

# **$\alpha$ -WISKUNDE**

**Alpha Wiskunde Graad 10 / *Alpha Mathematics Grade 10***

**Finale eksamen 2020 / *Final examination 2020***

## **MEMORANDUM**

**Totaal / *Total*: 130 punte / *marks***

**Eksaminator / *Examiner*: Lanice Liebenberg**

**Moderator: Rika Grobler**

**Hierdie memorandum bestaan uit 13 bladsye. /**

***This memorandum consists of 13 pages.***

Vraag / Question 1

[20 punte / marks]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

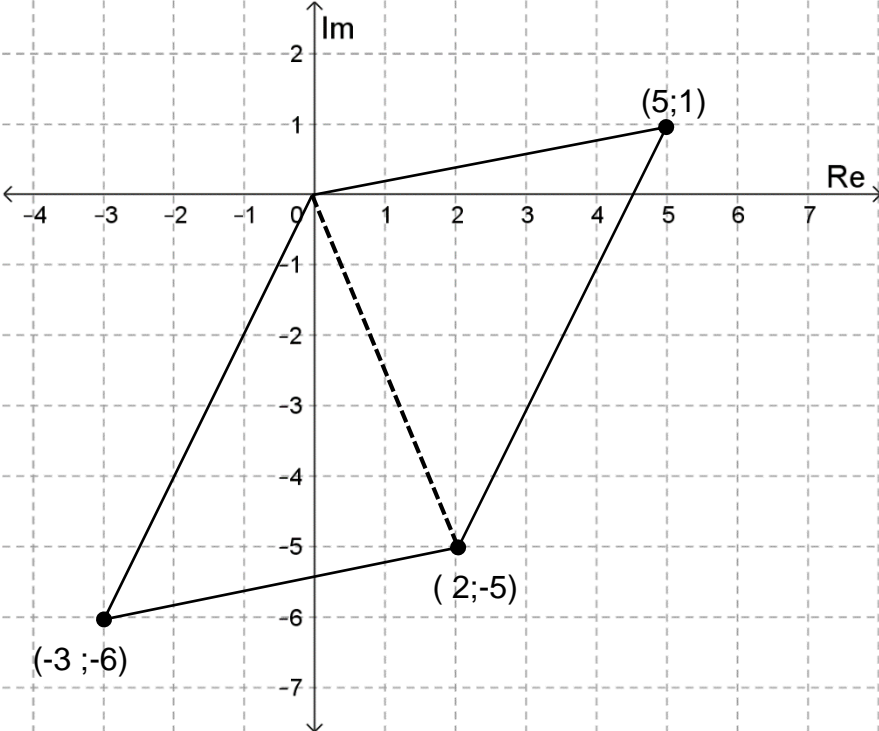
NR. NO	ANTWOORD ANSWER	BEREKENINGE (nie vir nasien doeleindes nie) CALCULATIONS (not for marking purpose)	PUNTE MARKS
1.1	C	$i^{20} = i^4 = 1$	(2)
1.2	D	$\arctan 1 = \frac{\pi}{4}$	(2)
1.3	C	$(x + 2)$ and $(x - 3)$ are factors	(2)
1.4	A	$F(x) = 2(x - 1)^2 + \frac{\sqrt{(x - 1)^2}}{3(x - 1)^2}$ $= 2(x - 1)^2 + \frac{(x - 1)}{3(x - 1)^2}$ $= 2(x - 1)^2 + \frac{1}{3(x - 1)}$ $= 2(x - 1)^2 + \frac{1}{3x - 3}$	(2)
1.5	B	$g(x) = x^{-\frac{1}{2}}$ $g'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$	(2)
1.6	B	$[-8]$	(2)

NR. NO	ANTWOORD ANSWER	BEREKENINGE (nie vir nasien doeleindes nie) CALCULATIONS (not for marking purpose)	PUNTE MARKS
1.7	A	$a$ and $b \perp$ when $a \cdot b = 0$	(2)
1.8	D	$k(2 + i) + 3i$ $= 2k + (k + 3)i$ $\rightarrow k + 3 = 0$ $\therefore k = -3$	(2)
1.9	B	$\int_0^1 x^2 dx$ $= \frac{x^3}{3} \Big _0^1$ $= \frac{1}{3} \text{units}^2$	(2)
1.10	D	$\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$ $= 1(-1) - 2(-2)$ $= 3$	(2)

Vraag / Question 2

[20 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
2.1	$z^* = 3 + 2i$	✓Answer(1)
2.2.1	$i^{25}(6 - 3i)$ $= i(6 - 3i)$ $= 6i - 3i^2$ $= 6i - 3(-1)$ $= 3 + 6i$	$✓i^{25} = i$  $✓i^2 = -1$ <b>(2)</b>
2.2.2	$(1 - i)^3 + 2i + 2$ $= (1 - 2i + i^2)(1 - i) + 2i + 2$ $= (-2i)(1 - i) + 2i + 2$ $= -2i + 2i^2 + 2i + 2$ $= -2 + 2$ $= 0$	$✓(1 - 2i + i^2)$ $✓-2i$ $✓-2$ $✓Answer$ <b>(4)</b>
2.3	$LHS = \frac{4 - 2i}{1 - i} \times \frac{1 + i}{1 + i}$ $= \frac{4 + 2i - 2i^2}{1 - i^2}$ $= \frac{4 + 2i + 2}{1 + 1}$ $= \frac{6 + 2i}{2}$ $= 3 + i$  $LHS = RHS$	$✓ \times \frac{1+i}{1+i}$  $✓Simplify \frac{6+2i}{2}$  $✓✓i^2(-i - 3)$  $✓LHS = RHS (5)$
2.4	$(a + 3i)(1 - 4i) = 14 - 5i$ $a - 4ai + 3i - 12i^2 = 14 - 5i$ $(a + 12) + (-4a + 3)i = 14 - 5i$  <p style="text-align: center;">Alternatively</p> $\therefore a + 12 = 14$ $\therefore a = 2$	$✓ a - 4ai + 3i - 12i^2$ $✓(a + 12) + (-4a + 3)i$  $✓Equation$ $✓Solution (4)$

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
2.5		✓(5 ; 1) ✓(-3 ; -6) ✓✓(2 ; -5) (4)

Vraag / Question 3

[10 punte / marks]

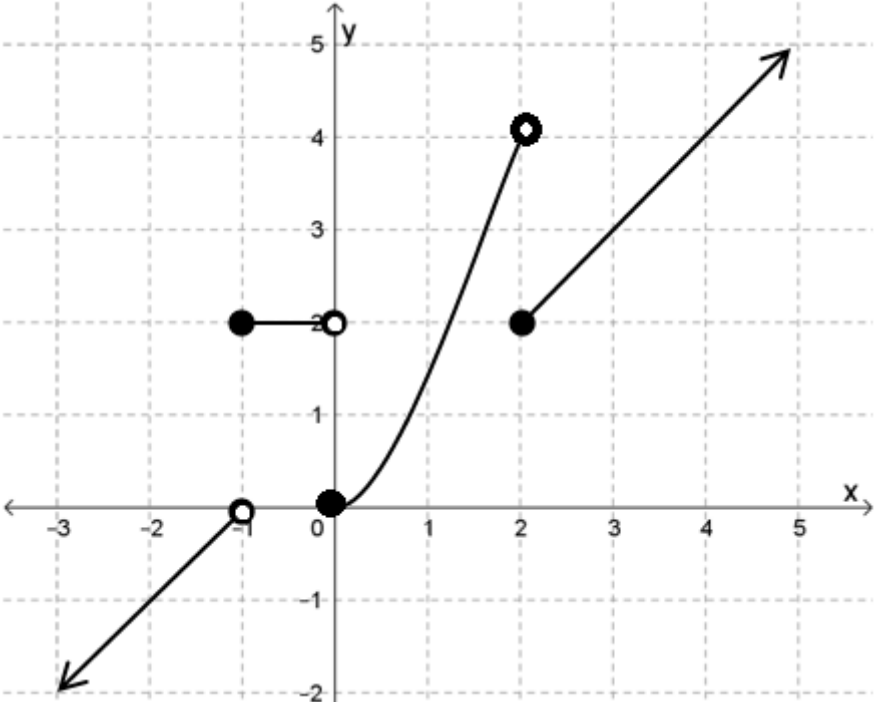
NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
3	$\frac{5x - 2}{(x - 1)(x^2 - 4x + 4)}$ $= \frac{5x - 2}{(x - 1)(x - 2)^2} \equiv \frac{A}{(x - 1)} + \frac{B}{(x - 2)} + \frac{C}{(x - 2)^2}$ $5x - 2 \equiv A(x - 2)^2 + B(x - 1)(x - 2) + C(x - 1)$ <p>Let <math>x = 1</math>    <math>A = 3</math> Let <math>x = 2</math>    <math>C = 8</math></p> $5x - 2 \equiv Ax^2 - 4Ax + 4A + Bx^2 - 3Bx + 2B + Cx - C$ $-2 = 4A + 2B - C$ $-2 = 4(3) + 2B - (8)$ $\therefore B = -3$ $\therefore \frac{5x - 2}{(x - 1)(x^2 - 4x + 4)} \equiv \frac{3}{x - 1} - \frac{3}{(x - 2)} + \frac{8}{(x - 2)^2}$	<p>✓Factorise</p> $\checkmark \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ <p>✓Multiply by LCM</p> <p>✓Value of A ✓Value of C</p> <p>✓✓Simplify</p> <p>✓Grouping ✓Value of B</p> <p>✓Solution <b>(10)</b></p>
OF / OR		
	$\frac{5x - 2}{(x - 1)(x^2 - 4x + 4)}$ $= \frac{5x - 2}{(x - 1)(x - 2)^2} \equiv \frac{A}{(x - 1)} + \frac{B}{(x - 2)} + \frac{C}{(x - 2)^2}$ $5x - 2 \equiv A(x - 2)^2 + B(x - 1)(x - 2) + C(x - 1)$ $5x - 2 \equiv Ax^2 - 4Ax + 4A + Bx^2 - 3Bx + 2B + Cx - C$ $0 = A + B$ $\therefore A = -B \dots \textcircled{1}$ $5 = -4A - 3B + C \dots \textcircled{2}$ <p>Sub <math>\textcircled{1}</math> into <math>\textcircled{2}</math></p> $5 = -4(-B) - 3B + C$ $B = 5 - C \dots \textcircled{3}$ $-2 = 4A + 2B - C \dots \textcircled{4}$	<p>OF / OR</p> <p>✓Factorise</p> $\checkmark \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ <p>✓Multiply by LCM</p> <p>✓✓Simplify</p> <p>✓Grouping</p>

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
	Sub ① and ③ into ④ $-2 = -4(5 - C) + 2(5 - C) - C$ $-2 = -20 + 4C + 10 - 2C - C$ $C = 8$ $B = 5 - 8$ $B = -3$ $A = -(-3)$ $A = 3$ $\therefore \frac{5x - 2}{(x - 1)(x^2 - 4x + 4)} \equiv \frac{3}{x - 1} - \frac{3}{x - 2} + \frac{8}{(x - 2)^2}$	✓Value of C  ✓Value of B  ✓Value of A  ✓Solution <b>(10)</b>

**Vraag / Question 4**

**[14 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
4.1.1	$f(x) = \frac{1}{\sqrt{x}} + x$	✓ $\frac{1}{\sqrt{x}}$ ✓ $x$ <b>(2)</b>
4.1.2	$F(2) = \frac{1}{\sqrt{(2)^2 - 3}} + ((2)^2 - 3)$ $= 2$	✓Substitution ✓Solution <b>(2)</b>
4.1.3	$g'(x) = 2x$ $g'(3) = 2(3)$ $= 6$	✓Derivative ✓Solution <b>(2)</b>

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
4.2		<p><b>For each function</b></p> <ul style="list-style-type: none"><li>✓Shape</li><li>✓Begin and end point</li></ul> <p style="text-align: right;"><b>(8)</b></p>

**Vraag / Question 5**

**[9 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
	$BQ = \sqrt{6^2 - 3^2}$ $= 3\sqrt{3}$ <p>Pythagoras</p> $\text{Area } \Delta ABC = \frac{1}{2} \cdot b \cdot h$ $= \frac{1}{2} \cdot 6 \cdot 3\sqrt{3}$ $= 9\sqrt{3}$ $\text{Area sector } APQ = \frac{1}{2} r^2 (\widehat{PAQ})$ $= \frac{1}{2} \cdot 3^2 \cdot \frac{\pi}{3}$ $= \frac{3\pi}{2}$ $\text{Shaded Area} = 9\sqrt{3} - \frac{3\pi}{2}$ $\approx 10,88 \text{ units}^2$	<p>✓BQ</p> <p>✓Substitution into correct formula</p> <p>✓Area of <math>\Delta</math></p> <p>✓<math>r = 3</math></p> <p>✓<math>\widehat{PAQ} = \frac{\pi}{3}</math></p> <p>✓Area of sector</p> <p>✓Method</p> <p>✓✓Solution</p> <p><b>(9)</b></p>

**Vraag / Question 6**

**[20 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
6.1.1	$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \end{pmatrix} \begin{pmatrix} 4 & -4 \\ 3 & 0 \\ 1 & -1 \end{pmatrix}$ $= \begin{pmatrix} 9 & -3 \\ 9 & -6 \end{pmatrix}$	<p>✓For each element correct in solution</p> <p><b>(4)</b></p>
6.1.2	$\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix} + 2 \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ $= \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix}$ $= \begin{bmatrix} 7 & -4 \\ 8 & 7 \end{bmatrix}$	<p>✓Multiplication of scalar</p> <p>✓For each row correct in solution</p> <p><b>(3)</b></p>
6.1.3	$A^T = \begin{pmatrix} 10 & 13 \\ 12 & 15 \\ 11 & 14 \end{pmatrix}$	<p>✓For each column correct</p> <p><b>(2)</b></p>



Vraag / Question 7

[15 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
7.1	$\mathbf{u} \cdot \mathbf{v} = 0$ $(2 \times a) + (-1 \times 8) = 0$ $2a = 8$ $a = 4$	✓ Dot product = 0 ✓✓ Correct substitution into formula ✓ Solution (4)
7.2	$x = 20 \cos \frac{\pi}{2}$ $= 0$ $y = 20 \sin \frac{\pi}{2}$ $= 20$	✓ $x = 0$ ✓ $y = 20$ (2)
7.3	$ \mathbf{u}  \mathbf{v} \cos\theta = u_x v_x + u_y v_y$ $(\sqrt{4^2 + 3^2})(\sqrt{5^2 + (-3)^2})\cos\theta = 4(5) + 3(-3)$ $\cos\theta = \frac{4(5) + 3(-3)}{(\sqrt{4^2 + 3^2})(\sqrt{5^2 + (-3)^2})}$ $\theta \approx 1,18 \text{ rad}$	✓ Equation the 2 formulas ✓✓ Substitution ✓✓ Simplification ✓ Solution (6)
7.4	$ \mathbf{u}  = \sqrt{3^2 + 4^2}$ $= 5$ $\text{unit vector of } \mathbf{u} = \left(\frac{3}{5}; \frac{4}{5}\right)$	✓ Magnitude ✓ x-value ✓ y-value (3)

**Vraag / Question 8**

**[6 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
$x^2 - x - 2$	$2x^2 + 3x - 2$	✓✓✓Determining the other quadratic factor          ✓Factorising   ✓✓Values of $x$ . <b>(6)</b>
	$2x^4 + x^3 - 9x^2 - 4x + 4$	
	$-2x^4 + 2x^3 + 4x^2$	
	$3x^3 - 5x^2 - 4x + 4$	
	$-3x^3 + 3x^2 + 6x$	
	$-2x^2 + 2x + 4$	
	$+2x^2 - 2x - 4$	
	$0$	
	$\therefore 2x^4 + x^3 - 9x^2 - 4x + 4 = 0$	
	$\therefore (2x^2 + 3x - 2)(x^2 - x - 2) = 0$	
$\therefore (2x - 1)(x + 2)(x + 1)(x - 2) = 0$		
$\therefore x = -2; -1; \frac{1}{2}; 2$		

**Vraag / Question 9**

**[16 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
9.1	$V = \pi \int_a^b [f(x)]^2 dx$ $39\pi = \pi \int_0^3 (x + q)^2 dx$ $39 = \int_0^3 (x^2 + 2qx + q^2) dx$ $39 = \left( \frac{x^3}{3} + qx^2 + q^2x \right) \Big _0^3$ $39 = 9 + 9q + 3q^2$ $0 = 3q^2 + 9q - 30$ $0 = q^2 + 3q - 10$ $0 = (q + 5)(q - 2)$ $q = -5; 2$	✓✓Substitution into correct formula          $\sqrt{\frac{x^3}{3}}$ $\checkmark qx^2$ $\checkmark q^2x$ ✓Substitution of $x$ -values ✓Simplification ✓Factorisation ✓✓Solution <b>(10)</b>

9.2.1	$f(x) = 2x^{\frac{1}{4}} - 3x^{-3} - 2$ $f'(x) = \frac{1}{2}x^{-\frac{3}{4}} + 9x^{-4}$	✓Correct form $\checkmark \frac{1}{2}x^{-\frac{3}{4}}$ $\checkmark 9x^{-4}$ <b>(3)</b>
9.2.2	$\int (5x + 1)^5 dx$ $= \frac{(5x + 1)^6}{6.5}$	$\checkmark (5x + 1)^6$ $\checkmark 6$ $\checkmark 5$ <b>(3)</b>

**- EINDE VAN DIE MEMORANDUM / END OF THE MEMORANDUM -**