α-WISKUNDE

Alpha Wiskunde Graad 11 / ***Alpha Mathematics Grade 11***

Halfjaar eksamen 2020 / ***Half year examination 2019***

MEMORANDUM

Totaal / ***Total*****: 165 punte /** ***marks***

Hierdie memorandum bestaan uit 9 bladsye. /
This memorandum consists of 9 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	D	$-\|3-x\| < -5$ $\|3-x\| > 5$ $3-x > 5$ of $3-x < -5$ $x < -2$ of $x > 8$	**2**
1.2	B	$\binom{6}{2}(b)^2 = 60$ $\quad b^2 = 4$ $\quad b = \pm 2$	**2**
1.3	D	$(4-3x)^{-5} = \left(4\left(1-\frac{3}{4}x\right)\right)^{-5}$ $\therefore \left\|-\frac{3}{4}x\right\| < 1$ $\|x\| < \frac{4}{3}$	**2**
1.4	C	$0 \leq \frac{x}{2} \leq \pi$ $x \in [0; 2\pi]$	**2**
1.5	C	$\|x-2\| \leq 0$ slegs as $\|x-2\| = 0$, $x = 2$	**2**
1.6	B	Drie gelyk sektore: $\frac{2\pi}{3}$ Booglengte: $s = r\theta = \frac{3(2\pi)}{3} = 2\pi$	**2**
1.7	C	$-2+i$ dan is die ander wortel $-2-i$	**2**
1.8	B	$n = 3$, $r = 0$ tot 3, dus 4 terme	**2**
1.9	A	$f(0) = 0-4 = -4$ $f(5) = 5-4 = 1$ $f(10) = -10+6 = -4$ $-4+1-4 = -7$	**2**
1.10	C	$-bgcos(x)$ reflekteer op die x-as, $+\frac{\pi}{2}$ skuif op A: $bgcos(x) - \frac{\pi}{2}$ B: $\tan(x)$ D: $-\tan(x)$	**2**

Vraag / *Question* 2 **[33 punte / *marks*]**

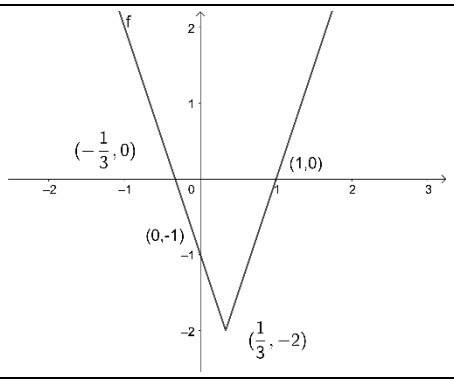
NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1(a)	$f(x) = 4x^4 - 4x^3 + 5x^2 - 4x + 1$ $1 - 2x = 0 \quad x = \frac{1}{2}$ $f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^3 + 5\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = 0$ $f\left(\frac{1}{2}\right) = 0$, dus is $2x - 1$ 'n faktor van $f(x)$	✓ $x = \frac{1}{2}$ ✓ $f\left(\frac{1}{2}\right) = 0$ ✓ Antwoorksin **[3 punte / *marks*]**
2.1(b)	$1 - 2x = 0 \quad x = \frac{1}{2}$ Kwosiënt is $4x^3 - 2x^2 + 4x - 2$ $\frac{1}{2}$ | 4 -4 5 -4 1 / 2 -1 2 -1 / 4 -2 4 -2 | 0	✓ Deler $\frac{1}{2}$ ✓ ✓✓ deling ✓ Antwoord **[5 punte / *marks*]**
2.1(c)	As Indien $x - i$ dan is Indien $x + i$ ook 'n faktor $(x - i)(x + i) = x^2 - i^2 = x^2 + 1$ Omdat $2x - 1$ reeds 'n faktor is: $(1 - 2x)(x^2 + 1) = -2x^3 + x^2 - 2x + 1$ $-2 + 1 - 2 + 1$ | -2 1 / 4 -4 5 -4 1 / 4 -2 4 -2 / -2 1 -2 1 / -2 1 -2 1 Dus ander wortel $-2x + 1 = (1 - 2x)$ Faktore $(1 - 2x)(1 - 2x)(x - i)(x + i)$ **OF** $1 + 0 + 1$ | 4 -4 1 / 4 -4 5 -4 1 / 4 0 4 / -4 1 -4 / -4 0 -4 / 1 0 1 / 1 0 1 $4x^2 + 4x + 1 = (2x - 1)(2x - 1)$ Faktore $(2x - 1)(2x - 1)(x - i)(x + i)$	✓ $x + i$ ✓ $x^2 - i^2$ ✓ $x^2 + 1$ ✓ $(1 - 2x)(x^2 + 1)$ ✓ $-2x^3 + x^2 - 2x + 1$ ✓ Deler $-2x^3 + x^2 - 2x + 1$ ✓ ✓Anwoord -2 en 1 ✓ Antwoord faktore **OF** ✓ Deel met $x^2 + 1$ ✓ ✓✓Antwoord $4 - 4 \quad 1$ ✓ Faktoriseer en kry $(2x - 1)$ ✓ Antwoord **[9 punte / *marks*]**

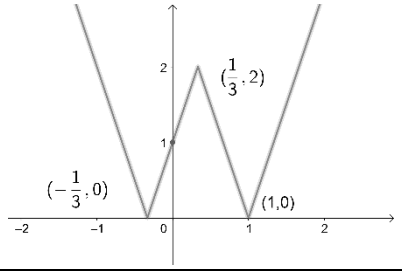
NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.2	$\frac{3\ -\ 4\,x^3}{(1\ -\ 2\,x)^2\,(x^2+\ 1)} \equiv \frac{A}{1-2\,x} + \frac{B}{(\,1-2\,x)^2} + \frac{Cx+D}{x^2+\ 1}$ $3\ -\ 4\,x^3 \equiv A\,(-1\ +\ 2\,x)(1\ +\ x^2) +\ B\,(1\ +\ x^2) -$ $(Cx+D)(\,2\,x-1\,)^2$ $3\ -\ 4\,x^3 \equiv A + B - D - 2Ax - Cx + 4Dx + Ax^2 + Bx^2 + 4Cx^2$ $- 4Dx^2 - 2Ax^3 - 4Cx^3$ $\equiv x^3(-2A-4C) + x^2(B+A+4C) + x(-2A-C+4D)$ $+ (B-A-D)$ **As $2x-1=0$ $\left(x=\frac{1}{2}\right)$:** $3-4\left(\frac{1}{2}\right)^3 = B\left(1+\left(\frac{1}{2}\right)^2\right)$ $B=2$ **Koëffisiënt van x^2:** $0=2+A+4C$ $A=-2-4C$ **Koëffisiënt van x^3:** $-4=-2A-4C$ $-4=-2(-2-4C)-4C$ $C=0$ $A=-2$ **As $x=0$:** $3=B-A-D$ $3=2+2-D$ $D=-1$ $A=-2$ $B=2$ $C=0$ $D=-1$ $\therefore \frac{3\ -\ 4\,x^3}{(1\ -\ 2\,x)^2\,(x^2+\ 1)} \equiv -\frac{2}{1-2\,x} + \frac{2}{(\,1-2\,x)^2} - \frac{1}{x^2+\ 1}$	✓ Ontbind in parsiële breuke ✓ Vermenigvuldig met KGV ✓ Vereenvoudig ✓ Groepeer volgens magte van x ✓ A ✓ B ✓ C ✓ D ✓ Finale antwoord **[9 punte / *marks*]**
2.3	$f(x) = x^3 - 17x^2 + 55x - 39$ As $2+\sqrt{1}$ nulpunt is dan is $2-\sqrt{1}$ ook 'n nulpunt. $x = 2 \pm \sqrt{1}$ $x-2 = \pm\sqrt{1}$ $(x-2)^2 = 1$ $x^2 - 4x + 3 = 0$ Wortels is: $(x-2+\sqrt{1})$ $(x-2-\sqrt{1})$ $(x-13)$ $1-4+3$ | 1 -13 +55 -39 -(1 -4 3) -13 52 -39 -(-13 52 -39) 1 0 1 0	Ander wortel ✓ $2-\sqrt{1}$ of $(x-2+\sqrt{1})$ ✓ $x = 2 \pm \sqrt{1}$ ✓ $(x-2)^2 = 1$ ✓ $x^2 - 4x + 3$ ✓ Deler $1-4+3$ ✓ 1 en -13 ✓ Antwoord $(x-13)$ **[7 punte / *marks*]**

Vraag / *Question* 3 **[19 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1	$2+(3\times 2)+(4\times 2^2)+\cdots+(n+1)\times 2^{n-1}=n\times 2^n$ **Stel / Let $n=1$**: LK $=2$ RK $=1\times 2^1=2$ $\therefore$ LK $=$ RK $\therefore$ bewering is waar vir / *statement is true for* $n=1$. **Aanvaar die bewering is waar as / *Accept the statement is true for* $n=k$** $2+(3\times 2)+(4\times 2^2)+\cdots+(k+1)\times 2^{k-1}=k\times 2^k$ **Beskou / *Consider* $n=k+1$**: LK $=k\times 2^k+(k+1)\times 2^{k+1}$ $=k.2^k+k.2^k+2.2^k$ $=2k.2^k+2^{k+1}$ $=k.2^{k+1}+2^{k+1}$ RK $=(k+1)\times 2^{k+1}$ $\therefore$ LK $=$ RK Volgens die beginsel van wiskundige induksie is die bewering waar vir alle $n\in\mathbb{N}$. / *The statement is true for* $n=1$. *If the statement is true for* $n=k$, *it is also true for* $n=k+1$. *Thus, the statement is true for all* $n\in\mathbb{N}$.	✓ LK $=$ RK $=2$ ✓ Rede ✓ Aanvaar... ✓ Stelling met $n=k$ ✓ Eerste k terme ✓ $(k+1)$de term ✓ Vervang eerste k terme met $k\times 2^k$ ✓ $(k+2)\times 2^k=k.2^k+2.2^k$ ✓ $k.2^k+k.2^k=2k.2^k$ ✓ Antwoord van LK ✓ Antwoord van RK ✓ Storie **[12 punte / *marks*]**
3.2	**Beskou / *Consider* $n=k+1$**: LK $=1(1!)+2(2!)+3(3!)+\cdots+k(k!)+(k+1)\big((k+1)!\big)$ $=(k+1)!-1+(k+1)\big((k+1)!\big)$ $=(k+1)!\big(1+(k+1)\big)-1$ $=(k+1)!\,(k+2)-1$ $=(k+2)!-1$ RK $=(k+2)!-1$ $\therefore$ LK $=$ RK	✓ Eerste k terme ✓ $(k+1)$de term ✓ Vervang eerste k terme met $(k+1)!-1$ ✓ Faktoriseer $(k+1)!$ ✓ $(k+1)!\,(k+2)=(k+2)!$ ✓ Antwoord van RK **[6 punte / *marks*]**
3.3	$4^{2n}-1$ **Stel / Let $n=1$**: $4^{2(1)}-1=15$ $\therefore 15$ is deelbaar deur 5 $\therefore$ bewering is waar vir / *statement is true for* $n=1$. **Aanvaar die bewering is waar as / *Accept the statement is true for* $n=k$** $4^{2k}-1$ is deelbaar/is divisable **Beskou / *Consider* $n=k+1$**: $4^{2(k+1)}-1$ $=4^{2k+2}-1$ $=16.4^{2k}-1$ $=16(4^{2k}-1)+15$ maar $(4^{2k}-1)$ is deelbaar deur 5 $\therefore$ Deelbaar deur 5 want $16(4^{2k}-1)$ is veelvoud van 5 en 15 is deelbaar Die bewering is waar as $n=1$. As die bewering waar is vir $n=k$, is dit ook waar vir $n=k+1$. Dus is die bewering waar vir alle $n\in\mathbb{N}$. / *The statement is true for* $n=1$. *If the statement is true for* $n=k$, *it is also true for* $n=k+1$. *Thus, the statement is true for all* $n\in\mathbb{N}$.	✓ 15 ✓ 15 deelbaar deur 5 ✓ Aanvaar... ✓ Stelling met $n=k$ ✓ $(k+1)$de term ✓ 4^{2k+2} ✓ 16.4^{2k} ✓ $16(4^{2k}-1)+15$ ✓ Rede vir deelbaarheid ✓ Storie . **[10 punte / *marks*]**

