

2021

Alpha Mathematics



Name: _____

ID:

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School: _____

Answer sheet: QUESTION 1

Marks

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

QUESTION	Possible Mark	Mark
1	20	
2	18	
3	21	
4	20	
5	20	
6	24	
7	17	
8	21	
9	20	
10	19	
Total	200	

α -MATHEMATICS

Alpha Mathematics FINAL EXAM PAPER

22 October 2021

Grade 12

Time: 3 hours

Total: 200 marks

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the question paper:

1. Answer all 10 questions on this exam paper.
2. Write your name and ID number on the front page of the question paper.
3. Non-programmable calculators may be used, unless otherwise indicated at a specific question.
4. Unless indicated otherwise, all answers, where applicable, must be given correct to two decimal places.
5. The diagrams in the question paper are not necessarily drawn to scale.
6. All angles are given in radians. Answers must be given in radians where applicable.
7. This question paper consists of a front page, 24 pages and a formula sheet of 3 pages.
8. Question 1 consists of 10 multiple choice questions. Answer it on the answer sheet. This answer sheet is at the front of the paper.
Do not remove the answer sheet from the question paper.
9. For all other questions, all necessary calculations must be shown clearly. The correct answer on its own will not necessarily lead to full marks.
10. Additional writing space is provided at the end of this question paper. Clearly indicate if you made use of this to complete a question.
11. Write neatly and legibly.

1.6 Solve for x : $\frac{5}{|x-1|} < -1$

(A) $x < -4$ or $x > 6$

(B) $-4 < x < 6$

(C) No solution

(D) $x \in \mathbb{R}$

1.7 If $f(x) = g(x) + 7$ for $x \in [0; 2]$, then $\int_0^2 (f(x) + g(x)) dx =$

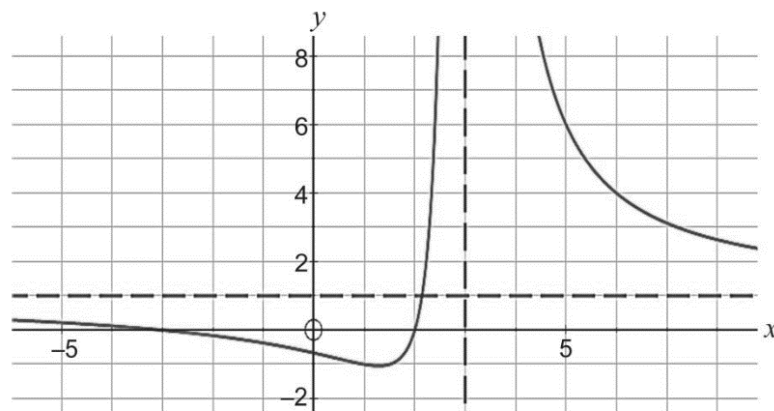
(A) $2 \int_0^2 g(x) dx + 7$

(B) $2 \int_0^2 g(x) dx + \frac{7}{2}$

(C) $2 \int_0^2 g(x) dx + 14$

(D) $\int_0^2 g(x) dx + 14$

1.8 Look at the following graph. Which one of the equations can be the equation of this graph?



(A) $\frac{x^2-4}{x^2-9}$

(B) $\frac{x^2+x-6}{x-3}$

(C) $\frac{x-2}{x^2-x-6}$

(D) $\frac{x^2+x-6}{x^2-6x+9}$

1.9 Which of the following is equal to $\int_0^\pi \sin x dx$?

(A) $\int_0^\pi \cos x dx$

(B) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$

(C) $\int_{-\pi}^0 \sin x dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x dx$

1.10 $F(x) = (f \circ g)(x)$, with $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, $g'(5) = 6$. Determine the value of $F'(5)$.

(A) 24

(B) 8

(C) 12

(D) 20

Answer the following questions **on the exam paper** on the lines provided at the end of each question. Clearly indicate if you use the additional writing space at the end of the paper to complete a question.

QUESTION 2 [18 MARKS]

2.1 An air conditioner is switched on in a room. When the air conditioner is switched on, the equation $T(t) = 15 + 9e^{-0.3t}$ applies, where $T(t)$ is the temperature in $^{\circ}\text{C}$ in the room t minutes after it was switched on.

- (a) Determine the initial temperature in the room. (1)

- (b) What is the minimum temperature which the room can reach? (1)

- (c) After how many minutes (rounded to the nearest minute) will the temperature in the room be 20°C ? (4)

- (d) Calculate the rate of change of the temperature 5 minutes after it was switched on. (3)

3.2 The following system of equations is given:

$$2x + y + z = 0$$

$$x + ay - z = 5$$

$$4x + ay + 2z = 1$$

(a) Write the equations in matrix form: $Ax = B$ (2)

(b) (i) Calculate $|A|$, the determinant of A in terms of the given a . (2)

(ii) Use Cramer's rule and write down matrix A_y . (1)

(iii) Determine the value of a with Cramer's rule if it is also given that $|A_y| = 3$ and $y = 1$. (2)

3.3 Given $f(x) = \sin\left(x - \frac{\pi}{6}\right) + \frac{1}{2}$

- (a) Determine the equation of $f^{-1}(x)$, the inverse function of f . (3)

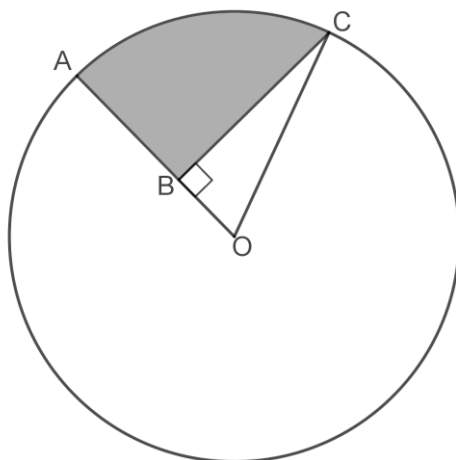
- (b) Give the range of f^{-1} . (2)

QUESTION 4 [20 MARKS]

4.1 $\mathbf{u} = (2; y; -1)$ and $\mathbf{v} = (0; -2; 5)$

- (a) Determine the value of y if a vector which is perpendicular to \mathbf{u} and \mathbf{v} is equal to $18\mathbf{i} - 10\mathbf{j} - 4\mathbf{k}$. (4)

- 4.2 The sketch shows a circle with centre O and radius 10. A and C lie on the circumference of the circle. B lies on OA so that BC is perpendicular to AO . $\widehat{OCB} = \frac{\pi}{6}$ radians.



- (a) Give the size of \widehat{AOC} in terms of π . (1)

- (b) Determine the area of the small sector AOC . (2)

- (c) Hence, calculate the area of the shaded area ABC .
Leave the answer in surd form and in terms of π where applicable. (4)

QUESTION 7 [17 MARKS]

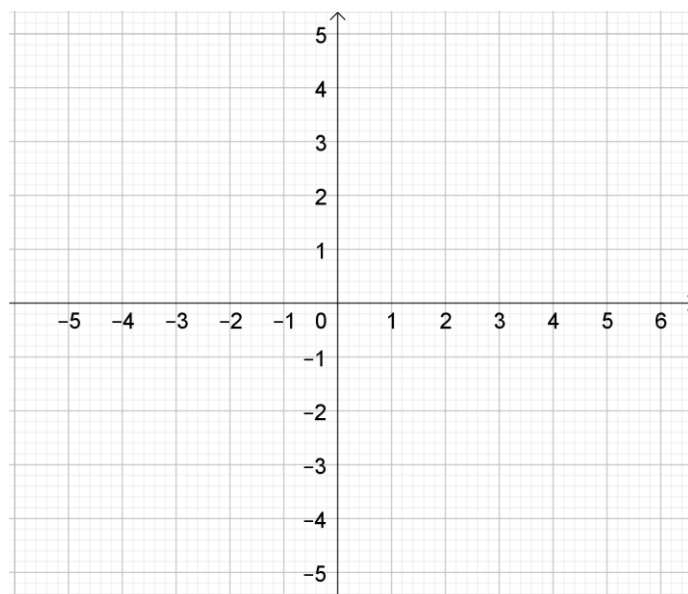
Given: $f(x) = \frac{x^2+2x+2}{x+1} = x + 1 + \frac{1}{x+1}$

7.1 Give the intercept of f with the y -axis. (1)

7.2 Give the equations of the asymptotes of f and say which type of asymptote they are. (4)

7.3 Calculate the co-ordinates of the turning points of f . (6)

- 7.4 Make a sketch graph of f on the axes below. Clearly show the asymptotes and the turning points. (6)



QUESTION 8 [21 MARKS]

- 8.1 Differentiate the following functions:

(a) If $f(x) = 5^x \times \log x$, determine $f'(x)$ (3)

(b) $D_x \left[\tan \left(e^{\frac{x}{2}+1} \right) \right]$ (4)
