α-MATHEMATICS

Alpha Mathematics

June examination 2020

Grade 12 **Time: 3 ure**

Examiner: Pieter of Onselen **Total: 190 marks**

INSTRUCTIONS EN INFORMATION

Carefully read through the following instructions before answering the question paper:

1. This question paper consists of 8 pages and a formula sheet of 3 pages and an answer sheet of 2 pages.
2. Answer ALL 9 questions.
3. Use the same numbering for the answers as it appears in the questions.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to **two decimal places**.
6. Except for question one, **clearly show all calculations**, diagrams, graphs et cetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians if necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neatly and legibly.

Question 1 [20 marks]

This question must be answered **on the answer sheet**. Every question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with an **X** on the answer sheet.

1.1 If a perpendicular line is drawn for $y = e^x$ at any point $(x; y)$, the gradient at that point would be ...

(A) -1 **(B)** y

(C) $-y$ **(D)** $-\frac{1}{y}$

1.2 For which values of x shall the function $f(x) = x^2 + 2x - 15$ be decreasing?

(A) $x \leq -5$ of $x \geq 3$ **(B)** $x \leq -1$

(C) $x \geq -1$ **(D)** $-5 \leq x \leq 3$

1.3 Given $f(x) = \log_3(e^x)$, then $f'(x) =$...

(A) $\frac{1}{\ln 3}$ **(B)** $\frac{1}{e^x \ln 3}$

(C) $\frac{1}{e^x}$ **(D)** $\frac{e^x}{\ln 3}$

1.4 Given $f(x) = 3e^{2x}$. Then the inverse of f in the form $f^{-1}(x) = \cdots$

(A) $\frac{1}{2} ln\left(\frac{x}{3}\right)$ **(B)** $\log_2\left(\frac{x}{3}\right)$

(C) $ln\left(\frac{x}{3}\right)$ **(D)** $\frac{1}{4} ln\left(\frac{x}{3}\right)$

1.5 For which values of a and b is the point $(1; 1)$ an inflection point of the function $f(x) = x^3 + ax^2 + bx + 1$?

(A) $a = 1$; $b = 1$ **(B)** $a = -3$; $b = 2$

(C) $a = -3$; $b = 5$ **(D)** $a = 2$; $b = -3$

1.6 Which of the following is equal to $ln5$?

(A) $ln4 + ln1$ **(B)** $\frac{ln10}{ln2}$

(C) $\int_1^5 lnx\, dx$ **(D)** $\int_1^5 \frac{1}{x}\, dx$

1.7 Find the horizontal asymptote, if there is one, ~~of~~ for $\frac{x^2+x}{3x^3-2x^2+1}$.

(A) $y = 1$ **(B)** $y = \frac{1}{3}$

(C) $y = 0$ **(D)** No horizontal asymptote.

1.8 If $ln\, f(x) = e^x\, ln\, x$, then $f'(1) = \cdots$

(A) $-\frac{1}{e}$ **(B)** $-e$

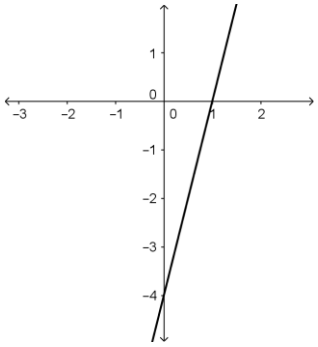
(C) $\frac{1}{e}$ **(D)** e

1.9 The equation of the tangent of $f(x) = e^{2x}$ at $x = 0$ is ...

(A) $y = 2x + 1$ **(B)** $y = 2x - 1$

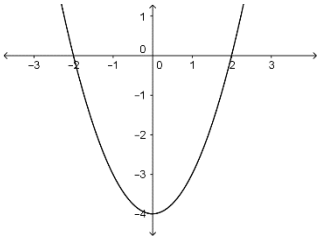
(C) $y = -2x + 1$ **(D)** $y = -2x - 1$

1.10 The sketch shows the graph for~~of~~ $y = g'(x)$, the derivative for $y = g(x)$.

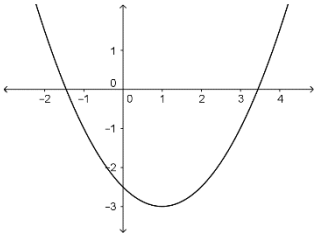


Which of the following can be the sketch for $y = g(x)$?

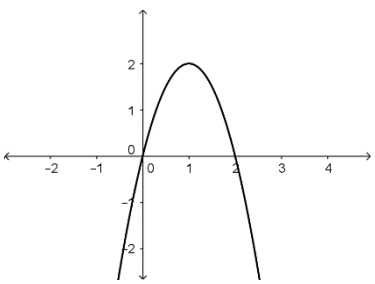
(A)



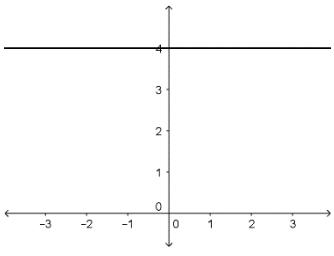
(B)



(C)



(D)



Question 2 [29 marks]

2.1 The dispersion of the COVID-19 virus in Italy can be determined with the Law of growth:

$$N(t) = N_0 e^{kt}$$

Where :

- N_0 is the number of people infected with the virus
- t is the time in days
- k is the constant of the growth

(a) The initial infections in Italy was 230 at the start of the epidemic. After 4 weeks the number of infections was 6500. Determine the growth constant k. (4)

(b) How many people will be infected after 50 days? Given that $k = 0{,}12$. (2)

(c) Determine the rate at which the virus will spread after 10 days. (3)

(d) Scientists calculated that the dispersion can be determined more accurately with:

$$N(t) = 230(2^{0{,}17t})$$

Using this formula what would the rate of dispersion be after 10 days? (3)

2.2 Solve for $x \in \mathbb{R}$ in:

(a) $e^{1-x} = \frac{5}{e^{2x}}$ (4)

(b) $\ln\left(\frac{1}{e^3}\right) = e^{\ln(1-2x)}$ (3)

2.3 Given the function $f(x) = \ln(2x+1) - \frac{e}{2}$

(a) Determine the inverse $f^{-1}(x)$ (4)

(b) Determine the x- and y-intercepts of $f(x)$ (4)

(c) Find the equation of the vertical asymptote. (2)

Question 3 [24 marks]

3.1 Given $(i-1)^3 = a\,\text{cis}\,3\theta$.

(a) Express $i-1$ in polar form. (4)

(b) Determine $(i-1)^3$ using **de Moivre's theorem**. Do all calculations in polar form. (3)

(c) Using **de Moivre's theorem** and the **binomial theorem** show that

$$\sin 3\theta = (\cos^2\theta\, sin\theta - sin^2\theta)$$ **(**6)

3.2 (a) Convert $2(\sqrt{3}-i)$ to **exponential** form.

Leave your answer in surd form and π. (4)

(b) Use the De Moivre theorem and determine the value of $\frac{\sqrt{2}(i-1)}{\sqrt{2(\sqrt{3}-i)}}$.

Do all calculations in exponential form and give the answer in rectangle form. (7)

Question 4 [24 marks]

4.1 Given the equation $\sin y = \frac{e^x}{x} - \ln(x\,y^2)$:

(a) Use implicit differentiation and show that $\frac{dy}{dx} = \frac{ye^x\,(x-1)-xy}{x^2\,(2+y\,cos(y))}$. (11)

(b) Determine the gradient of the line at point $(1;\frac{15}{e})$ (2)

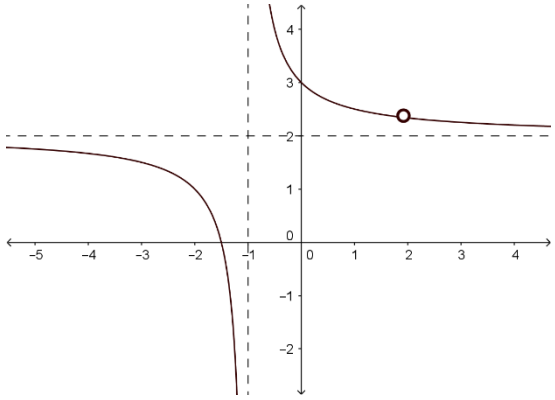
4.2 Given the equation: $x^2 + y^2 - 2x + 3y = 8$:

(a) Determine the derivative of the equation. (5)

(b) Determine the value of $\frac{d^2x}{dy^2}$ at the point $(-2;0)$. (6)

Question 5 [23 marks]

The graph shows the function $f(x) = \frac{(ax+3)(x+b)}{(x+c)(x+d)}$.



5.1 Determine the values of a, b, c *and* d given that $b = d$. (6)

5.2 Given that $d = -1$. Determine the assymptotes. (3)

5.3 Given that $a = 1$, $b = -1$, $c = 0$ and $d = 0$, determine the turning point of the function. (8)

5.4 Determine the nature of the turning point. (6)

Question 6 [22 marks]

Given the function $\frac{x^2 - 3x + 3}{x - 1}$ and that the function has no x-intercepts.

6.1 Determine the y-intercept. (2)

6.2 Determine the equations of the asymptotes and name them. (4)

6.3 Determine the coordinates of the turning point(s). (10)

6.4 Draw the rational function. Use **DIAGRAM SHEET 1** for the graph.
Show all intercepts with the axes, turning points and the equations of the asymptotes clearly. (6)

Question 7 [24 marks]

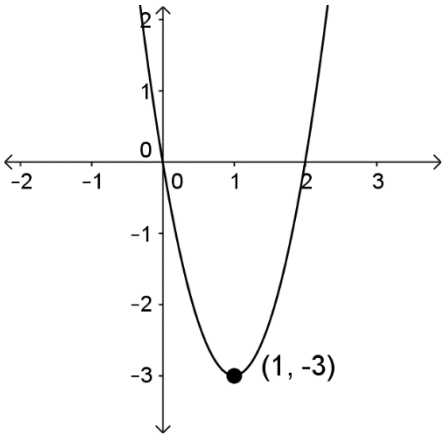
7.1 Given the gradient of the function $g(x)$ at the y-intercept is -36 and $g''(x) = 12x + 6$.

(a) Determine the stationary points of the graph and classify them. (9)

(b) Determine if $g(x)$ has an inflection point and if so, give the coordinates of the points of inflection and prove that it is indeed an inflection point. (5)

(c) Determine the interval where $g(x)$ is concave upward. (2)

7.2 Given the function $f'(x) = 3\,(-2\, +\, x)\, x$, the **derivative** of $f(x)$.



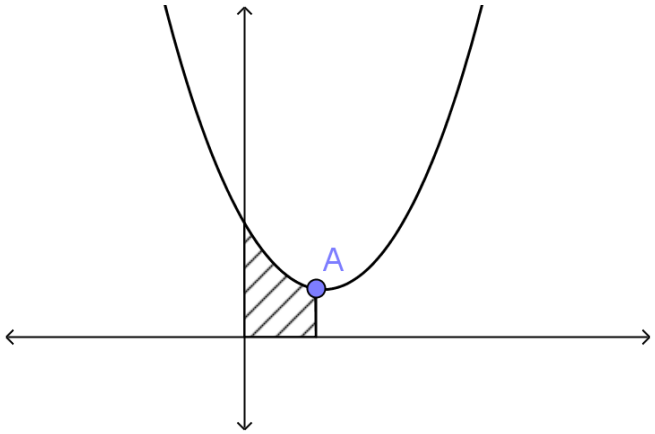
(a) Determine the stationary point(s) of $f(x)$ and clasify the point(s). (4)

(b) Determine the inflection point of $f(x)$. (2)

(c) Determine for which values of x the graph will be increasing. (2)

Question 8 [16 marks]

Given the function $f(x) = e^x + \frac{1}{e^{x-1}} - 3$.



8.1 Determine the x-coordinate of the inflection point of $f(x)$ at A. (5)

8.2 Determine the second derivative of the function $f(x)$. Use the second derivative to determine if the turning point(s) have a maximum or minimum value. (4)

8.3 Determine the area between the function f and the x-axis between $x = 0$ and the x-coordinate of A.

(7)

Question 9 [8 marks]

Determine the following integrals:

(a) $\int \frac{e}{2x-1}\ dx$ (2)

(b) $\int \frac{e}{(2x-1)ln3}\ dx$ (3)

(c) $\int 5^{2x-3}\ dx$ (3)

- END OF QUESTION PAPER-

ANSWER SHEET

Alpha Mathematics Grade 12

June examination 2020

Name and Surname:

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Question Total	1 [20]	2 [29]	3 [24]	4 [24]	5 [23]	6 [22]	7 [24]	8 [16]	9 [8]	TOTAL 190
Learner's mark										

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1