α-WISKUNDE

Alpha Wiskunde Graad 10 / *Alpha Mathematics Grade 11*

Halfjaar eksamen 2020 / *Half year examination 2020*

MEMORANDUM

Totaal / *Total*: 120 punte / *marks*

Eksaminator / *Examiner*: Pieter van Onselen

Hierdie memorandum bestaan uit 8 bladsye. /
This memorandum consists of 8 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	C	'n Getransponeerde matriks is 'n matriks waar die rye en kolomme omruil van die oorspronklike matriks.	**2**
1.2	B	$\left(\sqrt{-16}\right)\left(\sqrt{-4}\right) = 4i.2i = 8i^2 = -8$	**2**
1.3	C	$\frac{5\pi}{18} = 0{,}87$	**2**
1.4	C	Toegevoegde van $3 - 2i$ is $3 + 2i$	**2**
1.5	B	$(2;3)$ en $(4;-3)$ Puntproduk: $2 \times 4 + 3 \times -3 = -1$	**2**
1.6	B	$-2 \times x + 4 \times -3 = -2$ $-2x - 12 = -2$ $-2x = 10$ $x = -5$	**2**
1.7	A	$Im\left(\frac{1}{\sqrt{3}}i - \frac{2}{3}\right) = \frac{1}{\sqrt{3}}$	**2**
1.8	A	$f(2) = 2$	**2**
1.9	B	$\frac{1}{3} \times \frac{180^0}{\pi} = 19{,}10^0$	**2**
1.10	D	$(1-i)^8 = ((1-i)^2)^4 = (1 - 2i + i^2)^4 = (-2i)^4 = 16i^4 = 16$	**2**

Vraag / *Question* 2 **[24 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1(a)	$m = 3 + 5i$ en $n = 2 - i$ $2n^* - mi^{155}$ $\quad 155 \div 4 = 38\, res\, 3$ $= 2(2-i)^* - (3+5i)i^{154}$ $\quad i^{155} = i^3 = -i$ $= 2(2+i) - (3+5i)(-i)$ $= 4 + 2i + 3i + 5i^2$ $= -1 + 5i$	✓ $i^{155} = i^3 = -i$ ✓ $(2-i)^* = 2+i$ ✓ $4 + 2i$ ✓ $3i + 5i^2$ ✓ Antwoord **[5 punte / *marks*]**
2.1(b)	$(3+5i)(2-i)$ $= 6 - 3i + 10i - 5i^2$ $= 11 + 7i$	✓ $6 - 3i + 10i - 5i^2$ ✓ $-5i^2 = +5$ ✓ Antwoord **[3 punte / *marks*]**
2.1(c)	$\frac{2-i}{3i}$ $\frac{2-i}{3i} \times \frac{i}{i} = \frac{2-i^2}{3i^2}$ $= \frac{2i+1}{-3}$ $= -\frac{1}{3} - \frac{2i}{3}$	✓ $\times \frac{i}{i}$ ✓ $\frac{2-i^2}{3i^2}$ ✓ $i^2 = -1$ ✓ Antwoord **[4 punte / *marks*]**
2.2	$(a+bi)(1+3i) = (1-i)$ $(a+bi) = \frac{1-i}{(1+3i)}$ $= \frac{(1-i)}{(1+3i)} \times \frac{(1-3i)}{(1-3i)}$ $= \frac{1-3i-i+3i^2}{1-9i^2}$ $= \frac{-2-4i}{10}$ $= -\frac{1}{5} - \frac{2i}{5}$	✓ $\frac{1-i}{(1+3i)}$ ✓ $\times \frac{(1-3i)}{(1-3i)}$ ✓✓ $\frac{1-3i-i+3i^2}{1-9i^2}$ ✓ Vereenvoudig noemer ✓ Vereenvoudig teller ✓ Antwoord **[7 punte / *marks*]**
2.3	$(2x+i) - x(1-3i)$ $= 2x + i - x + 3ix$ $= (2x - x) + (1 + 3x)i$ Suiwer reëel: $1+3x = 0$ $x = -\frac{1}{3}$	✓ $2x + i - x + 3ix$ ✓ $1+3x = 0$ ✓ $x = -\frac{1}{3}$ **[3 punte / *marks*]**
2.4(a)	$(3+i)$ $(-1-3i)$	✓✓ Antwoord **[2 punte / *marks*]**
2.4(b)	$2 - 2i$	✓ **[1 punte / *marks*]**

Vraag / *Question* 3 **[9 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1	$\frac{3x^2-7x-2}{x^3-x}=\frac{3x^2-7x-2}{x(x-1)(x+1)}\equiv\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$ $3x^2-7x-2\equiv A(x-1)(x+1)+Bx(x+1)+Cx(x-1)$ $\equiv Ax^2-A+Bx^2+Bx+Cx^2-C$ $\equiv x^2(A+B+C)+x(B-C)-A$ **As $x=0$:** $-2=-A$ $\boldsymbol{A=-2}$ **As $x=1$:** $2B=-6$ $\boldsymbol{B=-3}$ **As $x=-1$:** $2C=8$ $\boldsymbol{C=4}$ $\frac{3x^2-7x-2}{x^3+x}\equiv-\frac{2}{x}-\frac{3}{x-1}+\frac{4}{x+1}$	✓ Faktoriseer ✓ Ontbind in parsiële breuke ✓ Vermenigvuldig met KGV ✓ Vereenvoudig ✓ Groepeer volgens magte van x ✓ A ✓ B ✓ C ✓ Finale antwoord **[9 punte / *marks*]**

Vraag / *Question* 4 **[15 punte / *marks*]**

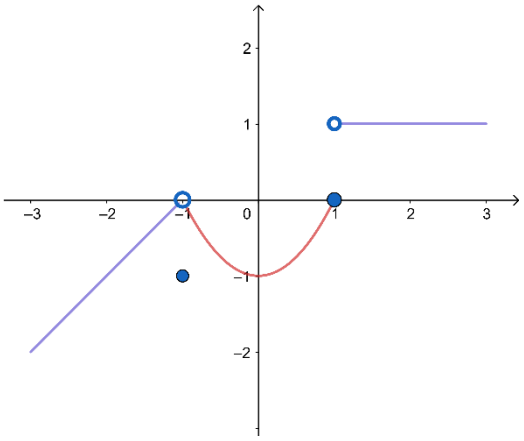
NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1(a)	$2\begin{pmatrix}2&-1\\3&4\end{pmatrix}-\begin{pmatrix}0&-2\\2&3\end{pmatrix}$ $=\begin{pmatrix}4&-2\\6&8\end{pmatrix}-\begin{pmatrix}0&-2\\2&3\end{pmatrix}$ $=\begin{pmatrix}4&0\\4&5\end{pmatrix}$	✓ $\begin{pmatrix}4&-2\\6&8\end{pmatrix}$ ✓ Finale antwoord

NR / *NO*	**ANTWOORD / *ANSWER***	**PUNTE / *MARKS***
		[2 punte / *marks*]
4.1(b)	$\begin{pmatrix}2 & -5\\1 & 2\end{pmatrix}\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}$ $=\begin{pmatrix}4-5 & -2-10\\2+2 & -1+4\end{pmatrix}$ $=\begin{pmatrix}-1 & -12\\4 & 3\end{pmatrix}$	✓ Elke term in antwoord **[4 punte / *marks*]**
4.1(c)	$(1 \quad -1)\begin{pmatrix}-4\\2\end{pmatrix}$ $=(-4-2)$ $=(-6)$	✓ $(-4-2)$ ✓ Antwoord **[2 punte / *marks*]**
4.2(a)	3×4	✓ Rye 3 ✓ Kolomme 4 **[2 punte / *marks*]**
4.2(b)	4×3	✓ Rye 4 ✓ Kolomme 3 **[2 punte / *marks*]**
4.2(c)	$A_{32} = -3$	✓ -3 **[1 punte / *marks*]**
4.2(d)	$(3 \times 4) \times A = (3 \times 2)$ A dimensies (4×2)	✓ Rye 4 ✓ Kolomme 2 **[2 punte / *marks*]**

Vraag / *Question* 5 [16 punte / *marks*]

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1 (a)	$\begin{pmatrix}1 & -2\\ 2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}8\\ 8\end{pmatrix}$	✓ $\begin{pmatrix}1 & -2\\ 2 & 4\end{pmatrix}$ ✓ $\begin{pmatrix}x\\ Y\end{pmatrix}$ ✓ $\begin{pmatrix}8\\ 8\end{pmatrix}$ **[3 punte / *marks*]**
5.1 (b)	$detA = det\begin{pmatrix}1 & -2\\ 2 & 4\end{pmatrix} = (1)(4)-(2)(-2)=8$ $detAx = det\begin{pmatrix}8 & -2\\ 8 & 4\end{pmatrix} = (8)(4)-(8)(-2)=48$ $detAy = det\begin{pmatrix}1 & 8\\ 2 & 8\end{pmatrix} = (1)(8)-(2)(8)=-8$ $x=\frac{detAx}{detA}=\frac{48}{8}=6$ $y=\frac{detAy}{detA}=-\frac{8}{8}=-1$	✓ $detA=8$ ✓ Ax ✓ detAx=48 ✓ Ay ✓ detAy ✓ Antwoord x ✓ Antwoord y **[7 punte / *marks*]**
5.2	$\begin{vmatrix}a & 3 & 2\\ 0 & -1 & a\\ -3 & -4 & a\end{vmatrix}=-12$ $a\begin{vmatrix}-1 & a\\ -4 & a\end{vmatrix}-3\begin{vmatrix}3 & 2\\ -1 & a\end{vmatrix}=-12$ $a(-a+4a)-3(3a+2)=-12$ $-a^2+4a^2-9a-6=-12$ $3a^2-9a+6=0$ $(a-2)(a-1)=0$ $a=2$ of $a=1$	✓ $a\begin{vmatrix}-1 & a\\ -4 & a\end{vmatrix}$ ✓ $-3\begin{vmatrix}3 & 2\\ -1 & a\end{vmatrix}$ ✓ $a\begin{vmatrix}-1 & a\\ -4 & a\end{vmatrix}=3a^2$ ✓ $-9a+6$ ✓ $a=0$ ✓ $a=1$ **[6 punte / *marks*]**

Vraag / *Question* 6 **[13 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1(a)	$(f \circ g)(x) = f\big(g(x)\big) = \left(\frac{1}{2\sqrt{x}} - 1\right)^2$	✓ $f\big(g(x)\big)$ ✓ $\left(\frac{1}{2\sqrt{x}} - 1\right)^2$ **[2 punte / *marks*]**
6.1(b)	$(\mathrm{g} \circ \mathrm{f})(x) = g\big(f(x)\big) = \frac{1}{2\sqrt{(x-1)^2}} = \frac{1}{2(x-1)}$	✓ $g\big(f(x)\big)$ ✓ $\frac{1}{2\sqrt{(x-1)^2}}$ **[2 punte / *marks*]**
6.2	$F(x) = \sqrt{x-1} + \frac{1}{\sqrt[3]{x-1}}$ $F(x) = f(g(x))$ $f(x) = \sqrt{x} + \frac{1}{\sqrt[3]{x}}$ $g(x) = x - 1$	✓ $f(x)$ ✓ $g(x)$ **[2 punte / *marks*]**
6.3		✓ Reguit lyn $(\infty; -1)$ ✓ OOP kol $(-1; 0)$ ✓ TOE kol $(-1; -1)$ ✓ Parabool $(-1; 1]$ ✓ TOE kol $(1; 0)$ ✓ OOP kol $(1; 1)$ ✓ Reguit line $(1; \infty)$ **[7 punte / *marks*]**

Vraag / *Question* 7 **[19 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1(a)	$\boldsymbol{u} = (2;4)$ $\boldsymbol{v} = (5;-1)$	✓ $(2;4)$ ✓ $(5;-1)$ **[2 punte / *marks*]**
7.1(b)	$\lvert\boldsymbol{u}\rvert = \sqrt{2^2+4^2} = 2\sqrt{5}$	✓$\sqrt{2^2+4^2}$ ✓ $2\sqrt{5}$ **[2 punte / *marks*]**
7.1(c)	$\boldsymbol{u}\cdot\boldsymbol{v} = (2)(5)+(4)(-1) = 6$ Indien $\boldsymbol{u}\cdot\boldsymbol{v} = 0$ dan **is** $\boldsymbol{u} \perp \boldsymbol{v} = 0$ Maar $\boldsymbol{u}\cdot\boldsymbol{v} \neq 0$, dus $\boldsymbol{u}$ is nie loodreg op $\boldsymbol{v}$ nie	✓ $(2)(5)+(4)(-1)$ ✓ 6 ✓ Rede/*reason* **[3 punte / *marks*]**
7.2(a)	$\boldsymbol{c} = \left(2;\frac{1}{2}\right)$ $\lvert\boldsymbol{c}\rvert = \sqrt{2^2+\left(\frac{1}{2}\right)^2} = \frac{\sqrt{17}}{2}$ Eenheidsvector van c $= \left(\frac{2}{\frac{\sqrt{17}}{2}};\frac{\frac{1}{2}}{\frac{\sqrt{17}}{2}}\right) = \left(\frac{4}{\sqrt{17}};\frac{2}{2\sqrt{17}}\right)$	✓ $\lvert\boldsymbol{c}\rvert = \frac{\sqrt{17}}{2}$ ✓ $\frac{2}{\frac{\sqrt{17}}{2}}$ ✓ $\frac{\frac{1}{2}}{\frac{\sqrt{17}}{2}}$ ✓ Vereenvoudig **[4 punte / *marks*]**
7.2(b)	$\boldsymbol{a}\cdot\boldsymbol{b} = (-3)(2)+(2)(5) = 4$ $\lvert\boldsymbol{a}\rvert = \sqrt{(-3)^2+2^2} = \sqrt{13}$ $\lvert\boldsymbol{b}\rvert = \sqrt{3^2+5^2} = \sqrt{34}$ $\boldsymbol{a}\bullet\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos\theta$ $\cos\theta = \frac{\boldsymbol{a}\bullet\boldsymbol{b}}{\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert} = \frac{4}{\sqrt{13}\sqrt{34}}$ $\theta = 1{,}37937 \approx 1{,}38$	✓ ✓$\boldsymbol{a}\cdot\boldsymbol{b}$ ✓ $\lvert\boldsymbol{a}\rvert$ ✓ $\lvert b\rvert$ ✓ Formule ✓ Antwoord afgerond 1,38 **[6 punte / *marks*]**
7.2(c)	$tan\theta = \frac{3}{5}$ $\theta = bgtan9$ $\theta = 0{,}54$ radiale	✓ $\tan\theta$ ✓ $\theta = 0{,}54$ radiale **[2 punte / *marks*]**

Vraag / *Question* 8 [10 punte / *marks*]

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1	$\theta = \frac{s}{r} = \frac{70{,}28}{60} = 1{,}17$ **Of** $OD = 60$ cm - 10 cm $= 50cm$ radius van sirkel $B\widehat{O}D = bgcos(\frac{50}{60}) = 0{,}5857$ $A\widehat{O}B = 2 \times B\widehat{O}D = 1{,}17$	✓ $\theta = \frac{s}{r}$ ✓ 1,17 **[2 punte / *marks*]**
8.2	$DB = \sqrt{OB^2 - OD^2} = \sqrt{60^2 - 50^2} = 10\sqrt{11}$ $OD = 60$ cm - 10 cm $= 50cm$ radius van sirkel Area ΔAOD $= \frac{1}{2}(DB)OD = \frac{1}{2}(10\sqrt{11})(50) = 250\sqrt{11}$ Area ΔABD$= 2\Delta\text{AOD} = 500\sqrt{11}$ cm^2	✓ DB ✓ $OD = 50\ cm$ ✓ ΔAOD ✓ Antwoord **[4 punte / *marks*]**
8.3	Area sektor OACB $= \frac{1}{2}r^2\theta = \frac{1}{2}60^2(1{,}17) = 2106\ cm^2$ Geskakeerde area: Area sektor OACB – area ΔABD$= 2106 - 500\sqrt{11} = 447{,}69$	✓ ✓Area sektor OACB ✓ Area sektor – driehoek ABC ✓ Antwoord **[4 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -