α-WISKUNDE

Alpha Wiskunde Graad 12 / *Alpha Mathematics Grade 12*

Rekord Eksamen 2017 / *Prelim Examination 2017*

MEMORANDUM

Totaal / *Total*: 200 punte / *marks*

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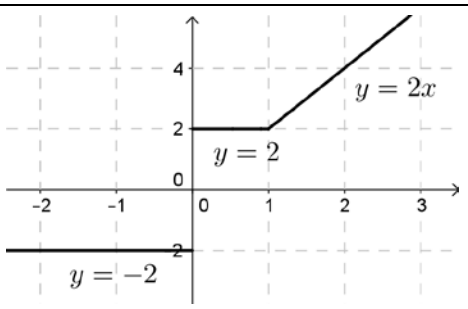
Moderators: Anna Muller en / *and* Rika Grobler

Hierdie memorandum bestaan uit 14 bladsye. /
This memorandum consists of 14 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

NR.	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	**C**	$-\|5-x\|<1 \implies \|5-x\|>-1 \implies x\in\mathbb{R}$	**2**
1.2	**D**	Definisie van kontinuïteit in die punt / *Definition of continuity in the point* $x=5$: $\lim\limits_{x\to 5} f(x)=f(5)$	**2**
1.3	**B**	$f'(2)=\lim\limits_{h\to 0}\dfrac{(2+h)^3-8}{h}=\lim\limits_{h\to 0}\dfrac{8+12h+6h^2+h^3-8}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(12+6h+h^2)}{h}=12$ **OF / *OR*** Sien dat / *See that* $f(x)=x^3$, dus / *thus* $f'(2)=3(2)^2=12$	**2**
1.4	**A**	$(4-2x)^{\frac{1}{2}}=4^{\frac{1}{2}}\left(1-\dfrac{2x}{4}\right)^{\frac{1}{2}}$ Dus / *Thus* $\left\|-\frac{2x}{4}\right\|<1 \implies \|x\|<2$	**2**
1.5	**B**	$3i^{41}(1+3i)+\sqrt{-25}$ $=3i(1+3i)+5i$ $=3i+9i^2+5i=-9+8i$ ... reële deel / *real part* is -9	**2**
1.6	**D**	$\int\frac{1}{x}\,dx=\log_e x+k.$ Onthou dat / *Remember that* $\log_e x=\ln x$	**2**
1.7	**A**	Gradiënt van kromme is gelyk aan die gradiënt van $y=3x$. / *Gradient of the curve is equal to the gradient of* $y=3x$. $\dfrac{dy}{dx}=4x-13=3 \implies x=4$	**2**
1.8	**C**	Lokale maksimum is waar / *Local maximum is where* $f'(x)=0$ en $f''(x)<0$. $f''(x)<0$ is waar $f'(x)$ verander van **positief** na **negatief**. / $f''(x)<0$ *is where* $f'(x)$ *changes from* ***positive*** *to* ***negative***.	**2**
1.9	**D**	$\int_2^6 f''(x)\,dx=f'(6)-f'(2)=11-5=6$	**2**
1.10	**B**	Sprong diskontinuïteit by $x=0$ in die gradiënt funksie. / *Jump discontinuity at* $x=0$ *in the gradient function.* 	**2**

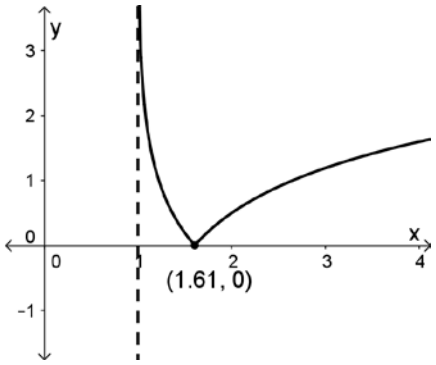
Vraag / *Question* 2 **[20 punte / *marks*]**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
2.1	$\|A\| = \begin{vmatrix} a & 4 & -1 \\ -1 & 1 & 2 \\ 2a & 6 & -3 \end{vmatrix} = 0$ ✓ $a\begin{vmatrix} 1 & 2 \\ 6 & -3 \end{vmatrix} - 4\begin{vmatrix} -1 & 2 \\ 2a & -3 \end{vmatrix} - 1\begin{vmatrix} -1 & 1 \\ 2a & 6 \end{vmatrix}$ ✓ $= 0$ (of enige ander ry/kolom *or any other row/column*) $a(-3-12) - 4(3-4a) - 1(-6-2a)$ ✓ $= 0$ $-15a - 12 + 16a + 6 + 2a = 0$ $\therefore a = 2$ ✓	**4**
2.2	$x = 2 - i$ is 'n nulpunt dus / *a zero thus* $x = 2 + i$ ✓ is ook 'n nulpunt / *also a zero.* $(x-2+i)(x-2-i)$ ✓ **OF** $x - 2 = \pm i$ **OF** enige ander metode $= (x-2)^2 - i^2$ $= x^2 - 4x + 5$ is 'n faktor / *a factor* ✓ Doen langdeling dan is / *Do long division, then*: $(x^2 - 4x + 5)(x^2 - 2x + 3)$ ✓✓✓ Indien verder gefaktoriseer / *If further factorized* – **max 5/6** Indien x opgelos / *If x is solved* – **max 4/6**	**6**
2.3	$\binom{18}{r}\left(\frac{x}{5}\right)^{18-r}$ ✓ $\left(\frac{5}{x^2}\right)^{r}$ ✓ (substitusie in formule / *substitute in formula*) $= \binom{18}{r} x^{18-r-2r} 5^{r-18+r}$ ✓ (vereenvoudig / *simplify*) Dus / *Thus* $18 - 3r = 0 \implies r = 6$ ✓ Die konstante / *The constant* term is $\binom{18}{6} 5^{2(6)-18} = 1.188$ ✓S (korrek tot 3 desimale / *correct to 3 decimals)* Indien r verkeerd is / *If r is incorrect* – **max 4/5** Indien aangeneem / *If assume $r = 6$* –**max 3/5**	**5**
2.4	$(1+2x)\sqrt{1+2x} = (1+2x)^{\frac{3}{2}}$ ✓ $= 1$ ✓ $+ (2x)\left(\frac{3}{2}\right)$ ✓ $+ \frac{\frac{3}{2}\times\frac{1}{2}}{2!}(2x)^2$ ✓ (substitusie in formule / *substitute in formula*) $= 1 + 3x + \frac{3}{2}x^2$ ✓ (vereenvoudig / *simplify*) **OF / OR** $(1+2x)^{\frac{1}{2}} = 1$ ✓ $+ (2x)\left(\frac{1}{2}\right)$ ✓ $+ \frac{\frac{1}{2}\times-\frac{1}{2}}{2!}(2x)^2$ ✓ (substitusie in formule / *substitute in formula*) $= 1 + x - \frac{1}{2}x^2$ Dus / *Thus* $(1+2x)\sqrt{1+2x} = (1+2x)\left(1 + x - \frac{1}{2}x^2 + \cdots\right)$ ✓ $= 1 + 3x + \frac{3}{2}x^2$ ✓ (vereenvoudig / *simplify*)	**5**

Vraag / *Question* 3 [16 punte / *marks*]

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
3.1	een-een-duidige **OF** een-tot-een / *one-to-one* ✓	**1**
3.2(a)	$y = -\frac{\pi}{2} + \frac{\pi}{8} = -\frac{3\pi}{8}$ ✓ $y = \frac{\pi}{2} + \frac{\pi}{8} = \frac{5\pi}{8}$ ✓	**2**
3.2(b)	$a = -1$ ✓	**1**
3.2(c)	$0 = -\arctan(4.414 + p) + \frac{\pi}{8}$ ✓ $\frac{\pi}{8} = \arctan(4.414 + p)$ $p = \tan\frac{\pi}{8} - 4.414 = -4$ ✓✓ arctan x is bgtan x in Afrikaans **OF / OR** Indien / *If* $a = +1$ gekies is by vraag / *was chosen at question* 3.2(b): $0 = \arctan(4.414 + p) + \frac{\pi}{8}$ ✓ $p = \tan\left(-\frac{\pi}{8}\right) - 4.414 = -5$ ✓✓	**3**
3.2(d)	$x = 4$ ✓ *(die p-waarde van vraag 3.2(c) / the p-value of question 3.2(c))* **OF / OR** $x = 5$ ✓ *(Indien/If $a = +1$ gekies is by vraag/was chosen at question 3.2(b))*	**1**
3.3(a)	Sprong diskontinuïteit / *Jump discontinuity* ✓	**1**
3.3(b)	$x = 2$ ✓, $x = 4$ ✓	**2**
3.3(c)	$x = 0$ ✓	**1**
3.3(d)	$x = -3.4$ ✓, $x = -0.6$ ✓ **Let Wel:** $x = 2$ is nie 'n antwoord nie, alhoewel $f(2) = 0$, bestaan $f'(2)$ nie omdat die funksie NIE differensieerbaar is in die punt nie. / ***Please note***: $x = 2$ *is not an answer, although* $f(2) = 0$, $f'(2)$ *does not exist, because the function is NOT differentiable in the point.* Indien $x = 2$ een van die leerder se antwoorde is / *If* $x = 2$ *is one of the answers of the student:* **minus 1 punt / *marks.***	**2**
3.3(e)	$x = 1$ ✓, $x = 5$ ✓	**2**

Vraag / *Question* 4 **[26 punte / *marks*]**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
4.1(a)	(i) x-afsnit / *intercept*: $ln(x-1)+\frac{1}{2}=0$ ✓ $e^{-\frac{1}{2}}=x-1$ ✓ $\Rightarrow x=1.61$ Dus / *Thus* $(1.61\ ;\ 0)$ ✓ (ii) Asimptoot / *Asymptote* $x=1$ ✓	**4**
4.1(b)	**DIAGRAMBLAD / *DIAGRAM SHEET* 1**  Reflekteer alles onder die x-as op / *Reflect everything under the x-axis up* ✓ Asimptoot / *Asymptote* $(x=1)$ by dieselfde / *remain the same* ✓ x-afsnit by dieselfde / *x-intercept remain the same* ✓	**3**
4.1(c)	$y\in\mathbb{R}\ ; y>1$ ✓	**1**
4.2	$\frac{\lvert x-2\rvert}{x+1}=-2x$ As / If $x\geq 2$: $x-2=-2x^2-2x$ ✓ $2x^2+3x-2=0$ $(2x-1)(x+2)$ ✓ $=0$ $x=\frac{1}{2}$ ✓ of $x=-2$ ✓ (N.V.T./ *N.A.*) As / If $x<2$: $-(x-2)=-2x^2-2x$ ✓ $2x^2+x+2=0$ ∴ geen oplossing / *no solution* ✓ Dus geen oplossing vir x. / *Thus no solution for x* ✓ Indien geen voorwaardes / *If no conditions* – **max 5/7**	**7**

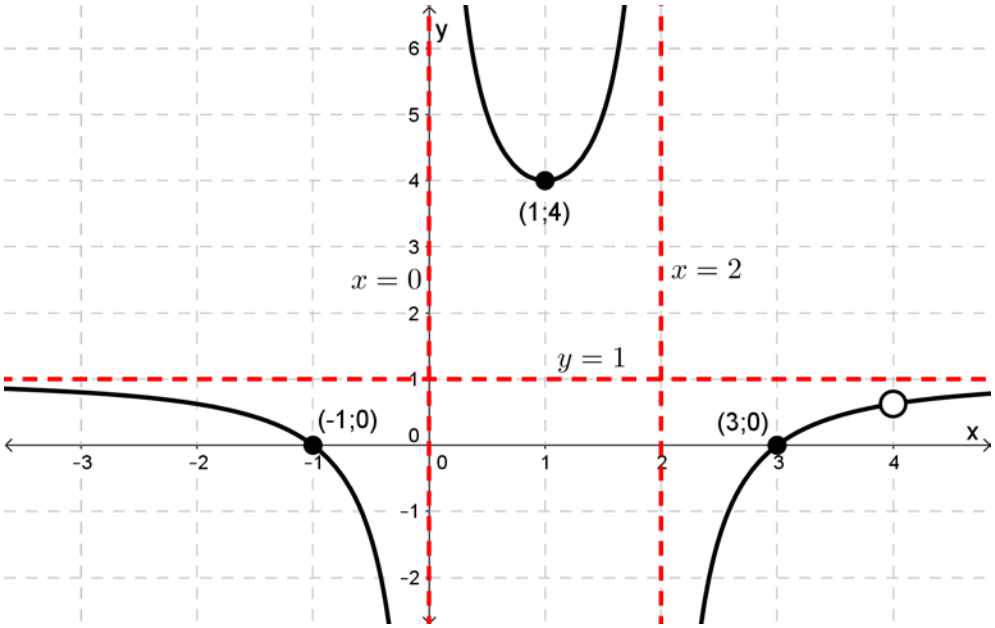
NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
4.3	$1.\ln x + 2.\ln x + 3.\ln x + \cdots + n.\ln x = \frac{n}{2}.\ln x^{n+1}$ **Stel / Let $n = 1$:** LK $= \ln x$ RK $= \frac{1}{2}\ln x^2 = \frac{1}{2} \times 2\ln x = \ln x$ ✓ $\therefore$ LK = RK dus bewering is waar vir / *thus statement is true for* $n = 1$. ✓ **Aanvaar die bewering is waar as $n = k \in \mathbb{Z}$ /** ***Accept the statement is true for*** $n = k \in \mathbb{Z}$ ✓ $1.\ln x + 2.\ln x + 3.\ln x + \cdots + k.\ln x = \frac{k}{2}.\ln x^{k+1}$ ✓ **Beskou / *Consider* $n = k + 1$:** LK $= 1.\ln x + 2.\ln x + 3.\ln x + \cdots + k.\ln x$ ✓ $+ (k+1)\ln x$ ✓ $= \frac{k}{2}\ln x^{k+1}$ ✓ $+ \ln x^{k+1}$ $= \ln x^{k+1}\left(\frac{k}{2} + 1\right)$ ✓ $= \ln x^{\left(\frac{k}{2}+1\right)(k+1)}$ $= \ln x^{\frac{1}{2}(k+2)(k+1)}$ ✓ $= \frac{k+1}{2}\ln x^{k+2}$ RK $= \frac{k+1}{2}\ln x^{k+2}$ ✓ $\therefore$ LK = RK Indien dadelik begin by tweede stap / *If immediately begins at second step* – **gee/*give* 2/2** **OF / OR** LK $= \frac{k(k+1)}{2}\ln x + (k+1)\ln x$ ✓ $= \frac{k+1}{2}(k+2).\ln x$ ✓ RK $= \frac{k+1}{2}\ln x^{k+2}$ $= \frac{k+1}{2}(k+2).\ln x$ ✓ Die bewering is waar as $n = 1$. As die bewering waar is vir $n = k$, is dit ook waar vir $n = k + 1$. Dus is die bewering waar vir alle $n \in \mathbb{N}$. / *The statement is true for* $n = 1$. *If the statement is true for* $n = k$, *it is also true for* $n = k + 1$. *Thus, the statement is true for all* $n \in \mathbb{N}$. ✓	**11**

Vraag / *Question* 5 [20 punte / *marks*]

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
5.1(a)	$m_1 = 4 \checkmark \operatorname{cis}\frac{\pi}{3} \checkmark$ $m_2 = \sqrt{2} \checkmark \operatorname{cis}\frac{3\pi}{4} \checkmark$ **Let op:** $\theta > 0$ en r in wortelvorm. Indien negatiewe hoeke gegee is / **Please note:** $\theta > 0$ and r in root form. If negative angles are given – **max 3/4**	**4**
5.1(b)	LK $= \left(\frac{\left(4 \operatorname{cis}\frac{\pi}{3}\right)\left(\sqrt{2} \operatorname{cis}\frac{3\pi}{4}\right)}{2i}\right)^4 \checkmark$ $= \left(\frac{4\sqrt{2} \checkmark \operatorname{cis}\frac{13\pi}{12} \checkmark}{2i}\right)^4$ $= \frac{1024 \operatorname{cis}\frac{13\pi}{3}}{16i^4} \checkmark = 64 \operatorname{cis}\frac{13\pi}{3} \checkmark = 64 \operatorname{cis}\frac{\pi}{3} =$ RK **OF / OR** LK $= \left(\frac{\left(4 \operatorname{cis}\frac{\pi}{3}\right)\left(\sqrt{2} \operatorname{cis}\frac{3\pi}{4}\right)}{\mathbf{2 \operatorname{cis}\frac{\pi}{2}}}\right)^4 \checkmark$ $= \left(\frac{4\sqrt{2} \checkmark \operatorname{cis}\frac{13\pi}{12} \checkmark}{2 \operatorname{cis}\frac{\pi}{2}}\right)^4$ $= \frac{1024 \operatorname{cis}\frac{13\pi}{3}}{16 \operatorname{cis} 2\pi} \checkmark = 64 \operatorname{cis}\frac{7\pi}{3} \checkmark = 64 \operatorname{cis}\frac{\pi}{3} =$ RK	**5**
5.2(a)	Area van sektor / *of sector* OCD $= \frac{1}{2}(5)^2(0.66) \checkmark = 8.25 \checkmark$ Area van / *of* ΔAOB $= \frac{1}{2}(8)(12)\sin 0.66 \checkmark = 29.429\ldots \checkmark$ Area van gearseerde deel / *of shaded area* is $= 21.18$ cm^2 $\checkmark$ (Finale antwoord twee desimale syfer. Ignoreer eenhede / *Final answer must be correct to two decimal places. Ignore units)*	**5**
5.2(b)	Omtrek van gearseerde deel/*Perimeter of shaded area* is $3 + 7 +$arc CD $+$ AB AB$^2 = 8^2 + 12^2 - 2(8)(12)\cos 0.66 \checkmark\checkmark = 56.321\ldots$ $\therefore$ AB $= 7.5$ $\therefore$ Omtrek / *Perimeter* $= \underbrace{3 + 7}_{\checkmark} + (5 \times 0.66) \checkmark + 7.5 \checkmark = 20.8$ cm $\checkmark$ (Finale antwoord een desimale syfer. Ignoreer eenhede / *Final answer must be correct to one decimal places. Ignore units)*	**6**

Vraag / *Question* 6 **[23 punte / *marks*]**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
6.1(a)	Vertikale asimptoot / *Vertical asymptote*: $4x-1=0 \implies x=\frac{1}{4}$ ✓ Skuins asimptoot / *Oblique asymptote*: $y=\frac{1}{2}x$ ✓ $+\frac{1}{8}$ ✓ (een punt vir elke term korrek / *one mark for each term correct)* $\begin{array}{r\|l} & \frac{1}{2}x \quad +\frac{1}{8} \\ \hline 4x-1 & 2x^2 \quad +0x \quad +3 \\ & 2x^2 \quad -\frac{1}{2}x \\ \hline & \frac{1}{2}x \\ & \frac{1}{2}x \quad -\frac{1}{8} \\ \hline & \frac{25}{8} \end{array}$ ✓ Indien skuins asimptoot se vergelyking verkeerd is gee **1 punt** indien langdeling gedoen is (metode-punt), andersins **volpunte**. / *If oblique asymptote's equation is wrong give **1 mark** if long division has been done (method-mark), otherwise **full marks.***	**4**
6.1(b)	**Kwosiënt reël / *Quotient rule*:** $f'(x)=\frac{4x(4x-1)\checkmark-(2x^2+3)(4)\checkmark}{(4x-1)^2}=0\checkmark$ $16x^2-4x-8x^2-12=0\checkmark$ $8x^2-4x-12=0$ $2x^2-x-3=0\checkmark$ $(x+1)(2x-3)=0$ $x=-1\checkmark$ of / *or* $x=\frac{3}{2}\checkmark$ **OF / OR** **Produkreël / *Product rule*:** $f(x)=(2x^3+3)(4x-1)^{-1}$ $f'(x)=(4x)(4x-1)^{-1}\checkmark+(2x^2+3)(-(4x-1)^{-2}.4)\checkmark=0\checkmark$ $4x(4x-1)-4(2x^2+3)=0\checkmark$ $16x^2-4x-8x^2-12=0$ $8x^2-4x-12=0$ $2x^2-x-3=0\checkmark$ $(x+1)(2x-3)=0$ $x=-1\checkmark$ of / *or* $x=\frac{3}{2}\checkmark$	**7**
6.1(c)	(i) $f''\left(\frac{3}{2}\right)=\frac{4}{5}\checkmark>0\checkmark$ Lokale minimum by / *Local minimum at* $x=\frac{3}{2}\checkmark$ (ii) Vals / *False* ✓	**4**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
6.2	**DIAGRAMBLAD / *DIAGRAM SHEET* 2**  x-afsnitte / *intercept* (-1;0) en / *and* (3;0) ✓ Draaipunt (lokale minimum) / *Turning point (local minimum)* (1 ; 4) ✓✓ Asimptoot / *Asymptote* $x = 0$ ✓ Asimptoot / *Asymptote* $x = 2$ ✓ Asimptoot / *Asymptote* $y = 1$ ✓ Oop kol "gat" by / *Open circle at* (4 ; $f(4)$) ✓ Vorm (en die neiging na die asimptote) / *Shape (and the tendency towards the asymptotes)* ✓	**8**

Vraag / *Question* 7 **[26 punte / *marks*]**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
7.1(a)	$h'(x) = 9^{\sin x}$ ✓$.\ln 9$ ✓$.\cos x$ ✓	**3**
7.1(b)	$10(x^3-2)^9$ ✓ $(3x^2)$✓ $\text{bgtan}\, x + (x^3-2)^{10}.\frac{1}{1+x^2}$ ✓ (oorspronklike funksies / *original function*) ✓ Indien nie produkreël gebruik het nie / *If product rule was not used* – **max 2/4**	**4**
7.1(c)	$\frac{dy}{dx} = \frac{2x}{x^2}$ ✓ $\frac{}{\ln 4}$ ✓ $= \frac{2}{x.\ln 4}$	**3**
7.2(a)	$(x^2+y^2)^2 = 16xy$ $\frac{d}{dx}: \quad 2(x^2+y^2)$✓$\left(2x$ ✓$+2y$ ✓$\frac{dy}{dx}$ ✓$\right) = 16y$ ✓$+16x\frac{dy}{dx}$ ✓ $4x(x^2+y^2)+4y(x^2+y^2)\frac{dy}{dx} = 16y+16x\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{16y-4x(x^2+y^2)}{4y(x^2+y^2)-16x} = \frac{4y-x(x^2+y^2)}{y(x^2+y^2)-4x}$ ✓ (vereenvoudig / *simplify*) $\left.\frac{dy}{dx}\right\|_{(2;2)} = -1$ ✓ **OF / OR** $y^4+2x^2y^2+x^4 = 16xy$ $\frac{d}{dx}: \quad 4y^3\frac{dy}{dx}$ ✓$+4xy^2$ ✓$+4x^2y\frac{dy}{dx}$ ✓$+4x^3$ ✓$= 16y$ ✓$+16x\frac{dy}{dx}$ ✓ $\frac{dy}{dx} = \frac{16y-4xy^2-4x^3}{4y^3+4x^2y-16x} = \frac{4y-xy^2-x^3}{y^3+x^2y-4x}$ ✓ (vereenvoudig / *simplify*) $\left.\frac{dy}{dx}\right\|_{(2;2)} = -1$ ✓ Indien nie $\frac{dy}{dx}$ vereenvoudig het nie / *If* $\frac{dy}{dx}$ *was not simplified* – **volpunte/*full marks***	**8**
7.2(b)	Gradiënt van die normaal is / *Gradient of the normal is* 1 ✓ $(m_1.m_2 = -1)$ $y-y_1 = m(x-x_1)$ $y-2 = x-2$ ✓ (vervang in formule / *substitute in formula*) $y = x$ ✓	**3**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
7.3	$f(x) = 3x - \tan x$ $f'(x) = 3 - \sec^2 x$ ✓ (afgeleide funksie / *derivative function*) $x_{n+1} = x_n - \dfrac{3x_n - \tan x_n}{3 - \sec^2 x_n}$ ✓✓ (substitusie in formule / *substitute in formula*) $x_0 = 1.2$ $x_1 = 1.422672$ $x_2 = 1.365960$ $x_3 = 1.332157$ $x_4 = 1.324497$ $x_5 = 1.324194$ $x_6 = 1.324194$ $\therefore x \approx 1.32419$ ✓✓ Indien antwoord korrek (volledige stappe is nie nodig nie) / *If answer is correct (complete calculations aren't necessary)* – **volpunte / *full marks*** Indien finale antwoord nie vyf desimale syfers is nie / *If final answer is not five decimal places* – **max 4/5**	**5**

Vraag / *Question* 8 **[22 punte / *marks*]**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
8.1(a)	$-\frac{1}{2}$ ✓ $\text{cosec}\,2x$ ✓ $+\frac{(2x+1)^7}{7\times 2}$ ✓ $+k$	**4**
8.1(b)	$f(x)=x$ ✓ $\quad g'(x)=\sec^2 3x$ ✓ $f'(x)=1$ ✓ $\quad g(x)=\frac{1}{3}\tan 3x$ ✓ $\int x.\sec^2 3x\ dx=\frac{1}{3}x\tan 3x-\int\frac{1}{3}\tan 3x\ dx$ ✓ (vervang in formule / *substitute in formula)* $=\frac{1}{3}x\tan 3x-\frac{1}{3}\int\frac{\sin 3x}{\cos 3x}$ ✓ dx $=\frac{1}{3}x\tan 3x+\frac{1}{9}$ ✓ $\ln\lvert\cos 3x\rvert$ ✓ $+k$	**8**
8.2(a)	$\frac{36x^2+2x+60}{(9x^2+16)(x-2)}\equiv\frac{A}{9x^2+16}+\frac{B}{x-2}$ $36x^2+2x+60\equiv A(x-2)+B(9x^2+16)$ ✓ Stel $x=2$: $\quad 208=52B$ ✓ $\implies B=4$ ✓ Stel $x=0$ **(enige waarde / *any value*)**: $60=-2A+16B$ ✓ $\implies A=2$ ✓ **OF / OR** $36x^2+2x+60\equiv A(x-2)+B(9x^2+16)$ ✓ $\equiv Ax-2A+9Bx^2+16B$ ✓ $\equiv(9B)x^2+(A)x-2A+16B$ ✓ $9B=36 \quad A=2$ ✓ $\quad -2A+16B=60$ $B=4$ ✓	**5**
8.2(b)	$\int\frac{36x^2+2x+60}{(9x^2+16)(x-2)}\ dx=\int\frac{2}{9x^2+16}+\frac{4}{x-2}\ dx$ $=\int\frac{2}{16}$ ✓ $\frac{dx}{\left(1+\left(\frac{3}{4}x\right)^2\right)}$ ✓ $+\int\frac{4}{x-2}\ dx$ $=\frac{\textbf{2}}{16}.\frac{4}{3}$ ✓ $\arctan\frac{3x}{4}$ ✓ $+\textbf{4}\ln\lvert x-2\rvert$ ✓ $+k$ (A, B) $=\frac{1}{6}\arctan\frac{3x}{4}+4\ln\lvert x-2\rvert+k$	**5**

Die omringde waardes gaan verskil indien vraag 8.2(a) se waardes verkeerd is / *The encircled values will differ if question 8.2(a)'s answers are incorrect.*

Vraag / *Question* 9 **[27 punte / *marks*]**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
9.1	$\Delta x_i = \frac{4}{n}$ ✓ $x_i = -2 + \frac{4}{n}i$ ✓ $f(x_i) = 2 + \left(-2 + \frac{4}{n}i\right)^2$ ✓ $= 6 - \frac{16}{n}i + \frac{16}{n^2}i^2$ ✓ $f(x_i)\Delta x_i = \frac{24}{n} - \frac{64}{n^2}i + \frac{64}{n^3}i^2$ ✓ $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{24}{n} - \frac{64}{n^2}i + \frac{64}{n^3}i^2\right)$ ✓ $= \lim_{n\to\infty}\left[\frac{24}{n}\sum_{i=1}^{n}1 - \frac{64}{n^2}\sum_{i=1}^{n}i + \frac{64}{n^3}\sum_{i=1}^{n}i^2\right]$ ✓ $= \lim_{n\to\infty}\left[\frac{24}{n}\times n - \frac{64}{n^2}\times\left(\frac{n^2}{2} + \frac{n}{2}\right)$ ✓ $+ \frac{64}{n^3}\times\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$ ✓$\right]$ $= \lim_{n\to\infty}$ ✓ $\left[24 - 32 - \frac{32}{n} + \frac{64}{3} + \frac{32}{n} + \frac{32}{3n^2}\right]$ $= \frac{40}{3} = 13.33$ ✓ (twee desimale syfers / *two decimal places*)	**11**
9.2	$\int_0^2 e^{x-2}\,dx$ ✓ $+ \int_2^3 2^{-x+3} - 1\,dx$ ✓ $= [e^{x-2}]_0^2$ ✓ $+ \left[\frac{2^{-x+3}\text{✓}}{-\ln 2\text{✓}} - x\text{✓}\right]_2^3$ $= (e^0 - e^{-2}) + \left(-\frac{2^0}{\ln 2} - 3\right) - \left(-\frac{2^1}{\ln 2} - 2\right)$ ✓ $= 1.31$ ✓	**8**

NR.	ANTWOORD / *ANSWER*	PUNTE *MARKS*
9.3	Sonder substitusie-metode / *Without substitution-method)*: $\pi\int_{-1}^{a} x^2\sqrt{x^3+1}\,dx = 6\pi$ ✓ $\int_{-1}^{a} x^2(x^3+1)^{\frac{1}{2}}\,dx = 6$ $\frac{2}{3}$ ✓ $\frac{1}{3}$ ✓✓ $(x^3+1)^{\frac{3}{2}}$ ✓$\Big\vert_{-1}^{a} = 6$ $\frac{2}{9}(a^3+1)^{\frac{3}{2}} - 0$ ✓ $= 6$ $(a^3+1)^{\frac{3}{2}} = 27$ ✓ $a^3+1 = 9$ $a = 2$ ✓ **OF / OR** Substitusie-metode / *Substitution-method*: $\pi\int_{-1}^{a} x^2\sqrt{x^3+1}\,dx = 6\pi$ ✓ $\int_{-1}^{a} x^2(x^3+1)^{\frac{1}{2}}\,dx = 6$ Stel / *Let* $u = x^3+1$ ✓ $\Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow \frac{1}{3}du = x^2\,dx$ ✓ $\therefore \int \frac{1}{3}u^{\frac{1}{2}}\,du$ ✓ $= \frac{1}{3}.\frac{2}{3}u^{\frac{3}{2}} + k$ ✓ $\int_{-1}^{a} x^2(x^3+1)^{\frac{1}{2}}\,dx = \frac{2}{9}(x^3+1)^{\frac{3}{2}}\Big\vert_{-1}^{a}$ ✓ $= 6$ $\frac{2}{9}(a^3+1)^{\frac{3}{2}} - 0 = 6$ $(a^3+1)^{\frac{3}{2}} = 27$ } ✓ $a^3+1 = 9$ $a = 2$ ✓	**8**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -