α-WISKUNDE

Alpha Wiskunde Graad 12 / ***Alpha Mathematics Grade 12***

Finale eksamen 2018 / ***Final examination 2018***

MEMORANDUM

Totaal / ***Total*****: 200 punte /** ***marks***

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Hierdie memorandum bestaan uit 18 bladsye. /
This memorandum consists of 18 pages.

Vraag / *Question* 1 [20 punte / *marks*]

1.1	A	B	**C**	D
1.2	A	B	C	**D**
1.3	A	**B**	C	D
1.4	A	B	**C**	D
1.5	A	B	C	**D**
1.6	A	B	**C**	D
1.7	A	B	C	**D**
1.8	**A**	B	C	D
1.9	**A**	B	C	D
1.10	A	**B**	C	D

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	C	Knakpunt: $(-1;2)$ dus $y = \|x+1\| + 2$	**2**
1.2	D	$-1 \le 2x \le 1 \implies -\frac{1}{2}x \le x \le \frac{1}{2}$	**2**
1.3	B	$y = 2\,\text{bgsin}(-1) = -\pi$ $y = 2\,\text{bgsin}(1) = \pi$	**2**
1.4	C	Die funksie g het 'n verwyderbare diskontinuïteit by $x = -2$. Dus OOP kol.	**2**
1.5	D	$y = \frac{x+3}{(x-3)(x+3)} = \frac{1}{x-3}\,; x \neq -3$ Indien $x = -3$ is dan is $k = -\frac{1}{6}$	**2**
1.6	C	$\binom{9}{r}(3a)^{9-r}\left(-\frac{b}{3}\right)^r$ Term bevat a^5b^4 dus $9-r=5 \implies r=4$ $\binom{9}{4}(3a)^{9-4}\left(-\frac{b}{3}\right)^4 = 126(3^5)\left(\frac{1}{3}\right)^4 a^5b^4 = 378a^5b^4$	**2**
1.7	D	$f'(x) = 0$ $1 - x^{-2} = 0$ $x^2 - 1 = 0$ $(x-1)(x+1) = 0$ $x = 1$ of $x = -1$ $f''(x) = 2x^{-3}$ $f''(1) = 2 > 0$ lokale minimum $f''(-1) = -2 < 0$ lokale maksimum	**2**
1.8	A	$f'(x) > 0$ ... stygende funksie $f''(x) > 0$ ... konkaaf op	**2**
1.9	A	(A) $-e^{1-x} = -e^{1-x}$ (B) $e^{x-1} \neq -e^{x-1}$ (C) $e^{-x} \neq -1 + e^{-x}$ (D) $-e^{-x} \neq -e^{-x} + 1$	**2**
1.10	B	$\int_a^b \big(f(x)+g(x)\big)\,dx = \int_a^b f(x)\;dx + \int_a^b g(x)\;dx = 5 - 1 = 4$ Maar (I) is nie altyd waar nie, want f en g kan mekaar sny in die interval $[a;b]$.	**2**

Vraag / *Question* 2 **[20 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1 (a)	As $t = 0$ dan is $N = \frac{10094}{3+100e^{0}} = 98$ mense	✓Stel $t = 0$ ✓Antwoord **[2 punte / *marks*]**
2.1 (b)	$1265 = \frac{10094}{3+100e^{-0.5t}}$ $1265(3+100e^{-0.5t}) = 10094$ $3+100e^{-0.5t} = \frac{10094}{1265}$ $e^{-0.5t} = 0.04979\ldots$ $-0.5t = \ln(0.04979\ldots)$ $t = 6$ dae	✓1265 ✓Vereenvoudig ✓$\ln(0.04979\ldots)$ ✓Antwoord **[4 punte / *marks*]**
2.1 (c)	$\frac{dN}{dt} = \frac{0-10094(100e^{-0.5t})(-0.5)}{(3+100e^{-0.5t})^2}$	✓$-10094(3+100e^{-0.5t})^{-2}$ ✓$100e^{-0.5t}$ ✓-0.5 ✓$(3+100e^{-0.5t})^2$
	OF / *OR*	**OF / *OR***
	$N = 10094(3+100e^{-0.5t})^{-1}$ $\frac{dN}{dt} = -10094(3+100e^{-0.5t})^{-2}(100e^{-0.5t})(-0.5)$	✓Herskryf N ✓$-10094(3+100e^{-0.5t})^{-2}$ ✓$100e^{-0.5t}$ ✓-0.5 **[4 punte / *marks*]**
2.1 (d)	Meer mense word besmet. **REDE 1**: Want die afgeleide is POSITIEF, en dus is N 'n stygende funksie. **REDE 2**: Want by $t = 0$ was daar 98 mense besmet en na 6 dae is daar 1265 mense besmet. Dus die funksie neem toe oor tyd.	✓Meer ✓Enige rede **[2 punte / *marks*]**

NR. NO	**ANTWOORD / *ANSWER***	**PUNTE / *MARKS***
2.2	$x = -2 - i$ is ook 'n nulpunt. $(x + 2 - i)(x + 2 + i)$ **OF / *OR*** $x = -2 + i$ $= (x + 2)^2 - i^2$ $(x + 2)^2 = i^2$ $= x^2 + 4x + 5$ is 'n faktor $x^2 + 4x + 5 = 0$ Deur inspeksie / Langdeling: $(x + 3)(x^2 + 4x + 5) = 0$ $x = -3$	✓Metode ✓$4x$ ✓5 ✓$x = -3$ **[4 punte / *marks*]**
2.3	Vierde term in $(1 + 2x)^{\frac{1}{3}}$ is: $\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(2x)^3 = \frac{40}{81}x^3$ Dus $a = \frac{40}{81}$.	✓$\frac{1}{3}$ ✓$\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)$ ✓$(2x)^3$ ✓Antwoord **[4 punte / *marks*]**

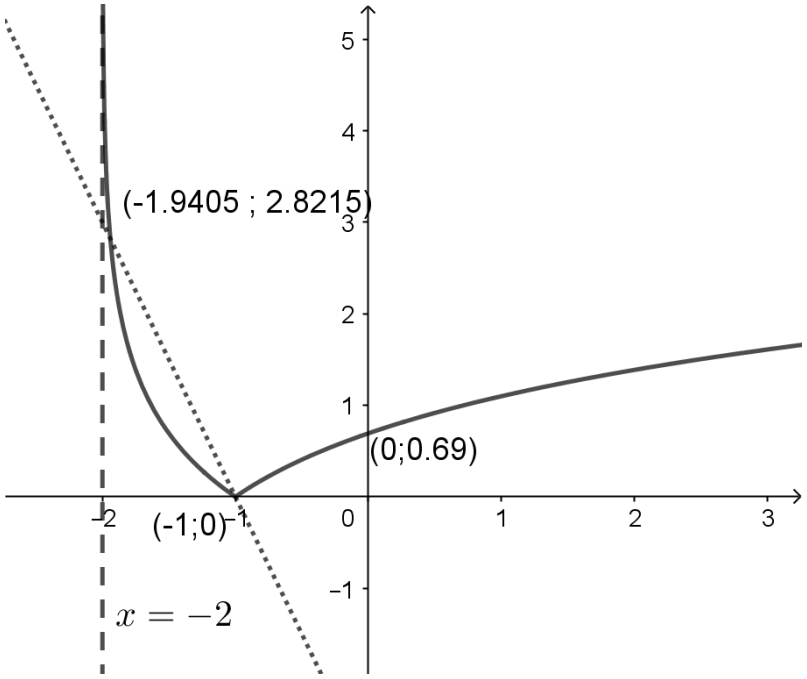
Vraag / *Question* 3 **[20 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1 (a)(i)	$\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{1+2i+i^2}{1-i^2}$ $=\frac{2i}{2}$ $=i$	✓$\frac{1+i}{1+i}$ ✓Vermenigvuldig uit ✓$\frac{2i}{2}$ **[3 punte / *marks*]**
3.1 (a)(ii)	$\frac{(1+i)^{16}(1+i)}{(1-i)^{16}}=\left(\frac{1+i}{1-i}\right)^{16}(1+i)$ $=i^{16}(1+i)$ $=1+i$	✓Herskryf ✓Vervang met i ✓Antwoord **[3 punte / *marks*]**
3.1 (b)(i)	$1+i=\sqrt{2}\,\text{cis}\frac{\pi}{4}$ $1-i=\sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)$ $=\sqrt{2}\,\text{cis}\frac{7\pi}{4}$	✓$\sqrt{2}$ ✓$\frac{\pi}{4}$ ✓$\sqrt{2}$ ✓$-\frac{\pi}{4}$ of $\frac{7\pi}{4}$ **[4 punte / *marks*]**
3.1 (b)(ii)	$\frac{\left(\sqrt{2}\,\text{cis}\frac{\pi}{4}\right)^{17}}{\left(\sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)\right)^{16}}=\frac{\left(\sqrt{2}\right)^{17}\text{cis}\left(17\times\frac{\pi}{4}\right)}{\left(\sqrt{2}\right)^{16}\text{cis}\left(-16\times\frac{\pi}{4}\right)}$ $=\sqrt{2}\,\text{cis}\frac{\pi}{4}$ $=1+i$	✓$\left(\sqrt{2}\right)^{17}$ en $\left(\sqrt{2}\right)^{16}$ ✓$\frac{17\pi}{4}$ en $-\frac{16\pi}{4}$ ✓$\frac{\pi}{4}$ ✓Antwoord
	OF / *OR*	**OF / *OR***
	$\frac{(1+i)^{16}(1+i)}{(1-i)^{16}}=\left(\frac{1+i}{1-i}\right)^{16}(1+i)$ $=\left(\frac{\sqrt{2}\,\text{cis}\frac{\pi}{4}}{\sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)}\right)^{16}\sqrt{2}\,\text{cis}\frac{\pi}{4}$ $=\left(\text{cis}\frac{\pi}{2}\right)^{16}\sqrt{2}\,\text{cis}\frac{\pi}{4}$ $=\text{cis}\,8\pi\times\sqrt{2}\,\text{cis}\frac{\pi}{4}=\sqrt{2}\,\text{cis}\frac{33\pi}{4}$ $x=\sqrt{2}\cos\frac{33\pi}{4}=1$ $y=\sqrt{2}\sin\frac{33\pi}{4}=1$ $\therefore\frac{(1+i)^{17}}{(1-i)^{16}}=1+i$	✓$\frac{\pi}{2}$ ✓8π ✓$\frac{33\pi}{4}$ ✓Antwoord **[4 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.2	LK $= x^0 + x^1 + x^2 + \cdots + x^{k-1} + x^{k+1-1}$ $= \frac{1-x^k}{1-x} + x^k$ $= \frac{1-x^k + x^k - x^1.x^k}{1-x}$ $= \frac{1-x^{k+1}}{1-x}$ RK $= \frac{1-x^{k+1}}{1-x}$	✓$\frac{1-x^k}{1-x}$ ✓x^k ✓$1-x^k+x^k$ ✓$-x^1.x^k$ ✓Antwoord van LK ✓Antwoord van RK **[6 punte / *marks*]**

Vraag / *Question* 4 **[19 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1	$\|\ln(x+2)\| = -3x-3$ $\ln(x+2) = -(-3x-3)$ of $\ln(x+2) = -3x-3$ $x+2 = e^{3x+3}$	✓Negatief ✓$\ln(x+2)$ ✓Antwoord **[3 punte / *marks*]**
4.2	$f(x) = x+2-e^{3x+3}$ $f'(x) = 1-3e^{3x+3}$ $x_{n+1} = x_n - \frac{x_n+2-e^{3x_n+3}}{1-3e^{3x_n+3}}$ $x_1 = -1.8$ $x_2 = -1.98014$ $x_3 = -1.94051$ $x_4 = -1.94048$ $x^5 = -1.94048$ $x \approx -1.9405$	✓$f(x)$ ✓$f'(x)$ ✓Vervang in formule ✓✓ Antwoord korrek tot 4 desimale syfers
	OF / *OR*	**OF / *OR***
	$f(x) = e^{3x+3}-x-2$ $f'(x) = 3e^{3x+3}-1$ $x_{n+1} = x_n - \frac{e^{3x_n+3}-x_n-2}{3e^{3x_n+3}-1}$ $x_1 = -1.8$ $x_2 = -1.95014$ $x_3 = -1.94051$ $x_4 = -1.94047$ $x^5 = -1.94047$ $x \approx -1.9405$	✓$f(x)$ ✓$f'(x)$ ✓Vervang in formule ✓✓ Antwoord korrek tot 4 desimale syfers **[5 punte / *marks*]**

NR. NO	**ANTWOORD / *ANSWER***	**PUNTE / *MARKS***
4.3 (a)		✓Asimptoot $x = -2$ ✓"Knakpunt" by $x = -1$ ✓Snypunt van f en g ✓Neig na asimptoot ✓Vorm regs van $x = -1$ **[5 punte / *marks*]**
4.3 (b)(i)	$-1.9 < x < -1$	✓-1.9 ✓-1 **[2 punte / *marks*]**
4.3 (b)(ii)	$-2 < x < -1$	✓-2 ✓-1 **[2 punte / *marks*]**
4.4	$y = \sin(x + 2)$ $x = \sin(y + 2)$ $y = \text{bgsin}\, x - 2$ $f^{-1}(x) = \text{bgsin}\, x - 2$	✓Ruil x en y om ✓Antwoord **[2 punte / *marks*]**

Vraag / *Question* 5 **[12 punte / *marks*]**

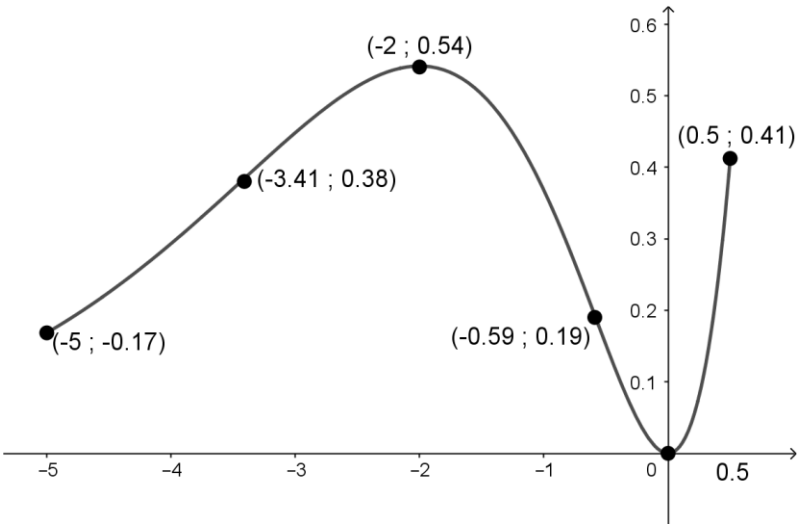
NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$\det A = \begin{vmatrix} a & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -1 & 0 \end{vmatrix} = a\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} - 0 + 2\begin{vmatrix} 0 & 2 \\ 4 & -1 \end{vmatrix}$ $= a(0+1) + 2(0-8)$ $= a - 16$ $\det A_x = \begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ 2a & -1 & 0 \end{vmatrix} = 9\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} - 0 + 2\begin{vmatrix} 8 & 2 \\ 2a & -1 \end{vmatrix}$ $= 9(0+1) + 2(-8-4a)$ $= 9 - 16 - 8a = -7 - 8a$ $\therefore x = \dfrac{-7-8a}{a-16} = 1$ $-7 - 8a = a - 16$ $-9a = -9$ $a = 1$	✓ $\det A$ ✓ Bepaal van determinante ✓ Antwoord ✓ $\det A_x$ ✓ Bepaal van determinante ✓ Antwoord ✓ Antwoord **[7 punte / *marks*]**
5.2 (a)	$S = r\theta = 24$ $\theta = \dfrac{24}{r}$ $A = \dfrac{1}{2}r^2\left(\dfrac{24}{r}\right) = 96$ $12r = 96$ $r = 8$	✓ Vervang in formule ✓ Herskryf ✓ Vervang in formule ✓ Antwoord **[4 punte / *marks*]**
5.2 (b)	$\theta = \dfrac{24}{8} = 3$ rad	✓ Antwoord **[1 punt / *mark*]**

Vraag / *Question* 6 **[25 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1	$f'(x) = \lim_{h\to 0} \dfrac{\dfrac{2}{1-4(x+h)} - \dfrac{2}{1-4x}}{h}$ $= \lim_{h\to 0} \dfrac{2(1-4x) - 2(1-4x-4h)}{h(1-4x-4h)(1-4x)}$ $= \lim_{h\to 0} \dfrac{2-8x-2+8x+8h}{h(1-4x-4h)(1-4x)}$ $= \lim_{h\to 0} \dfrac{8h}{h(1-4x-4h)(1-4x)}$ $= \dfrac{8}{(1-4x)^2}$	✓ $f(x+h)$ ✓ Vereenvoudig ✓ Noemer ✓ $8h$ ✓ Antwoord **[5 punte / *marks*]**
6.2 (a)	$f'(x) = \dfrac{2x}{x^2.\ln 10} \times \text{bgcos}\dfrac{4}{x} + \log x^2 \times \dfrac{-1}{\sqrt{1-\left(\frac{4}{x}\right)^2}} \times \dfrac{-4}{x^2}$	✓ $x^2 \ln 10$ ✓ $2x$ ✓ $\text{bgcos}\frac{4}{x} + \log x^2$ ✓ $\frac{-1}{\sqrt{1-\left(\frac{4}{x}\right)^2}}$ ✓ $\frac{-4}{x^2}$ **[5 punte / *marks*]**
6.2 (b)	$D_x[\ln(\tan x) - \ln(1+2x)]$ $= \dfrac{1}{\tan x}.\sec^2 x - \dfrac{1}{1+2x}.2$	✓ ln wet ✓ $\frac{1}{\tan x}$ ✓ $\sec^2 x$ ✓ $\frac{1}{1+2x}.2$
	OF / *OR*	**OF / *OR***
	$\dfrac{1}{\frac{\tan x}{1+2x}} \times \dfrac{\sec^2 x\,(1+2x) - \tan x\,(2)}{(1+2x)^2}$	✓ $\frac{1}{\frac{\tan x}{1+2x}}$ ✓ $\sec^2 x$ ✓ Kwosiënt reël ✓ 2 **[4 punte / *marks*]**
6.3 (a)	$e^{\sin y}\cos y\dfrac{dy}{dx} - 1 = 3x^2\sin y + x^3\cos y\dfrac{dy}{dx} + k\dfrac{dy}{dx}$ $\dfrac{dy}{dx} = \dfrac{3x^2\sin y + 1}{e^{\sin y}\cos y - x^3\cos y - k}$	✓ $e^{\sin y}\cos y\frac{dy}{dx}$ ✓ -1 ✓ $3x^2\sin y$ ✓ $x^3\cos y\frac{dy}{dx}$ ✓ $k\frac{dy}{dx}$ ✓ Antwoord **[6 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.3 (b)	$\left.\frac{dy}{dx}\right\|_{(1;\pi)} = 1$ $\frac{3\sin\pi + 1}{e^{\sin\pi}\cos\pi - \cos\pi - k} = 1$ $\frac{1}{-1+1-k} = 1$ $k = -1$	✓Vervang in f' ✓Gelyk aan 1 ✓$k = -1$
	Die punt $(1;\pi)$ lê op die oorspronklike funksie: $e^{\sin\pi} - 1 + a\pi = \sin\pi + \pi$ $1 - 1 + a\pi = \pi$ $a = 1$	✓Vervang in funksie ✓$a = 1$
	OF / *OR*	**OF / *OR***
	$e^{\sin\pi} - 1 + a\pi = \sin\pi + k\pi$ $\therefore a = k$	✓Vervang in funksie ✓$a = k$
	$\left.\frac{dy}{dx}\right\|_{(1;\pi)} = 1$ $\frac{3\sin\pi + 1}{e^{\sin\pi}\cos\pi - \cos\pi - k} = 1$ $\frac{1}{-1+1-k} = 1$ $k = -1$ $\therefore a = -1$	✓Vervang in f' ✓$k = -1$ ✓$a = -1$ **[5 punte / *marks*]**

Vraag / *Question 7* **[22 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	x-afsnit: $x^2e^x = 0$ $x = 0$ of $e^x = 0$ (N.V.T.) y-afsnit: $y = 0$	✓$x = 0$ ✓$y = 0$ **[2 punte / *marks*]**
7.2	$f'(x) = 0$ $2xe^x + x^2e^x = 0$ $xe^x(2 + x) = 0$ $x = 0$ of $x = -2$ of $e^x = 0$ (N.V.T.) Dus $a = -2$	✓$2xe^x$ ✓x^2e^x ✓Faktoriseer ✓Antwoord **[4 punte / *marks*]**
7.3	$f''(x) = 2e^x + 2xe^x + 2xe^x + x^2e^x$ $= 2e^x + 4xe^x + x^2e^x$ $f''(0) = 2 > 0$ $f''(-2) = -0.27 < 0$ Lokale minimum draaipunt by $x = 0$ Lokale maksimum draaipunt by $x = -2$	✓$2e^x + 2xe^x$ ✓$2xe^x + x^2e^x$ ✓$f''(0) = 2 > 0$ ✓$f''(-2) = -0.27 < 0$ ✓Minimum by $x = 0$ ✓Maksimum by $x = -2$ **[6 punte / *marks*]**
7.4	$f''(x) = 0$ $2e^x + 4xe^x + x^2e^x = 0$ $e^x(2 + 4x + x^2) = 0$ $x = \frac{-4 \pm \sqrt{4^2 - 4(2)}}{2}$ $= -2 \pm \sqrt{2}$ $= -0.59$ of -3.41	✓Stel gelyk aan nul ✓Faktoriseer ✓✓Antwoorde **[4 punte / *marks*]**
7.5		'n Punt by elk van die volgende punte op die skets: ✓$x = -5$ ✓$x = -3.41$ ✓$x = -2$ ✓$x = -0.59$ ✓$x = 0$ ✓Vorm **[6 punte / *marks*]**

Vraag / *Question* 8 **[26 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1 (a)	$f(x) = \dfrac{x^2+2}{x(x-4)}$ Vertikale asimptoot by $x = 0$ en $x = 4$ $y = \lim\limits_{h\to\infty} \dfrac{x^2\left(1+\frac{2}{x^2}\right)}{x^2\left(1+\frac{4}{x}\right)} = 1$ Horisontale asimptoot by $y = 1$	✓$x = 0$ ✓$x = 4$ ✓$y = 1$ **[3 punte / *marks*]**
8.1 (b)	$x \in \mathbb{R}; x \neq 0; x \neq 4$	✓$x \in \mathbb{R}$ ✓$x \neq 0; x \neq 4$ **[2 punte / *marks*]**
8.1 (c)	$f'(x) = 0$ $\dfrac{2x(x^2-4x)-(x^2+2)(2x-4)}{(x^2-4x)^2} = 0$ $2x^3-8x^2-2x^3+4x^2-4x+8=0$ $-4x^2-4x+8=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ of $x=1$ $f(-2)=\dfrac{1}{2}$ $f(1)=-1$ Waardeversameling is $y \leq -1$ of $y \geq \frac{1}{2}$	✓$f'(x)=0$ ✓$2x(x^2-4x)$ ✓$(x^2+2)(2x-4)$ ✓Vermenigvuldig hakies uit ✓Vereenvoudig ✓Antwoorde van x ✓Antwoorde van y ✓Waardeversameling **[8 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.2 (a)	$\int 9^x + x^9 - \frac{9}{x}\ dx$ $= \frac{9^x}{\ln 9} + \frac{x^{10}}{10} - 9\ln x + k$	✓$\frac{9^x}{\ln 9}$ ✓$\frac{x^{10}}{10}$ ✓$-9\ln x$ ✓k **[4 punte / *marks*]**
8.2 (b)	$\int \sin^5 2x \,.\cos 2x\ dx$ Stel $u = \sin 2x \implies \frac{du}{dx} = 2\cos 2x \implies \frac{1}{2}du = \cos 2x\ dx$ $\int \frac{1}{2}u^5\ du = \frac{1}{12}u^6 + k$ $\therefore \frac{1}{12}\sin^6 2x + k$	✓Stel $u = \sin 2x$ ✓$\frac{1}{2}du = \cos 2x\ dx$ ✓$\int \frac{1}{2}u^5\ du$ ✓$\frac{1}{12}u^6$ ✓$\frac{1}{12}\sin^6 2x + k$ Volpunte vir net antwoord. **[5 punte / *marks*]**
8.2 (c)	$\int \sin 5x \cos 2x\ \ dx$ $= \frac{1}{2}\int \sin 3x + \sin 7x\ \ dx$ $= -\frac{1}{2}.\frac{1}{3}\cos 3x - \frac{1}{2}.\frac{1}{7}\cos 7x + k$	✓Vervang in formule ✓$-\cos 3x$ ✓$-\cos 7x$ ✓$\frac{1}{3}$ en $\frac{1}{7}$ **[4 punte / *marks*]**

Vraag / *Question* 9 **[20 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
9.1 (a)	$\frac{16x^2-9x+3}{(x-1)(4x^2+1)} \equiv \frac{A}{x-1}+\frac{Bx+C}{4x^2+1}$ $16x^2-9x+3 \equiv A(4x^2+1)+(Bx+C)(x-1)$ Stel $x=1$: $10=5A \implies A=2$ Stel $x=0$: $3=A-C$ $3=2-C \implies C=-1$ Stel $x=-1$ (enige waarde): $28=5A+(-2)(-B+C)$ $28=5(2)+2B-2(-1)$ $B=8$ $\therefore \frac{16x^2-9x+3}{(x-1)(4x^2+1)} \equiv \frac{2}{x-1}+\frac{8x-1}{4x^2+1}$	✓$A(4x^2+1)$ ✓$(Bx+C)(x-1)$ ✓$A=2$ ✓$C=-1$ ✓$B=8$ ✓Antwoord
	OF / *OR*	**OF / *OR***
	$\frac{16x^2-9x+3}{(x-1)(4x^2+1)} \equiv \frac{A}{x-1}+\frac{Bx+C}{4x^2+1}$ $16x^2-9x+3 \equiv A(4x^2+1)+(Bx+C)(x-1)$ $\equiv 4Ax^2+A+Bx^2-Bx+Cx-C$ $\equiv x^2(4A+B)+x(C-B)+(A-C)$ Stel $x=1$: $10=5A \implies A=2$ $4A+B=16$ $C-B=-9$ $A-C=3$ $B=8$ $C=-1$ $\therefore \frac{16x^2-9x+3}{(x-1)(4x^2+1)} \equiv \frac{2}{x-1}+\frac{8x-1}{4x^2+1}$	✓$A(4x^2+1)$ ✓$(Bx+C)(x-1)$ ✓$A=2$ ✓$C=-1$ ✓$B=8$ ✓Antwoord **[6 punte / *marks*]**
9.1 (b)	$\int \frac{16x^2-9x+3}{(x-1)(4x^2+1)}dx$ $=\int \frac{2}{x-1}\,dx+\int \frac{8x}{4x^2+1}\,dx-\int \frac{1}{4x^2+1}\,dx$ $=2\ln\lvert x-1\rvert+\ln\lvert 4x^2+1\rvert-\frac{1}{2}\text{bgtan}\,2x+k$	✓Split tweede breuk ✓$2\ln\lvert x-1\rvert$ ✓$\ln\lvert 4x^2+1\rvert$ ✓$-\frac{1}{2}\text{bgtan}\,2x$ **[4 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
9.2	$\int_1^3 x^2 + x \; dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \,.\, \Delta x_i$	
	$\Delta x_i = \frac{3-1}{n} = \frac{2}{n}$	✓$\frac{2}{n}$
	$x_i = 1 + \frac{2}{n} i$	✓$1 + \frac{2}{n} i$
	$f(x_i) = \left(1 + \frac{2}{n} i\right)^2 + \left(1 + \frac{2}{n} i\right)$	✓$f(x_i)$
	$= 1 + \frac{4i}{n} + \frac{4i^2}{n^2} + 1 + \frac{2i}{n}$ $= 2 + \frac{6i}{n} + \frac{4i^2}{n^2}$	✓Vereenvoudig $f(x_i)$
	$f(x_i).\Delta x_i = \left(2 + \frac{6i}{n} + \frac{4i^2}{n^2}\right).\left(\frac{2}{n}\right) = \frac{4}{n} + \frac{12i}{n^2} + \frac{8i^2}{n^3}$	
	$\therefore \int_2^3 x^2 + x \; dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{4}{n} + \frac{12i}{n^2} + \frac{8i^2}{n^3}\right]$	
	$= \lim_{n\to\infty} \left[\frac{4}{n}\sum_{i=1}^{n} 1 + \frac{12}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$	✓n
	$= \lim_{n\to\infty} \left[\frac{4}{n} \times n + \frac{12}{n^2} \times \left(\frac{n^2}{2} + \frac{n}{2}\right)\right.$	✓$\frac{n^2}{2} + \frac{n}{2}$
	$\left. + \frac{8}{n^3} \times \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)\right]$	✓$\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$
	$= \lim_{n\to\infty} \left[4 + 6 + \frac{6}{n} + \frac{8}{3} + \frac{4}{n} + \frac{8}{6n^2}\right] = \frac{38}{3} = 12.67$	✓✓ Vereenvoudig ✓Antwoord
	OF	**[10 punte / *marks*]**
	$f(x_i) = 2 + \frac{6i}{n} + \frac{4i^2}{n^2}$	
	$\sum_{i=1}^{n} \left[2 + \frac{6i}{n} + \frac{4i^2}{n^2}\right] = 2n + \frac{6}{n}\left(\frac{n^2}{2} + \frac{n}{2}\right) + \frac{4}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$	
	$= \frac{19n}{3} + \frac{10}{n} + \frac{2}{3n}$	
	$\therefore \int_2^3 x^2 + x \; dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{38}{3} + \frac{10}{n} + \frac{4}{3n^2}\right]$	

Vraag / *Question* 10 [16 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
10.1 (a)	$f(x)=\ln x \qquad g'(x)=x$ $f'(x)=\frac{1}{x} \qquad g(x)=\frac{1}{2}x^2$ $\int x.\ln x\ dx=\frac{1}{2}x^2\ln x-\frac{1}{2}\int\frac{1}{x}.x^2\ dx$ $=\frac{1}{2}x^2\ln x-\frac{1}{2}\int x\ dx$ $=\frac{1}{2}x^2\ln x-\frac{1}{2}.\frac{1}{2}x^2+k$	✓$f(x)$ ✓$f'(x)$ ✓$g(x)$ ✓$g'(x)$ ✓Vervang in formule ✓$\frac{1}{2}\int x\ dx$ ✓Antwoord **[7 punte / *marks*]**
10.1 (b)	$\int_1^4 x.\ln x\ dx=\frac{1}{2}x^2\ln x-\frac{1}{2}.\frac{1}{2}x^2\Big\vert_1^4$ $=\frac{1}{2}(4)^2\ln 4-\frac{1}{4}(4)^2-\frac{1}{2}(1)^2\ln 1+\frac{1}{4}(1)^2$ $=8\ln 4-\frac{15}{4}=7.34$	✓Bepaalde integraal ✓Vervang grense in ✓Antwoord **[3 punte / *marks*]**
10.2	$72\pi=\pi\int_{-6}^{6}9-\frac{9}{a}x^2\ dx$ $72=\left[9x-\frac{9}{3a}x^3\right]_{-6}^{6}$ $72=9(6)-\frac{3}{a}(6)^3-\left[9(-6)-\frac{3}{a}(-6)^3\right]$ $72=108-\frac{648}{a}-\frac{648}{a}$ $-36=-\frac{1296}{a}$ $a=36$	✓Funksie ✓$9x$ ✓$-\frac{9}{3a}x^3$ ✓Vervang grense in ✓Vereenvoudig ✓Antwoord **[6 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -