Alpha Mathematics

Grade 12 Exam Paper

October 21, 2016

Total: 200 marks **Time: 3 hours**

INSTRUCTIONS

1. Answer all questions on this exam paper and hand it in. Remember to write your name and ID number on the front sheet.
2. Write neatly and legibly.
3. Non-programmable calculators may be used.
4. The diagrams on the question paper are not necessarily drawn to scale.
5. All angles are given in radians. Answers must be given in radians if necessary.
6. Unless otherwise indicated, all answers, where necessary must be given correct to two decimal figures.
7. This paper consists of 10 questions and 3 formula sheets.
8. Question 1 consists of 10 multiple choice questions. Answer it on the answer sheet. This answer sheet is in the front of the paper. Please do NOT remove it.
9. For all other questions, all necessary calculations must be shown. The correct answer alone will not necessarily lead to full marks.
10. There is extra space at the end of the paper. Clearly indicate if you use this space for a question.

QUESTION 1 [20 marks]

Answer this question on the answer sheet in front by making an X on either A, B, C or D. Please don't remove this page from the paper. These questions count 2 marks each.

1.1 If $f(x) = \begin{cases} lnx & if\ 0 < x \leq 1 \\ x^2 & if\ x \geq 1 \end{cases}$, then

(A) f is continuous in the point $x = 1$.

(B) f is differentiable in the point $x = 1$.

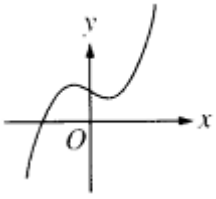
(C) there exists a removable discontinuity at $x = 1$.

(D) there exists a jump discontinuity at $x = 1$.

1.2 The binomial expansion of $(a + b)^{20}$ will have ..... terms.

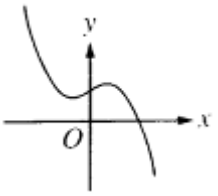
(A) an infinite number of (B) 19 (C) 20 (D) 21

1.3 The graph of $y = h(x)$ is shown:

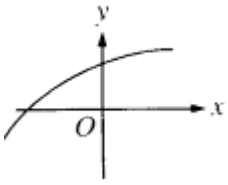


Which of the following can be the graph of $y = h'(x)$?

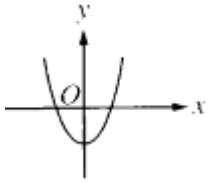
(A)



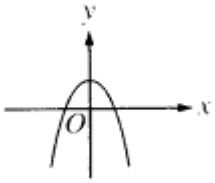
(B)



(C)



(D)



1.4 Given $f(x) = tanx$. If $0 \leq k \leq \frac{\pi}{2}$ and $f'(k) = 2$, then $k =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

1.5 If $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$, which statement must be true:

(A) $f(2)$ is undefined (B) $\lim_{x \to 2} f(x) = 5$ (C) $f(2) = 5$ (D) $f'(2) = 5$

1.6 The number of real solutions that $|x+1| = -x^2 - 1$ will have, is

(A) 0 (B) 1 (C) 2 (D) 3

1.7 Given: $y = f(x)$ is differentiable for all $x \in \mathbb{R}$. It is further given that $f'(2) = 0$ and $f''(2) > 0$. then at $x = 2$ f will have a

(A) point of inflection. (B) local maximum.

(C) local minimum. (D) stationary point of inflection.

1.8 The area included by the graph $y = cosx$ and the x-axis between the points $x = 0$ and $x = \pi$ is equal to

(A) 3,14 (B) 2 (C) 1 (D) 0

1.9 Use the laws of logarithms and expand $\boldsymbol{ln}\left(\frac{\mathbf{1}}{\boldsymbol{y^2}}\right)$.

(A) $2lny$ (B) $-2lny$

(C) $ln1 - (lny)^2$ (D) $-(lny)^2$

1.10 $(f \circ g)(x) = x^2 + x$. If $g(2) = 4$ and $g'(2) = 5$, determine $f'(4)$.

(A) 1 (B) 2 (C) 5 (D) 6

Answer the following questions on the question paper on the lines provided below question.

QUESTION 2 [19 marks]

2.1 Newton's law of cooling of a liquid, like soup, is given with the equation

$$T(t) = 20 + 60e^{-0,054t}$$

where T gives the temperature, measured in ℃, of the soup after **t** minutes.

(a) Determine the initial temperature of the soup. (2)

(b) Determine the temperature of the soup after 10 minutes. Give the answer as an integer. (2)

(c) Calculate after how many minutes the soup will get to a temperature of 40℃. (3)

(d) The asymptote of the graph of $y = T(t)$ is $T = 20$. Wat is the practical meaning of this answer? (1)

2.2 Use mathematical induction and prove that , for all $n \in \mathbb{N}$, will

$$\frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \cdots + \frac{1}{(n+1)(n+2)} = \frac{n}{2(n+2)}$$ (11)

QUESTION 3 [21 marks]

3.1 Use the binomial theorem and determine the coefficient of x^2 in $(1+x^2)\left(\frac{x}{2}-\frac{4}{x}\right)^6$ (8)

3.2 The following expansion is done by using the binomial theorem:
$(1-i)^4 = 1 + 4(-i) + 6(-i)^2 + 4(-i)^3 + (-i)^4$

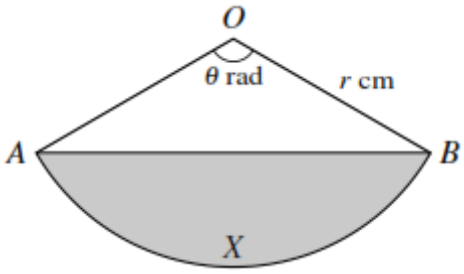
a) Simplify this expansion to the form $x + yi$. (2)

b) Now use **de Moiré's theorem** and simplify $(1-i)^4$. Work in polar form and use root form and π if necessary. the answer must be given in rectangular form. How does this answer compare to the answer of Question 3.2 (a)? (6)

3.3 Give the first three terms in the power extension of $\frac{1}{(1-x)^2}$ and then use these three terms to calculate an approximate value of $\frac{1}{0,8^2}$ (5)

QUESTION 4 [20 marks]

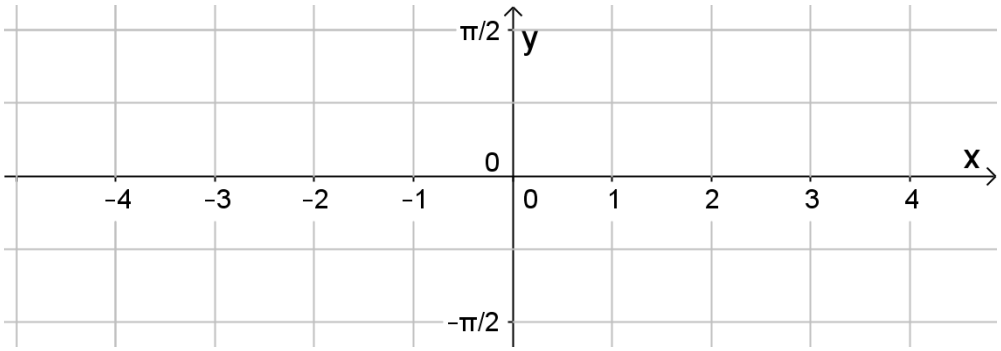
4.1 This ketch shows a sector of a circle with centre O and radius r. Angle AOB = θ radians.
Segment AXB is shaded.



a) If the areas of the segment AXB and of the triangle OAB are equal, determine the value of p if $\boldsymbol{\theta = psin\theta}$ (5)

b) If $p = 2$, use **Newton's method** to calculate the value of θ, correct to **four** decimal digits. Use $\theta_0 = 2$ as starting value. Clearly show the formula that you used with Newton's method. (5)

4.2 a) Use the given system of axes below and make a sketch graph of $f(x) = arctanx$ and $g(x) = -arctan(x - 2)$. Clearly show the asymptotes and intercepts with the axes. (5)



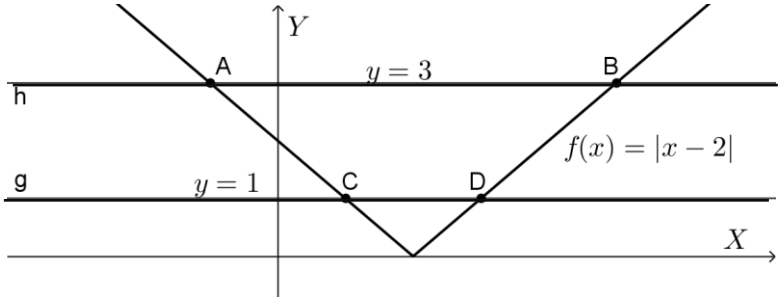
b) Use your graph and give the values of $\boldsymbol{x}$ for which $f(x) \geq g(x)$. (2)

c) Give the corresponding values of y for which the inequality in Question 4.2 (b) applies. (3)

QUESTION 5 [24 marks]

5.1 Solve for x with $x \in \mathbb{C}$ and $\boldsymbol{x^4 + 5x^3 + 27x^2 + 5x - 174 = 0}$ if one of the solutions is $\boldsymbol{x = -2 - 5i}$ (8)

5.2 The ketch below shows the graph of $f(x) = |x - 2|$ as well as the lines $y = 1$ and $y = 3$. These lines intersect the graph in the points A, B, C and D.



(a) Determine the x-values of A, B, C and D only. (6)

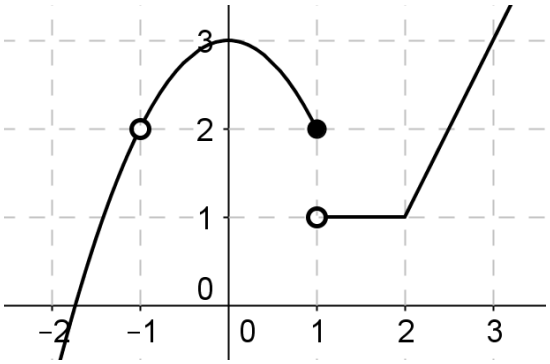
(b) Hence use the graph and your answered in Question 5.2 (a) and write down the solution of $1 \leq |x - 2| \leq 3$ (4)

5.3 The matrix formed by a system of equations is $M = \begin{pmatrix} \boldsymbol{a} & -2 & -\boldsymbol{a} \\ 1 & a & 2 \\ -1 & 1 & 1 \end{pmatrix}$.

Determine the value(s) of $\boldsymbol{a}$ for which this system has no solution. (6)

QUESTION 6 [16 marks]

6.1 The sketch shows the function $y = f(x)$. Use the sketch and answer the questions that follow.



(a) Give the x-values for which f is not continuous. Motivate your answer algebraically why the function is not continuous in these points and state the type of discontinuity. (6)

(b) Give a reason why f is not differentiable in the points given in Question 7.1 (a). (1)

(c) Name one other point where f is not differentiable and motivate your answer. (2)

(d) The function has a local maximum turning point at $x = 0$. Give possible values of $f'(0)$ and $f''(0)$. (2)

6.2 Determine $f'(x)$ from first principles if $f(x) = \frac{1}{1-x}$ (5)

QUESTION 7 [23 marks]

7.1 Differentiate the following functions. It is not necessary to simplify your answers.

(a) $f(x) = arcsin(\sqrt{x})$ (3)

(b) $f(x) = tan4x \times (e^x + x^2)^{10}$ (5)

7.2 Use implicit differentiation and determine $\frac{dy}{dx}$ if $\boldsymbol{ln}\left(\frac{\mathbf{1}}{\boldsymbol{y^2}}\right) = \frac{\boldsymbol{x}}{\boldsymbol{y}}$ for $y \neq 0$. (7)

7.3 A graph's equation is $y = e^{x-2} - 2x + 6$. The x- value of the stationary point is of the form $x = ln(p) + p$.

(a) Determine the value of p. (4)

(b) Determine the nature of the stationary point. (4)

QUESTION 8 [20 marks]

8.1 Determine the following integrals:

(a) $\int cosec^2(4x)\,dx$ (3)

(b) $\int_1^e x.lnx\,dx$ by using factor (piecewise) integration and **give the answer in terms of e**. Show clearly how you have used factor integration. (6)

(c) $\int \frac{2x^2+x+1}{(x^2+1)(x-1)}\,dx$ by using partial fractions. (7)

8.2 If $y = f(x)$, $\frac{dy}{dx} = 12e^{2x+4}$ and $f(-2) = 10$, determine the equation of f. (4)

QUESTION 9 [27 marks]

9.1 Determine the following integral with an infinite Riemann sum: $\int_2^3 (3x^2 - 6)dx$ (12)

9.2 $f(x) = x.\sqrt{25 - x^2}$

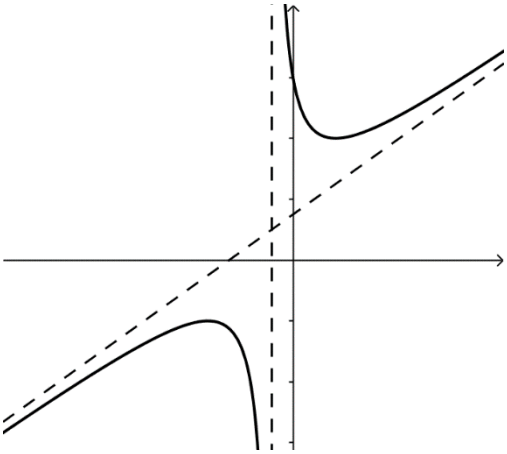
(a) The area bounded by the graph $y = f(x)$, the lines $x = a$ with $0 < a < 5$, $x = 5$ and the x-axis is equal to $\frac{64}{3}$. Determine the value of a. Show all calculations. No marks will be given for the answer only. (8)

(b) Calculate the volume of the solid of revolution that is formed when $y = f(x)$ rotates around the x-axis with $-1 \leq x \leq 0$. Show all your calculations. No marks will be given for the answer only. (7)

QUESTION 10 [10 marks]

The sketch shows the graph of $f(x) = \frac{2x^2+4x+6}{2x+1}$ and the asymptotes of f

10.1 Determine the equations of the asymptotes of f (4)



10.2 Determine the range of f (6)

TOTAL: 200

Extra Space