

# $\alpha$ -MATHEMATICS

## Alpha Mathematics Prelim Examination paper

**Time: 3 hours**

**Grade 12**

**Total: 200 marks**

**26 September 2017**

### **INSTRUCTIONS AND INFORMATION**

Read the instructions carefully before answering the questions:

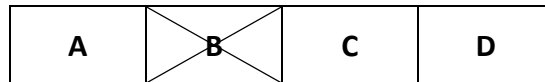
1. This question paper consists of 10 pages, an Answer sheet of 2 pages and a Formula sheet of 3 pages
2. Answer ALL nine questions.
3. Number the answers exactly the way the questions are numbered.
4. Non-programmable calculators may be used.
5. Unless otherwise indicated, all answers, where necessary, must be given correct to TWO decimal places.
6. ALL necessary calculations must be shown clearly, except in question one. No marks will be awarded if only the answer is given.
7. The diagrams in the question paper are not necessarily drawn to scale.
8. All angles are given in radians. Answers must also be given in radians if necessary.
9. Write neatly and legible.

**Question 1****[20 marks]**

This question must be answered on the **ANSWER SHEET**.

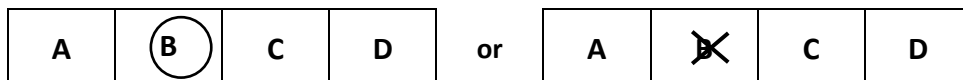
Each question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with an **X** on the answer sheet.

**CORRECT MANNER:**



**INCORRECT MANNER:**

The following indication of answers are **NOT** acceptable:



1.1 Solve for  $x$  in  $2 - |5 - x| < 3$

- (A)  $4 < x < 6$   
 (B)  $x < 0$  or  $x > 10$   
 (C)  $x \in \mathbb{R}$   
 (D) No solution for  $x$

1.2 Which equation below describes the following statement the best:

**“A function  $f$  is continuous in the point 5.”**

- (A)  $\lim_{x \rightarrow 0} f(x) = 5$   
 (B)  $\lim_{x \rightarrow 5} f(x) = 0$   
 (C)  $\lim_{x \rightarrow 0} f(x) = f(5)$   
 (D)  $\lim_{x \rightarrow 5} f(x) = f(5)$

1.3 Given:  $f'(2) = \lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$

What is the value of  $f'(2)$  ?

- (A) 14  
 (B) 12  
 (C) 6  
 (D) 2

1.4 The expansion of  $\sqrt{4 - 2x}$  will be valid if ...

- (A)  $|x| < 2$
- (B)  $|x| < \frac{1}{2}$
- (C)  $|x| > 2$
- (D)  $|x| > -\frac{1}{2}$

1.5 What is the real part of  $3i^{41}(1 + 3i) + \sqrt{-25}$  ?

- (A) 17
- (B) -9
- (C) 8
- (D) -4

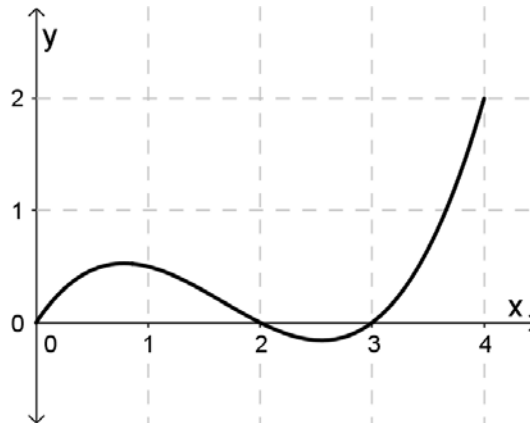
1.6 Which expression is correct?

- (A)  $\log_a x = \frac{1}{x \cdot \ln a}$
- (B)  $\frac{d}{dx}(\log_a x) = \frac{1}{x}$  for all  $x \in \mathbb{R}; a \neq e$
- (C)  $\int \ln x \, dx = \frac{1}{x} + k$
- (D)  $\int \frac{1}{x} \, dx = \log_e x + k$

1.7 Determine the  $x$ -coordinate of the point on the curve  $y = 2x^2 - 13x + 5$  where the tangent to the curve is **parallel** to the line  $y = 3x$ .

- (A)  $x = 4$
- (B)  $x = \frac{5}{2}$
- (C)  $x = 1$
- (D)  $x = 8$

- 1.8 The function  $y = f(x)$  is defined for  $x \in [0; 4]$ . The **derivative** of  $f(x)$ ,  $y = f'(x)$ , is sketched below. At which point does  $f(x)$  has a **local maximum** value.



- (A)  $x = 0$   
 (B)  $x = 1$   
 (C)  $x = 2$   
 (D)  $x = 3$
- 1.9 Given the following information regarding  $f(x)$ :

$f(2) = 4$	$f'(2) = 5$	$f''(2) = 6$
$f(6) = 12$	$f'(6) = 11$	$f''(6) = 10$

The value of  $\int_2^6 f''(x) dx =$

- (A) 8  
 (B) 2  
 (C) 4  
 (D) 6
- 1.10 Given the function:  $y = \begin{cases} |2x| & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$

Determine the  $x$ -value where the **gradient** (derivative) **function** has a jump discontinuity

- (A)  $x = -1$   
 (B)  $x = 0$   
 (C)  $x = 1$   
 (D)  $x = 2$

**Question 2****[20 marks]**

2.1 The following system of equations are given:

$$ax + 4y - z = 7$$

$$-x + y + 2z = 3$$

$$2ax + 6y - 3z = 0$$

Determine the value of  $a$  so that this system of equations have no solution, by using Cramer's rule. (4)

2.2 Given that  $P(x) = x^4 - 6x^3 + 16x^2 - 22x + 15$  and  $P(2 - i) = 0$ .

Factorize  $P(x)$  completely over  $\mathbb{Z}[x]$ . (6)

2.3 Determine the term in this expansion  $\left(\frac{x}{5} + \frac{5}{x^2}\right)^{18}$  which is **independent** of  $x$ .

Leave your answer correct to THREE decimal places. (5)

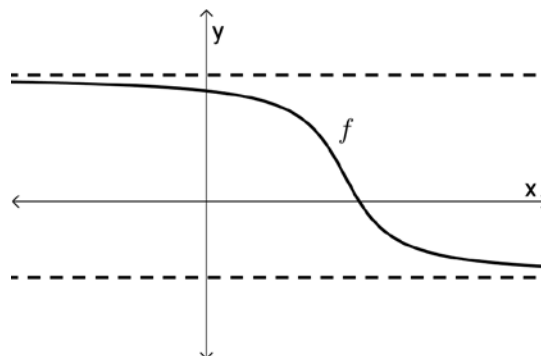
2.4 Determine the **first 3 terms**, in ascending powers of  $x$ , of this expansion:

$$(1 + 2x)\sqrt{1 + 2x} \quad (5)$$

**Question 3****[16 marks]**

3.1 Complete the following:

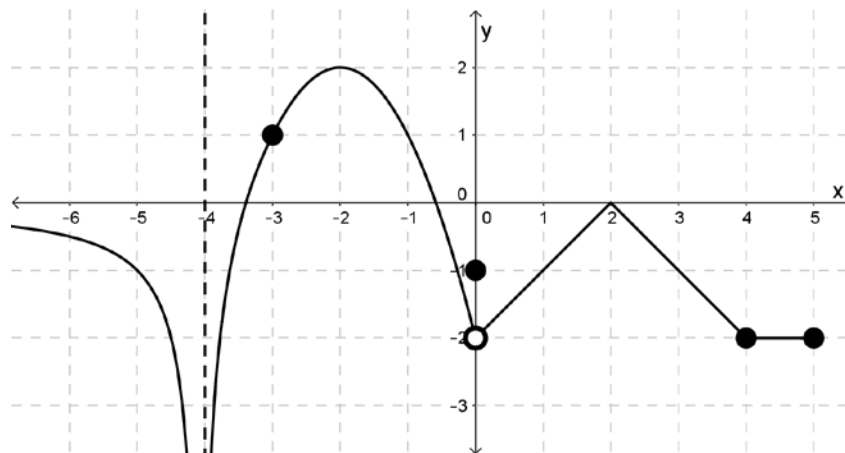
The inverse function can only be determined if the function is a/an \_\_\_\_\_ function. (1)

3.2 The sketch shows the graph of  $f(x) = a \arctan(x + p) + \frac{\pi}{8}$ (a) Determine the equations of the asymptotes of  $f$ . (2)(b) If  $a$  can be only one of the following two values, which one describes the sketch of  $f$  the best:

$$a = +1 \text{ or } a = -1 \quad (1)$$

(c) Hence, determine the value of  $p$  if the  $x$ -intercept of  $f$  is 4.414, correct to the nearest integer. (3)(d) **Write** down the  $x$ -value where  $f$  changes in convexity. (1)

- 3.3 The piecewise function of  $f$ , is sketched below. The function has  $x$ -intercepts at  $x = -3.4$ ,  $x = -0.6$  and  $x = 2$ . The function has a **vertical asymptote** at  $x = -4$ .



- (a) What type of discontinuity occurs at  $x = -4$ ? (1)

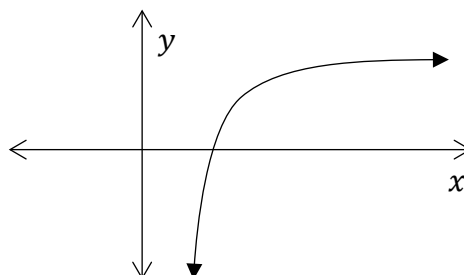
Write down the value(s) of  $x$  for which the following statements will apply:

- (b)  $f$  is not differentiable, but continuous. (2)
- (c) The limit exists and the function value exists in the point but they are not equal. (1)
- (d)  $\frac{f(x)}{f'(x)} = 0$  (2)
- (e)  $f(x) = f(x - 1)$  where  $x \in \mathbb{Z}$  (2)

#### Question 4

[26 marks]

- 4.1 The sketch of  $f(x) = \ln(x - 1) + \frac{1}{2}$  is sketched below.



- (a) Determine the:
- (i) coordinates of the  $x$ -intercept of  $f$ .
- (ii) equation of the asymptote of  $f$ . (4)
- (b) Hence, sketch the graph of  $y = |f(x)|$  on **DIAGRAM SHEET 1**. Show all intercepts with the axes and asymptotes clearly on your sketch. (3)
- (c) Write down the range of the inverse function of  $f(x)$ . (1)

4.2 Solve for  $x \in \mathbb{R}$  in  $\frac{|x-2|}{x+1} = -2x$  (7)

4.3 Use Mathematical Induction and prove that the following statement applies to all  $n \in \mathbb{N}$ , where  $n \geq 1$ :

$$1. \ln x + 2. \ln x + 3. \ln x + \dots + n. \ln x = \frac{n}{2} \cdot \ln x^{n+1}$$

**HINT:** make use of the  $\ln$ -laws. (11)

### Question 5

[20 marks]

5.1 Given:  $m_1 = 2 + 2\sqrt{3}i$  and  $m_2 = -1 + i$ .

(a) Convert  $m_1$  and  $m_2$  into polar form,  $r \operatorname{cis} \theta$ , where  $r$  is in root form, if necessary, and  $\theta > 0$  must be given in terms of  $\pi$ . (4)

(b) Use De Moivre's theorem and hence show that

$$\left(\frac{m_1 \times m_2}{2i}\right)^4 = 64 \operatorname{cis} \frac{\pi}{3} \quad (5)$$

5.2 In the diagram C is a point on OA and D is a point on OB. OCD is a sector, with midpoint O, inside a triangle AOB.

The following measurements are given:

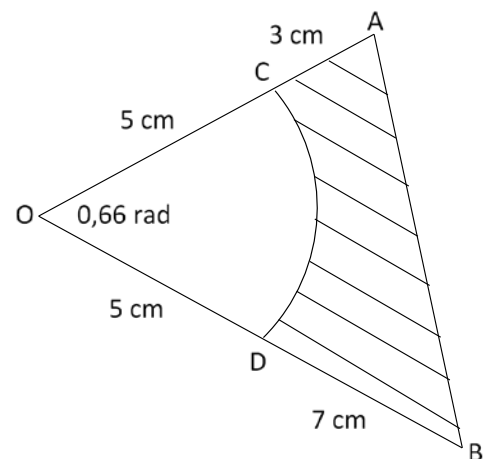
$$OC = OD = 5 \text{ cm}$$

$$AC = 3 \text{ cm}$$

$$BD = 7 \text{ cm}$$

$$\widehat{AOB} = 0,66 \text{ radians}$$

ACDB is the shaded area on the diagram.



(a) Determine the **surface area** of the shaded area. (5)

(b) Determine the **circumference** (perimeter) of the shaded area, correct to **ONE** decimal places. (6)

**Question 6****[23 marks]**

6.1 Given the function:  $f(x) = \frac{2x^2 + 3}{4x - 1}$

(a) Determine the equation of the vertical- and oblique asymptote of  $f$ . (4)

(b) Calculate the  $x$ -value(s) of the stationary point(s) of  $f$ . (7)

(c) Given that  $f''(x) = \frac{100}{(4x - 1)^3}$

(i) One of the stationary points is at  $x = \frac{3}{2}$ . **Classify** this stationary point. (3)

(ii) Is the following statement **TRUE** or **FALSE**:  
“ $f(x)$  has a point of inflection.” (1)

6.2 Draw a sketch of the rational function,  $f(x)$ , with the following properties. Clearly show all intercepts, turning points and asymptotes on your sketch, if there is enough information given. Use **DIAGRAM SHEET 2** for this sketch.

- The domain of  $f$  is  $x \in \mathbb{R}; x \neq 0; x \neq 2$  and  $x \neq 4$
- $f$  has only **ONE** stationary point and **TWO**  $x$ -intercepts.
- $f(1) = 4$                        $f'(1) = 0$                        $f''(1) = 6$
- $\lim_{x \rightarrow -\infty} f(x) = 1$                        $\lim_{x \rightarrow +\infty} f(x) = 1$
- $\lim_{x \rightarrow 0^-} f(x) = -\infty$                        $\lim_{x \rightarrow 0^+} f(x) = +\infty$
- $\lim_{x \rightarrow 2^-} f(x) = +\infty$                        $\lim_{x \rightarrow 2^+} f(x) = -\infty$
- $f(-1) = f(3) = 0$
- $f$  has a **removable** discontinuity at  $x = 4$  (8)

**Question 7****[26 marks]**7.1 Determine the following (*it is not necessary to simplify*):

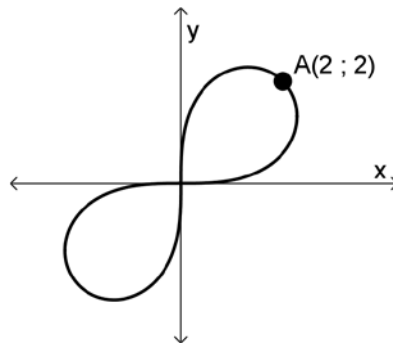
(a)  $h'(x)$  if  $h(x) = 9^{\sin x}$  (3)

(b)  $\frac{d}{dx} [(x^3 - 2)^{10} \times \arctan x]$  (4)

(c)  $\frac{dy}{dx}$  if  $y = \log_4(x^2)$  (3)

7.2 The “Limniscate of Bernoulli” is an adjusted ellipse, that looks like an “infinity” sign. The equation of the graph below is:

$$(x^2 + y^2)^2 = 16xy$$



(a) Determine the value of  $\frac{dy}{dx}$  in the point A, by using **implicit differentiation**. (8)

(b) If  $\left. \frac{dy}{dx} \right|_{(2;2)} = -1$ , determine the equation of the **normal** in the point A. (3)

7.3 Given:  $f(x) = 3x - \tan x$ Use the Newton-Raphson-method to determine one of the zeros of  $f(x)$ , correct to five decimal places. Use  $x_0 = 1.2$  as first approach. (5)

**Question 8****[22 marks]**

8.1 Determine the following integrals:

(a) 
$$\int \cot 2x \operatorname{cosec} 2x + (2x + 1)^6 dx \quad (4)$$

(b) 
$$\int x \cdot \sec^2 3x dx \quad (8)$$

*HINT: factor integration/piecewise integration*

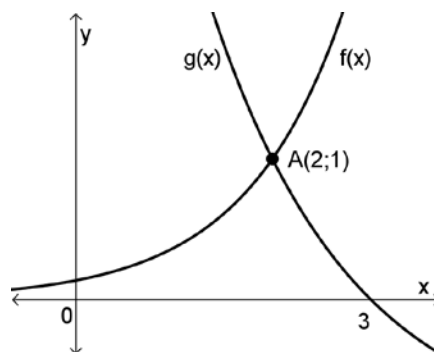
8.2 Given that 
$$\frac{36x^2 + 2x + 60}{(9x^2 + 16)(x - 2)} \equiv \frac{A}{9x^2 + 16} + \frac{B}{x - 2}$$

(a) Determine the values of A and B. (5)

(b) Hence, determine 
$$\int \frac{36x^2 + 2x + 60}{(9x^2 + 16)(x - 2)} dx \quad (5)$$

**Question 9****[27 marks]**

9.1 Use an infinite Riemann sum and calculate the value of 
$$\int_{-2}^2 2 + x^2 dx \quad (11)$$

9.2 Determine the area under the functions  $f(x) = e^{x-2}$  and  $g(x) = 2^{3-x} - 1$  and above the  $x$ -axis between the values  $x = 0$  and  $x = 3$ . The intersection of  $f$  and  $g$  is  $A(2; 1)$ .

(8)

9.3 The volume of the revolution that exists if the area under the function  $y^2 = x^2\sqrt{x^3 + 1}$  rotates around the  $x$ -axis between  $x = -1$  and  $x = a$  is  $6\pi$ . Determine the value of  $a$ . Show all calculations.*HINT: substitution-method may be used.* (8)**- END OF THE EXAMINATION PAPER -**

## Alpha Mathematics Grade 12 – Prelim Examination 2017

### ANSWER SHEET

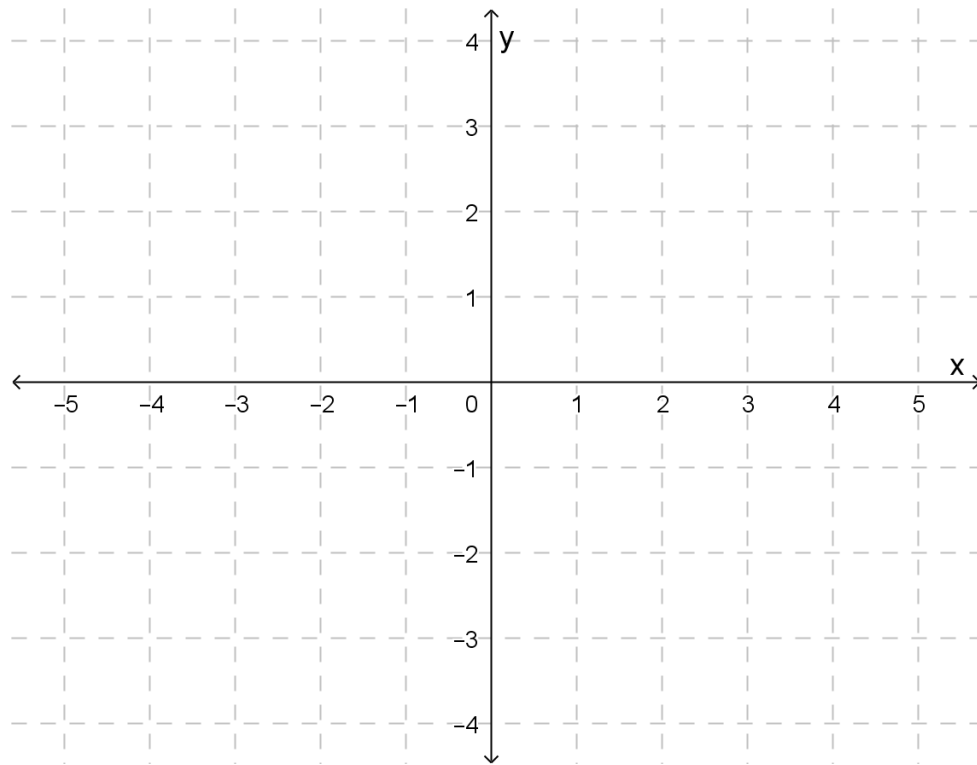
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Question [Total]	1 [20]	2 [20]	3 [16]	4 [26]	5 [20]	6 [23]	7 [26]	8 [22]	9 [27]	TOTAL 200
Learner's mark										

#### Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

### DIAGRAM SHEET 1



### DIAGRAM SHEET 2

