \sqrt[3]{2} x^{-\frac{2}{3}}$ ✓ $(\sqrt[3]{2x})^2$</p> <p>[5 punte / marks]</p>

Vraag / Question 8

[34 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
8.1(a)	$\int (\tan^2 2x + \sin^2 x) dx$ $= \int (\sec^2 x - 1 + \frac{1}{2}(1 - \cos 2x)) dx$ $= \frac{\tan 2x}{2} - \frac{1}{2}x - \frac{1}{4}\sin 2x + k$	<p>✓ $\sec^2 x - 1$</p> <p>✓ $\frac{1}{2}(1 - \cos 2x)$</p> <p>✓ $\frac{\tan 2x}{2}$</p> <p>✓ $-\frac{1}{2}x$</p> <p>✓ $-\frac{1}{4}\sin 2x$</p> <p style="text-align: right;">[5 punte / marks]</p>
8.1(b)	$\int \frac{b \sin 4x}{\sqrt{1-16x^2}} dx$ $u = b \sin 4x$ $\frac{du}{4} = \frac{1}{\sqrt{1-(4x)^2}} dx$ $\frac{1}{4} \int u du = \frac{1}{4} \frac{u^2}{2} + k$ $\int \frac{b \sin 4x}{\sqrt{1-16x^2}} dx = \frac{(b \sin(4x))^2}{8} + k$	<p>✓ $u = b \sin 4x$</p> <p>✓ $\frac{du}{4} = \frac{1}{\sqrt{1-(4x)^2}} dx$</p> <p>✓ $\frac{1}{4} \int u du$</p> <p>✓ $\frac{1}{4} \frac{u^2}{2} + k$</p> <p>✓ $\frac{(b \sin(4x))^2}{8}$</p> <p style="text-align: right;">[5 punte / marks]</p>
8.1(c)	$\int \left(\frac{e}{(x) \ln 3} \right) dx$ $= e \log_3 x + k$	<p>✓ $\log_3 2x$</p> <p>✓ $\frac{e}{2}$</p> <p style="text-align: right;">[2 punte / marks]</p>
8.2(a)	$4x^2 - 2x - 7 = \frac{(Ax + B)}{x^2 + 1} + \frac{C}{(x - 2)}$ $4x^2 - 2x - 7 \equiv (Ax + B)(x - 2) + C(x^2 + 1)$ $4x^2 - 2x - 7 \equiv (A + C)x^2 + (B - 2A)x + (C - 2B)$ $x = 2: 4(2)^2 - 2(2) - 7 = C(2^2 + 1)$ $5 = 5C \Rightarrow C = 1$ $x^2: 4 = A + C$ $A = 3$ $x: -2 = -2A + B$ $B = 4$ $\frac{4x^2 - 2x - 7}{(x^2 + 1)(x - 2)} \equiv \frac{3x + 4}{x^2 + 1} + \frac{1}{x - 2}$	<p>✓ Breuke</p> <p>✓ × KGV/LCD</p> <p>✓ $(A + C) \quad (B - 2A)$</p> <p>✓ $(C - 2B)$</p> <p>✓ $C = 1$</p> <p>✓ $A = 3$</p> <p>✓ $B = 4$</p> <p>✓</p> <p style="text-align: right;">[7 punte / marks]</p>
8.2(b)	$\int \frac{3x}{x^2+1} dx \quad u = x^2 + 1$ $\frac{3}{2} du = 3x dx$ $\frac{3}{2} \int \frac{1}{u} du = \frac{3}{2} \ln x^2 + 1 + k$ $\int \frac{3x}{x^2+1} dx + \int \frac{4}{x^2+1} dx + \int \frac{1}{x-2} + k$ $= \frac{3}{2} \ln x^2 + 1 + 4b \tan(x) + \ln(x + 2) + k$	<p>✓ u</p> <p>✓ $\frac{3}{2} du = 3x dx$</p> <p>✓ $\frac{3}{2} \ln x^2 + 1$</p> <p>✓ $4b \tan(x)$</p> <p>✓ $\ln(x + 2)$</p> <p style="text-align: right;">[5 punte / marks]</p>

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
8.3	$\int \frac{\ln x}{\sqrt{x}} dx$ $f(x) = \ln x \quad f'(x) = \frac{1}{x}$ $g(x) = x^{-\frac{1}{2}} \quad g'(x) = 2x^{-\frac{1}{2}}$ $\int \frac{\ln x}{\sqrt{x}} dx = 2x^{\frac{1}{2}} \ln x - \int 2x^{\frac{1}{2}} \frac{1}{x} dx + k$ $= 2x^{\frac{1}{2}} \ln x - \int 2x^{-\frac{1}{2}} dx + k$ $= 2x^{\frac{1}{2}} \ln x - 4x^{\frac{1}{2}} + k$	<p style="text-align: right;">[7 punte / marks]</p>
8.4	$x_i = a + \frac{b-a}{n} i$ $= 1 + \frac{2}{n} i$ $a = 1$ $\Delta x_i = \frac{2}{n} = \frac{b-a}{n}$ $b = 2 + a$ $= 3$	<p>✓ $a = 1$ ✓ $\frac{2}{n} = \frac{b-a}{n}$ ✓ $b = 3$</p> <p style="text-align: right;">[3 punte / marks]</p>

Vraag / Question 9

[15 punte / marks]

NR / NO	ANTWOORD / ANSWER	PUNTE / MARKS
9.1(a)	$x(x^2 - 2) = 0$ $x = 0 \quad x = \pm\sqrt{2}$ $y = 0$	✓ $x = \pm\sqrt{2}$ ✓ $x = 0$ ✓ $y = 0$ [3 punte / marks]
9.1(b)	$ \begin{array}{r rrrr} & & 1 & & \\ 1 & 0 & -1 & 1 & 0 & -2 & 0 \\ & & & 1 & 0 & -1 & \\ \hline & & & . & 0 & -1 & 0 \end{array} $ $\frac{x^3 - 2x}{x^2 - 1} = x - \frac{x}{x^2 - 1}$ Vertikale asimptote/vertical asymptotes: $x = 1 \quad x = -1$ Skuins asimptoot/skew asymptote: $y = x$	✓ Deling ✓ $x = 1$ ✓ $x = -1$ ✓ $y = x$ [4 punte / marks]
9.2	Lyn/line: $y = -x + 3$ $\pi \int_0^p [f(x)]^2 dx - \pi \int_0^p [g(x)]^2 dx$ $\pi \int_0^p [3 - x]^2 dx - \pi \int_0^p [\sqrt{x}]^2 dx \quad \checkmark$ $= \pi \int_0^p (9 - 6x + x^2) dx - \pi \int_0^p x dx \quad \checkmark$ $\frac{20\pi}{3} = \pi \left(9p - \frac{6p^2}{2} - \frac{p^3}{3} \right) - \pi \left(\frac{p^2}{2} \right) \quad \checkmark \quad \checkmark$ OF/OR $\pi \int_0^p ((3 - x)^2 - x) dx$ $\pi \int_0^p (9 - 7x + x^2) dx \quad \checkmark$ $\frac{20\pi}{3} = \pi \left[9p - \frac{7p^2}{2} - \frac{p^3}{3} \right]_0^p \quad \checkmark$ $\frac{40}{6} = \frac{2p^3}{6} - \frac{21p^2}{6} + \frac{54p}{6}$ $2p^3 - 21p^2 + 54p - 40 = 0 \quad \checkmark$ Substitusie/substitution: $p > 1$, toets/test $p = 2$: $2(2)^3 - 21(2)^2 + 54(2) - 40 \quad \checkmark$ $= 0$ $p = 2 \quad \checkmark$	[8 punte / marks]

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* –