Vraag / *Question* 4 **[32 punte / *marks*]**

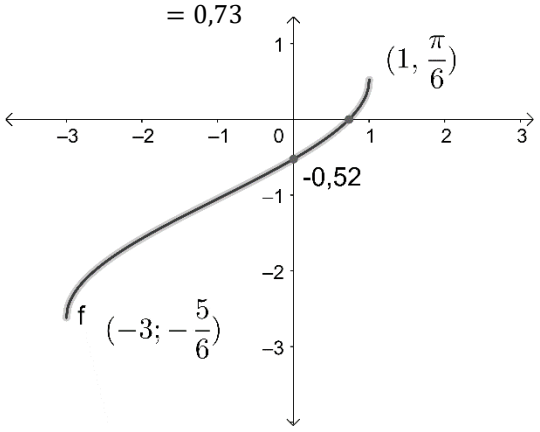
NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1 (a)	$2\|x-1\|-4-(x-2)=0$ $2\|x-1\|-4-x+2=0$ $2\|x-1\|=x+2$ $\|x-1\|=\frac{1}{2}x+1$ **As $x \geq 1$:** $x-1=\frac{1}{2}x+1$ $\frac{1}{2}x=2$ $x=4$ **As $x<1$:** $x-1=-\frac{1}{2}x-1$ $\frac{3}{2}x=0$ $x=0$ $A(0,0)\quad B(4,0)$	✓ $f(x)-g(x)=0$ ✓ $2\|x-1\|=x+2$ ✓ Deel deur 2 ✓ Vergelyking van $x \geq 1$ ✓ $x=4$ ✓ Vergelyking van $x<1$ ✓ $x=0$ ✓ $A(0,0)$ ✓ $B(4,0)$ **[9 punte / *marks*]**
4.1 (b)	$2\|x-1\|>x-2$ $x>4$ of $x<0$	✓ $x>4$ ✓ $x<0$ **[2 punte / *marks*]**
4.1 (c)	Vergelyking: $2\|x-1\|-4-x+2$ $x-1=0$ $x=1$ y-waarde: $y=2\|1-1\|-4-1+2$ $y=-3$ $C(1,-3)$	✓ $x=1$ ✓ $y=-3$ ✓ $C(1,-3)$ **[3 punte / *marks*]**
4.2 (a)	$-\|x-7\| \leq -2$ $\|x-7\| \geq 2$ $-2 \geq x-7 \geq 2$ $5 \geq x \geq 9$	✓ $-\|x-7\| \leq -2$ ✓✓ $5 \geq x \geq 9$ **[3 punte / *marks*]**
4.2 (b)	$\left\|\frac{x+1}{x-2}\right\|=2$ $\frac{x+1}{x-2}=2$ of $\frac{x+1}{x-2}=-2$ $x+1=2(x-2)$ $x+1=2x-4$ $x=5$ $x+1=-2(x-2)$ $x+1=-2x+4$ $3x=3$ $x=1$	✓ $=2$ ✓ $=-2$ ✓ $x=5$ ✓ $x=1$ **[4 punte / *marks*]**
4.2 (c)	$\|x-3\|>-2$ Maar $\|x\| \geq 0$ Dus $\|x-3\|>-2$ vir alle $x \in \mathbb{R}$	✓ $\|x\| \geq 0$ ✓ $x \in \mathbb{R}$ **[2 punte / *marks*]**
4.3 (a)		✓✓ x-afsnitte $\left(-\frac{1}{3};0\right)$ en $(1;0)$ ✓ y-afsnit $(0;-1)$ ✓✓ Knakpunt $\left(\frac{1}{3};-2\right)$ ✓ Vorm **[6 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.3 (b)	 **Onthou op dieselfde skets**	✓ Knakpunt $\left(\frac{1}{3};2\right)$ ✓ y-afsnit $(0;1)$ ✓ Vorm **[3 punte / *marks*]**

Vraag / *Question* 5 **[29 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$\binom{15}{r}(x^2)^{15-r}\left(-\frac{1}{2x}\right)^r$ $=\binom{15}{r}\left(-\frac{1}{2}\right)^r x^{30-2r-r}$ $\therefore 30-2r-r=0 \implies r=10$ $=\binom{15}{10}\left(-\frac{1}{2}\right)^{10}x^0$ $=\frac{3004}{1024}$	✓ ✓✓ Elke hakie ✓ Vereenvoudig ✓ $r=10$ ✓ Vervang r in ✓ Antwoord **[7 punte / *marks*]**
5.2	$(2-5x)^7=\binom{7}{0}(2)^{7-0}+\binom{7}{1}(2)^{7-1}(-5x)^1+\binom{7}{2}(2)^{7-2}(-5x)^2$ $=128-2240x+16800x^2$	✓ ✓✓ Elke koëffisiënt ✓ Magte van x toenemend **[4 punte / *marks*]**
5.3	$(2-5x)^{-7}=1+(-7)(-5x)+\frac{(-7)(-8)(-5x)^2}{2!}+\cdots$ $=1+35x+700x^2$	✓ Mag -7 ✓ ✓✓ Elke term uitgebrei ✓ Vereenvoudig **[5 punte / *marks*]**
5.4 (a)	$\sqrt[3]{8-x}=\sqrt[3]{8\left(1-\frac{x}{8}\right)}=2\left(1-\frac{x}{8}\right)^{\frac{1}{3}}$ $=2(1+\left(\frac{1}{3}\right)\left(-\frac{x}{8}\right)+\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{x}{8}\right)^2+\cdots)$ $=2(1-\frac{x}{24}-\frac{x^2}{288}+\cdots)$ $=2-\frac{x}{12}-\frac{x^2}{144}+\cdots$	✓ $\sqrt[3]{8\left(1-\frac{x}{8}\right)}$ ✓ $2\left(1-\frac{x}{8}\right)^{\frac{1}{3}}$ ✓ ✓✓ Elke term ✓ Vereenvoudig ✓ Vermenigvuldig **[7 punte / *marks*]**
5.4 (b)	$\left\lvert-\frac{x}{8}\right\rvert<1 \qquad x<8$	$\left\lvert-\frac{x}{8}\right\rvert<1$ ✓ $x<8$ ✓ **[2 punte / *marks*]**
5.4 (c)	$\sqrt[3]{1000(8-0{,}1)}=10\sqrt[3]{(8-0{,}1)}$ $=10\left(2-\frac{0{,}1}{12}-\frac{(0{,}1)^2}{144}\right)=19.915972\approx 19.916$	✓ 10 ✓ Vervang in 5.4 (a) ✓ Vermenigvuldig met 10 ✓ Antwoord afgerond **[4 punte / *marks*]**

Vraag / *Question* 6 **[16 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1	$y = 2\sin\left(x+\frac{\pi}{3}\right)-1$ $y+1 = 2\sin\left(x+\frac{\pi}{3}\right)$ $\frac{y+1}{2} = \sin\left(x+\frac{\pi}{3}\right)$ $bgsin\left(\frac{y+1}{2}\right) = x+\frac{\pi}{3}$ $bgsin\left(\frac{y+1}{2}\right)-\frac{\pi}{3} = x$ $f^{-1}(x) = bgsin\left(\frac{x+1}{2}\right)-\frac{\pi}{3}$	✓ Kry sin alleen ✓ Verander na bgsin ✓ Maak x die onderwerp ✓ Ruil y en x om **[4 punte / *marks*]**
6.2	$-\frac{\pi}{2} \le x+\frac{\pi}{3} \le \frac{\pi}{2}$ $-\frac{\pi}{2}-\frac{\pi}{3} \le x \le \frac{\pi}{2}-\frac{\pi}{3}$ $-\frac{5\pi}{6} \le x \le \frac{\pi}{6}$	✓✓$-\frac{\pi}{2}$ $\frac{\pi}{2}$ ✓ $-\frac{5\pi}{6} \le x \le \frac{\pi}{6}$ **[3 punte / *marks*]**
6.3	$x = 2\sin\left(y+\frac{\pi}{3}\right)-1$ $-\frac{5\pi}{6} \le x \le \frac{\pi}{6}$ $x = 2\sin\left(-\frac{5\pi}{6}+\frac{\pi}{3}\right)-1 = -3$ $x = 2\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)-1 = 1$ $y \in [-3,1]$ y-afsnit: $bgsin\left(\frac{1}{2}\right)-\frac{\pi}{3} = -0{,}52$ x-afsnit: $y = 2\sin\left(\frac{\pi}{3}\right)-1$ $= \sqrt{3}-1$ $= 0{,}73$  	✓ $\left(1;\frac{\pi}{6}\right)$ ✓ $\left(-3;-\frac{5\pi}{6}\right)$ ✓ x-afsnit: $(0{,}73;\ 0)$ ✓ y-afrsnit $(0;\ -0{,}52)$ ✓ Vorm **[5 punte / *marks*]**
6.4	$q+\frac{\pi}{2} = \frac{7\pi}{18}$ $q = -\frac{2}{18}\pi = -\frac{\pi}{9}$ $bgtan(\frac{3}{2}-p)-\frac{\pi}{9} = \frac{5}{36}\pi$ $bgtan(\left(\frac{3}{2}-p\right) = \frac{9\pi}{36}$ $p = \frac{3}{2}-\tan\left(\frac{\pi}{4}\right) = \frac{1}{2}$	✓ $q+\frac{\pi}{2}$ ✓ $q = -\frac{\pi}{9}$ ✓ $bgtan\left(\frac{3}{2}-p\right)-\frac{\pi}{9} = \frac{5}{36}\pi$ ✓ $p = \frac{1}{2}$ **[4 punte / *marks*]**

Vraag / *Question* 7 **[12 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	$\hat{O} = O\hat{A}D = O\hat{D}A = \frac{\pi}{3}$ Gelyksidge driehoek $O\hat{D}A = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ Hoeke op 'n reguit lyn.	✓ $O\hat{D}A = \frac{\pi}{3}$ ✓ Antwoord **[2 punte / *marks*]**
7.2	Booglengte AC: $s_{AC} = r\theta = 8cm\left(\frac{2\pi}{3}\right) = \frac{16\pi}{3}\text{cm}$ Booglengt BC: $s_{BC} = r\theta = 16cm\ \left(\frac{\pi}{3}\right) = \frac{16\pi}{3}\text{cm}$ Omtrek ABC: $s_{AC} + s_{BC} + AB = \frac{16\pi}{3}\text{cm} + \frac{16\pi}{3}\text{cm} + 8\text{cm} = 41.51cm$	✓ s_{AB} ✓ s_{BC} ✓ Omtrek getalsin ✓ Antwoord **[4 punte / *marks*]**
7.3	Area ΔAOD $= \frac{1}{2}(8cm)(8cm)\sin\frac{\pi}{3} = 16\sqrt{3}\ cm^2$ Area sektor DAC: $\frac{1}{2}r^2\theta = \frac{1}{2}8^2\left(\frac{2\pi}{3}\right) = \frac{64\pi}{3}cm^2$ Area sektor OBC: $\frac{1}{2}r^2\theta = \frac{1}{2}16^2\left(\frac{\pi}{3}\right) = \frac{128\pi}{3}\ cm^2$ Geskakeerde area: Area OBC – area DAC - ΔAOD $= \frac{128\pi}{3} - \frac{64\pi}{3} - 16\sqrt{3} = 39.31\ cm^2$	✓ ✓Area driehoek ✓ ✓Area sektor DAC ✓ ✓Area sektor OBC ✓ Antwoord **[7 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -