α-WISKUNDE

Alpha Wiskunde Graad 12 / *Alpha Mathematics Grade 12*

Rekord eksamen 2020 / *Prelim examination 2020*

MEMORANDUM

Totaal / *Total*: 200 punte / *marks*

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Moderator: Anna Muller

Hierdie memorandum bestaan uit 11 bladsye. /
This memorandum consists of 11 pages.

Vraag / *Question* 1 [20 punte / *marks*]

1.1	A	B	C	**D**
1.2	A	**B**	C	D
1.3	A	B	**C**	D
1.4	**A**	B	C	D
1.5	A	B	C	**D**
1.6	**A**	B	C	D
1.7	A	B	C	**D**
1.8	A	**B**	C	D
1.9	A	B	**C**	D
1.10	A	**B**	C	D

Vraag / *Question* 2 **[23 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1 (a)	$P(0) = 20 - (2 \times 4^0) = 18$ miljoen	✔ $t = 0$ ✔ antwoord/answer **[2 punte / *marks*]**
2.1 (b)	$10 = 20 - (2 \times 4^{0,1t})$ $-\frac{10}{2} = -4^{0,1t}$ $0,1t = \log_4 5$ $t = 11,61$	✔ 10 ✔ Vereenvoudig/simplify ✔ log ✔ antwoord/answer **[4 punte / *marks*]**
2.1 (c)	$P'(t) = -(2 \times 4^{0,1t}.ln4.0,1)$ $P'(10) = -\left(4^{0,1(10)}.ln4.0,2\right)$ $P'(t) = -1,11$ Verminder	✔ differensieer/differentiate ✔ x0,1 ✔ vereenvoudig/simplify ✔ antwoord/answer ✔ verminder/decreases **[5 punte / *marks*]**
2.2	$x \geq 5$ $\qquad$ $x < 5$ $x - 5 = 2x - 1$ $\qquad$ $5 - x = 2x - 1$ $x = -4$, nvt $\qquad$ $x = 2$	✔ voorwaardes/restrictions ✔ Lk som/ LHS sum ✔ antwoord/answer ✔ nvt/not appl ✔ Rk som/ RHF sum ✔ antwoord/answer **[6 punte / *marks*]**
2.3	$n = 9$ en $r = 5$ $\binom{9}{5}\left(-\frac{x}{3}\right)^4\left(\frac{2}{x^2}\right)^5 = \frac{448}{9x^6}$	✔ 9 ✔ 5 ✔ 1 ✔ $\left(-\frac{x}{3}\right)^4$ ✔ $\left(\frac{2}{x^2}\right)^5$ ✔ $\frac{1}{x^6}$ **[6 punte / *marks*]**

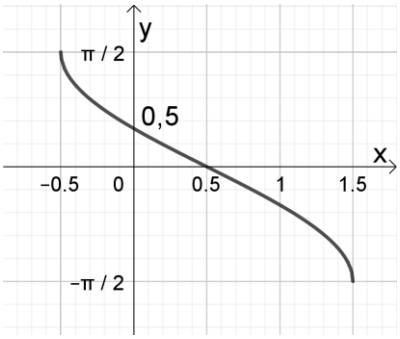
Vraag / *Question* 3 **[20 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1	$x-1=\pm i$ $x^2-2x+1=-1$ x^2-2x+2 is 'n faktor $\therefore (x^2-2x+2)(x^2-8)=0$ $x=1\pm i; x=\pm\sqrt{8}$	✔ $\pm i\theta$ ✔ kwadreer/square ✔ faktor/factor ✔ ✓✓ander faktor/ other factor ✔ ✓antwoorde/answers **[8 punte / *marks*]**
3.2 (a)	$-1+i$: $r=\sqrt{2}$ en $\theta=\frac{3\pi}{4}$ $\sqrt{2}^6\left(cis\frac{3\pi}{4}\right)^6=8cis\frac{9\pi}{2}$ $x=8\cos\frac{9\pi}{2}=0$ en $y=8\sin\frac{9\pi}{2}=8$ $\therefore 8i$ Dus altyd nie-reëel	✔ r en/and ✓ θ ✔ $\sqrt{2}^6$ ✔ $is\frac{9\pi}{2}$ ✔ x en y ✔ $8i$ **[6 punte / *marks*]**
3.2 (b)	$\sqrt{3}-i=2cis(-\frac{\pi}{6})$ $\frac{8cis\frac{9\pi}{2}}{2cis\,(-\frac{\pi}{6})}$ $= 4cis\ (\frac{14\pi}{3})$ $= -2+2\sqrt{3}i$	✔ 2 en/and ✓ $-\frac{\pi}{6}$ ✔ $\frac{8cis\frac{9\pi}{2}}{2cis\,(-\frac{\pi}{6})}$ ✔ $4cis\ (\frac{14\pi}{3})$ ✔ -2 en/and ✓$2\sqrt{3}$ **[6 punte / *marks*]**

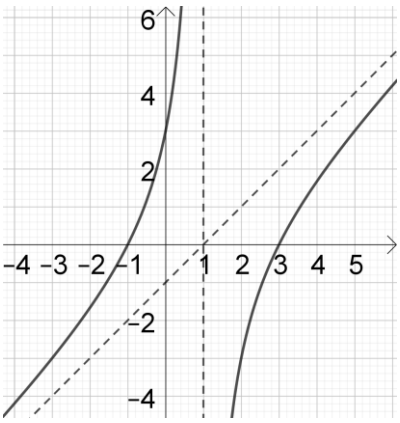
Vraag / *Question* 4 **[15 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1 (a)	$2^2 + a^2 + 2^2 = 9$ $a = \pm 1$	✔ ✓ $2^2 + a^2 + 2^2 = 9$ ✔ ✓ $a = \pm 1$ **[4 punte / *marks*]**
4.1 (b)	$\lvert v \rvert = \sqrt{16 + 4 + 1} = \sqrt{21}$ $u.v = 8 + 2 - 2 = 8$ $\cos\theta = \frac{8}{3 \times \sqrt{21}}$ $\theta = 0{,}95$	✔ $\sqrt{21}$ ✔ ✓ $u.v$ ✔ formule vervang substitude formula ✔ antwoord/answer **[5 punte / *marks*]**
4.2	$\begin{vmatrix} i & j & k \\ -2 & -1 & 4 \\ 0 & -5 & 2 \end{vmatrix}$ $= i(-2 + 20) - j(-4 - 0) + k(10 - 0)$ $= 18i + 4j + 10k$	✓ determinant ✓ ✓✓ bereken determinant calculate determinant ✓ formule reg/ formula correct ✓antwoord/answer **[6 punte / *marks*]**

Vraag / *Question* 5 **[19 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1 (a)	$\hat{O} = \frac{2\pi}{3}$ mdpt < = 2x omtr < $OAB = 5 + 5 + \frac{5.2\pi}{3}$ $= 10 + \frac{10\pi}{3}$ (of 20,47)	✔ hoek/angle ✔ bereken omtrek/ calculate circumference ✔ antwoord/answer **[3 punte / *marks*]**
5.1 (b)	$\frac{1}{2}.5^2.\frac{2\pi}{3}$ $= \frac{25\pi}{3}$ (of 26,18)	✔ formule/formula ✔ vervang reg/substitude correct ✔ antwoord/answer **[3 punte / *marks*]**
5.2		✔ vorm/form ✔ beginpunt/starting point ✔ y-afsnit/intercept ✔ x-afsnit/intercept ✔ Eindpunt/end point **[5 punte / *marks*]**
5.3	$\begin{vmatrix} 0 & 1 & 1 \\ p & -2 & -1 \\ 1 & 3 & 2 \end{vmatrix}$ $= 0(-4+3) - 1(2p+1) + 1(3p+2)$ $= -2p - 1 + 3p + 2$ $= p + 1$ Dus $\frac{p+1}{3} = 2$ $p = 5$	✔ skryf in determin vorm write in determ form ✓✓✓ haal uit determinant take out of determinant ✔ vereenvoudig/simplify ✔ $5p + 1$ ✔ $\frac{5p+1}{3} = 2$ ✔ antwoord/answer **[8 punte / *marks*]**

Vraag / *Question* 6 **[23 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1	$y = 3$; $(x-3)(x+1) = 0$, dus $x = 3$ of $x = -1$	✓y ✓✓✓ x **[4 punte / *marks*]**
6.2	Vertiaal: $x = 1$, Skuins: langdeling en $y = x - 1$	✓vertikaal/vertical ✓langdeling/long division ✓✓ skuins asimptoot oblique asymptote **[4 punte / *marks*]**
6.3	Asimptotiese diskontinuïteit, linker en regter limiet bestaan nie	✓tipe/type ✓rede/reason
6.4	$f'(x) = \frac{1(x^2-2x-3-(x-1)(2x-2))}{(x-1)^2} = 0$ Dus $x^2 - 2x - 3 - 2x^2 + 4x - 2 = 0$ $x^2 - 2x + 5 = 0$ Geen oplossing	✓✓ ✓afgeleide = 0/ derivative =0 ✓maal hakies uit/ multiply brackets ✓✓vereenvoudig/ simplify ✓geen opl/no solution **[7 punte / *marks*]**
6.5		✓✓asimptote/asymptotes ✓✓✓afsitte/intercepts ✓vorm/form **[6 punte / *marks*]**

Vraag / *Question 7* **[19 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	Stel/set $n = 1$: LK = 1; RK = 1; Dus waar vir $n = 1$/true for.. Aanvaar bewering waar vir $n = k$:/ Accept statement true for ... $1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$ Stel/set $n = k + 1$: RK = $\frac{(k+1)(k+2)(2k+3)}{6}$ LK= $\frac{k(k+1)(2k+1)}{6} + (k+1)^2$ = $\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$ = $\frac{(k+1)(k(2k+1)+6(k+1))}{6}$ = $\frac{(k+1)(2k^2+k+6k+6)}{6}$ =$\frac{(k+1)(k+2)(2k+3)}{6}$=RK Volgens beginsel van wiskundige induksie waar vir $n\epsilon\mathbb{N}$ According to the principle of mathematical induction, true for	✓n=1 ✓ aanvaar/accept ✓vergelyking ✓RK ✓✓LK ✓✓vereenvoudig na RK ✓storie **[9 punte / *marks*]**
7.2	**S 1** $\Delta x_i = \frac{2}{n}$ **S 2** $x_i = 1 + \frac{2}{n}.i$ **S 3** $f(x_i) = 3(1 + \frac{2}{n}.i)^2 = 3 + \frac{12i}{n} + \frac{12i^2}{n^2}$ **S 4** $\sum_{i=1}^{n}(3 + \frac{12i}{n} + \frac{12i^2}{n^2}) = \frac{12}{n^2}\sum_{i=1}^{n} i^2 + \frac{12}{n}\sum_{i=1}^{n} i + 3\sum_{i=1}^{n} 1$ $= \frac{12}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + \frac{12}{n}\left(\frac{n^2}{2} + \frac{n}{2}\right) + (3n)$ $= 13n + 12 + \frac{2}{n}$ **S 5** Oppervlakte = basis x hoogte/ Area = base x hight = **STAP 1 x STAP 4/ STEP1 x STEP 4** $= \frac{2}{n} \times \left(13n + 12 + \frac{2}{n}\right) = 26 + \frac{24}{n} + \frac{4}{n^2}$ **S 6** $\int_1^3 (2 + x^2)\,dx = \lim_{n\to\infty} 26 + \frac{24}{n} + \frac{4}{n^2}) = 26$	✔ $\frac{2}{n}$ ✔ $1 + \frac{2}{n}i$ ✔ $f(x_i)$ ✔ vereenvoudig $f(x_i)$ ✔ sigma ✔ ✓vervang ✔ ✓ vereenvoudig ✔ antwoord/answer **[10 punte / *marks*]**

Vraag / *Question* 8 **[22 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1 (a)	$\lim_{x\to 0^-} f(x) = 1$ en $\lim_{x\to 0^+} f(x) = 1$ $f(0) = 1$, dus kontinu	✓✓ limiete/limits ✓$f(0)$ ✓notasie/notation **[4 punte / *marks*]**
8.1 (b)	$\lim_{x\to 0^-} f'(x) = \frac{1}{1+0^2} = 1$ en $\lim_{x\to 0^+} f'(x) = 1$ Dus differensieerbaar	✓ ✓differensieer en 1/ differentiate and 1 ✓ 1 ✓ antwoord/answer **[4 punte / *marks*]**
8.2 (a)	$f'(x) = \frac{1}{x}.\log x + lnx.\frac{1}{x.ln10}$	✓ 1e term ✓ 2e term ✓ Produkreël/product rule **[3 punte / *marks*]**
8.2 (b)	$D_x[\sec(2x^{-1})] = \sec(2x^{-1}).\tan(2x^{-1}).(-2x^{-2})$	✓ $(2x^{-1})$ ✓ sec.tan ✓ $.(-2x^{-2})$ **[3 punte / *marks*]**
8.2 (c)	$y' = e^x + xe^x + y'e^y + ye^y.y'$ $y'(1 - e^y - ye^y) = e^x + xe^x$ $y' = \frac{e^x + xe^x}{1 - e^y - ye^y}$	✓ ✓✓✓✓elke term/each term ✓ ✓LK en RK ✓ antwoord/answer **[8 punte / *marks*]**

Vraag / *Question* 9 **[18 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
9.1	$\frac{2x^3}{3}+\frac{2^x}{ln2}-e^{-x}+k$	✓✓✓✓elke term **[4 punte / *marks*]**
9.2	$\frac{x^2-5x+6}{(x^2+1)(x-1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-1}$ $x^2-5x+6=(Ax+B)(x-1)+C(x^2+1)$ Stel $x=1: 2=2C; C=1$ $x^2: \ 1=A+C; A=0$ $x: \ -5=B$ $\int\left(-\frac{5}{x^2+1}+\frac{1}{x-1}\right)dx=-5\text{bgtan}x+ln(x-1)+k$	✓ uitbreiding/expansion ✓ ✓ X kgv/lcm ✓ C=1 ✓ A=0 ✓ B= -5 ✓ integraal reg/integral correct ✓ bgtan/arctan ✓ ln **[9 punte / *marks*]**
9.3	Stel $\sin 5x=u$ Dan $5\cos 5x\,dx=du$ $\frac{1}{5}\int u^5du$ $=\frac{u^6}{30}+k$ Dus $\frac{(\sin(5x))^6}{30}+k$	✓ $\sin 5x=u$ ✓differensieer/ differentiate ✓vervang/substitude ✓integreer/integrate ✓antwoord/answer slegs antwoord tel volpunte/ Only answer full marks **[5 punte / *marks*]**

Vraag / *Question* 10 [21 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
10.1	$f'(x) = \sqrt{2}x^{-\frac{1}{2}}.\frac{1}{2} - \cos x$ $x_0 = 2{,}5$ $x_{n+1} = x_n - \frac{\sqrt{2x_n} - \sin x_n - 2}{\sqrt{2}x_n{}^{-\frac{1}{2}}.\frac{1}{2} - \cos x_n}$ $x \approx 2{,}7769$	✔ afgeleide/derivative ✔ Newton ✔ antwoord/answer ✔ Korrek/correct 4 des **[4 punte / *marks*]**
10.2 (a)	$x = 1$ want $f' = 0$ Minimum	✔ 1 ✔ Rede/reason ✔ minimum **[3 punte / *marks*]**
10.2 (b)	$x = -2$, want $f'' = 0$ en f'' pos voor en neg na, en $f' = 0$	✔ $x = -2$ ✔ $f'' = 0$ ✔ f'' pos voor en neg na/ pos before and neg after ✔ $f' = 0$ [**4 punte / *marks***]
10.2 (c)	$x \in (-2; 0)$ want dan is $f'' < 0$	✔ ✓interval ✔ Rede/reason [**3 punte / *marks***]
10.3	$f(x) = lnx \qquad g'(x) = 1$ $f'(x) = \frac{1}{x} \qquad g(x) = x$ $\int lnx\ dx = xlnx - \int \frac{1}{x}.x\ dx$ $= xlnx - x + k$ Dus $\int_1^6 lnx\ dx = 6ln6 - 6 - 1ln1 + 1 = 6ln6 - 5$	✔ f ✔ g' ✔ f' ✔ g ✔ formule/formula ✔ integraal/integral ✔ antwoord/answer **[7 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -