α-MATHEMATICS

Alpha Mathematics

Prelim examination paper

August 2019 **Time: 3 hours**

Grade 12

Examiner: Me R Grobler
Moderator: Me A Muller **Total: 200 marks**

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the examination paper:

1. Answer ALL 10 questions on this examination paper.
2. Write your name and ID number on the front page of the answer sheet.
3. Non-programmable calculators may be used, unless otherwise indicated in the question.
4. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places
5. The diagrams on the question paper are not necessarily drawn to scale.
6. All angles are given in radians. Answers must be given in radians if necessary.
7. This examination paper consists of 8 pages and a formula sheet of 3 pages. There is also an answer sheet consisting of 2 pages.
8. Question 1 consists of 10 multiple choice questions. Answer it on the answer sheet.
9. For all other questions, all necessary calculations must be shown clearly. The correct answer alone will not necessarily lead to full marks.
10. Write neatly and legibly.

Question 1 [20 marks]

Answer this question **on the answer sheet** by marking a X (cross) on A, B, C or D. Each question counts 2 marks.

1.1 For which value(s) of x is $-\frac{6}{|x-2|} \leq 3$:

(A) No real values

(B) $x < 4$ of $x > 0$

(C) $-4 < x < 4$

(D) $x \in \mathbb{R}, x \neq 2$

1.2 Which of the following graphs has an asymptote $y = 1$?

(A) $y = lnx - 1$ **(B)** $y = \frac{x}{(x+1)}$ **(C)** $y = e^x$ **(D)** $y = e^{x-1}$

1.3 The graph of $y = arctan(x + 2) - \frac{\pi}{2}$ has a horizontal asymptote at

(A) $x = 0$ **(B)** $x = \pi$ **(C)** $x = 2$ **(D)** $x = -2$

1.4 For a continuous and differential function $y = f(x)$ the following is given: $f'(4) = 0$ and $f''(4) = 2$. Which statement is true: f has a … … at $x = 4$:

(A) point of inflection **(B)** maximum turning point

(C) minimum turning point **(D)** x-intercept

1.5 What is the value of r in the expansion of $\left(4x - \frac{2}{x}\right)^8$ when the coefficient of x^2 must be determined?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

1.6 The function f is defined as follow:

$$f(x) = \begin{cases} \sin x, & x < 0 \\ x^2, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x < 2 \\ x - 3, & x \geq 2 \end{cases}$$

For which value of x will f NOT be continuous?

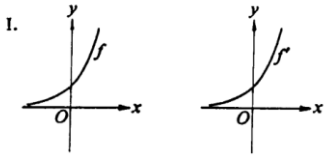
(A) 0 **(B)** 1 **(C)** 2 **(D)** none

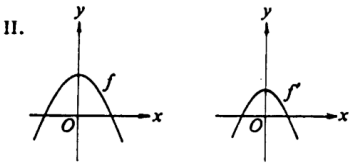
1.7 If $\lim_{x\to 3} f(x) = 7$, which statement is true:

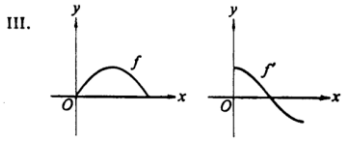
(A) f is continuous at $x = 3$. **(B)** f is differentiable at $x = 3$.

(C) $f(3) = 7$. **(D)** None of A. B or C is true.

1.8 Which of the following pairs of graphs can represent the graph of a function **AS WELL AS** the graph of its derivative:

I. 

II. 

III. 

(A) Only I **(B)** All

(C) I and II **(D)** I and III

1.9 If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]\,dx =$:

(A) $\int_3^5 g(x)dx + 7$ **(B)** $2\int_3^5 g(x)dx + 7$

(C) $2\int_3^5 g(x)dx + 14$ **(D)** $2\int_3^5 g(x)dx$

1.10 Given that $f(2) = 1$ and $f'(2) = 2$. The function f is differentiable for all real values x. Determine the derivative of $ln(f(x))$ at $x = 2$.

(A) 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{ln2}$

Question 2 [20 marks]

2.1 A new fridge is turned on. The temperature inside the refrigerator can be determined with the equation:

$$T(t) = ae^{-\frac{t}{2}} + 6$$

where T is the temperature in °C inside the refrigerator t hours after it is switched on.

(a) The initial temperature in the refrigerator is 18°C. Determine the value of a. (2)

(b) Determine the temperature in the refrigerator after 2 hours.
Use the value $a = 12$. (2)

(c) Calculate after how many hours the refrigerator will reach a temperature of 7°C.
Give the answer as an integer. (4)

(d) What is die minimum temperature that the refrigerator can reach? (1)

2.2 The function $f(x) = 3x^3 - 11x^2 - 2x + 20$ has a zero at $x = 1 + \sqrt{5}$.
Determine the rational x-intercept of the graph of f. (5)

2.3 Expand $\frac{1}{\sqrt[3]{1+3x}}$ to four terms by using the power series. Simplify your answer. (6)

Question 3 [16 marks]

3.1 (a) Given that $z = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$. Write z in polar form as $r(cos\theta + isin\theta)$.
Use root form and π if necessary. (2)

(b) Hence determine $z - \frac{1}{z}$ in polar form by using de Moivre's theorem. (4)

(c) Convert the answer to rectangular form. (1)

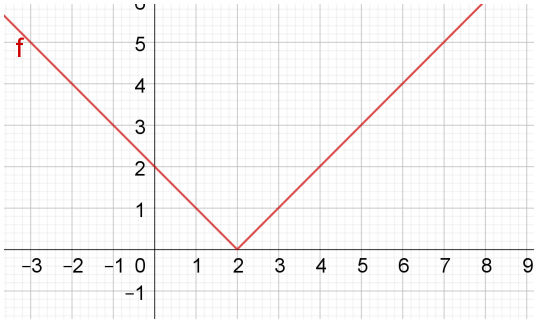
3.2 Use **mathematical induction** and prove that

$$(1 \times 0) + (2 \times 1) + (3 \times 2) + \cdots . + n(n-1) = \frac{n(n^2-1)}{3}$$

(9)

Question 4 [21 marks]

4.1 (a) Do this question on the sketch on the answer sheet.
The following sketch shows the graph of $f(x) = |x - 2|$.



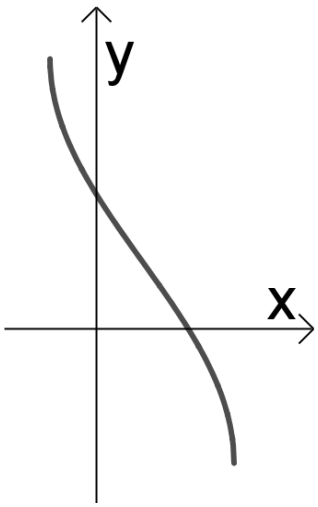
Sketch on the same set of axes the graph of $g(x) = -|x - 3| + 3$.
Clearly show the intercepts with the axes and the coordinates of the salient point. (6)

(b) Use the sketch and give the values of x for which $f(x) \leq g(x)$. (2)

4.2 Given the system of equations:

$ax + y = 8$ and $3x + 2ay = 15$ with solution $y = 9$ and a an integer.
Use Cramer's method and determine the value of a. Clearly show the determinants which you use and how you use Cramer's method. (6)

4.3



The sketch shows the graph of $f(x) = 2bgcos\left(x - \frac{1}{2}\right) - \frac{2\pi}{3}$.
Calculate the intercepts with the axes as well as the coordinates of the end points.
Give the answers in terms of π if necessary. (7)

Question 5 [19 marks]

5.1 Given vectors $\overline{\boldsymbol{a}} = 2i - 4j - 3k$ and $\overline{\boldsymbol{b}} = -i - 2j + 5k$.

(a) Determine the magnitudes of both vectors. (3)

(b) Use the point product and determine the angle between the vectors in radians. (4)

(c) Determine the angle between $\overline{\boldsymbol{a}} - \overline{\boldsymbol{b}}$ and the y-axis. (3)

5.2 (a) If $\overline{\boldsymbol{u}} = (a; -2; 3)$ and $\overline{\boldsymbol{v}} = (2; 4; -3)$, determine $\overline{\boldsymbol{u}} \times \overline{\boldsymbol{v}}$. (5)

(b) Hence determine the value of a if $\overline{\boldsymbol{u}} \times \overline{\boldsymbol{v}}$ is perpendicular to $\overline{\boldsymbol{r}} = 15i + 5j$. (4)

Question 6 [20 marks]

6.1 Differentiate the following. It is not necessary to simplify the answers.

(a) $f'(x)$ if $f(x) = 5^{2x} \times ln(x^2)$ (5)

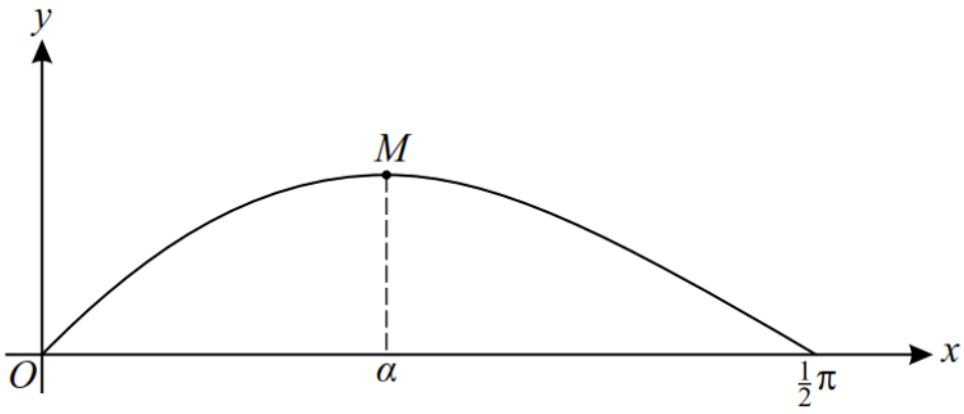
(b) $D_x\left[arctan(e^x + e^{-x})\right]$ (4)

6.2 (a) $x\sqrt[3]{y} - y\sqrt[3]{x} = 0$. Use **implicit differentiation** and determine $\frac{dy}{dx}$. (7)

(b) Determine the equation of the tangent to the graph at the point (1; 1). (4)

Question 7 [22 marks]

7.1



The diagram shows the graph of $y = \frac{sin2x}{x+2}$ for $0 \leq x \leq \frac{\pi}{2}$.

(a) Determine $\frac{dy}{dx}$. (4)

(b) Show that $\frac{dy}{dx} = 0$ can simplify to $tan2x = 2x + 4$. (3)

(c) The x-coordinate of the maximum M in this region is $\boldsymbol{\alpha}$.
Show that α, which is this equation's solution, lies between 0,6 and 0,7. (2)

(d) Use Newton's method and the equation in question 7.1(b) to determine the value of α, correct to four decimal digits. Clearly show how you use Newton's method. (4)

7.2 (a) $f(x) = ln(7 - x^3)$. Determine $f''(x)$ and use it to show that the x-value of a point of inflection of f can be $x = 0$. (6)

(b) Show that this is indeed a point of inflection. (3)

Question 8 [14 marks]

$f(x) = \frac{(x-2)(x^2+x-1)}{(x+2)(x-2)}$

8.1 Give a reason why f is not continuous in the point $x = 2$. (2)

8.2 Determine all possible asymptotes of f. (4)

8.3 Do this question on the answer sheet on the graph paper provided.

The x-intercepts of $f(x)$ are at $x = 0{,}6$ and $x = -1{,}6$. The y-intercept is at $y = -0{,}5$. The function has a maximum turning point at $(-3; -5)$ and a minimum turning point at $(-1; -1)$. Use this information as well as the information obtained in questions 8.1 and 8.2 and sketch the graph of $y = f(x)$.

Clearly show the intercepts with the axes, the asymptotes and the turning points on your sketch. (8)

Question 9 [35 marks]

9.1 Determine the following integrals:

(a) $\int \left(\frac{2}{xln5}\right) + \cot^2 x)dx$ (5)

(b) $\int (\frac{(ln(5x))^5}{x}\ dx$ (5)

(c) $\int \cos^2(2x)\ dx$ (4)

9.2 (a) Decompose $\frac{2x^2-5x+8}{x(x^2+4)}$ in partial fractions. (6)

(b) Hence determine $\int \frac{2x^2-5x+8}{x(x^2+4)}dx$ (5)

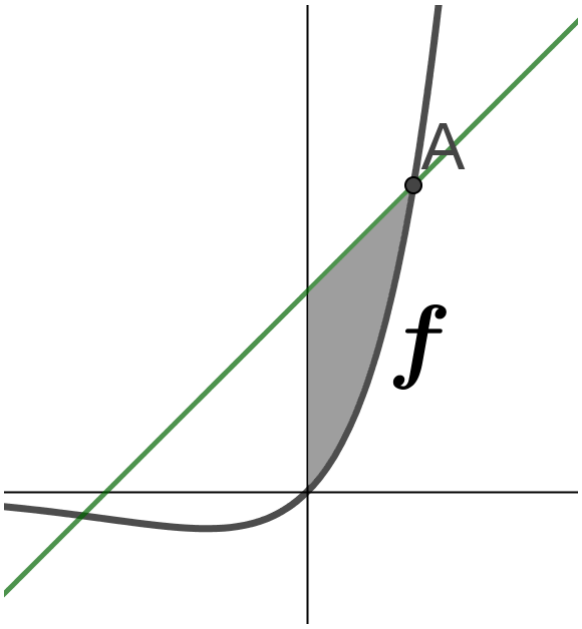
9.3 Determine the value of $\int_1^4 (2x^2 - 1)\ dx$ by using a **Riemann sum**. (10)

Question 10 [13 marks]

No calculators may be used in this question. Show all the steps of integration.

10.1 Use factor integration and determine $\int x.e^x \ dx$. (6)

10.2 The sketch shows the graph of $f(x) = x.e^x$, the line $g(x) = x + 2$ as well as the area included between the graphs and the y-axis. The graphs intersect at **A(1; 3)** (rounded to integers).



Use your answer determined in question 10.1 and calculate this area. (7)

- END OF THE QUESTION PAPER –

2019

Alpha Mathematics

Name: _____

ID:

School: _____

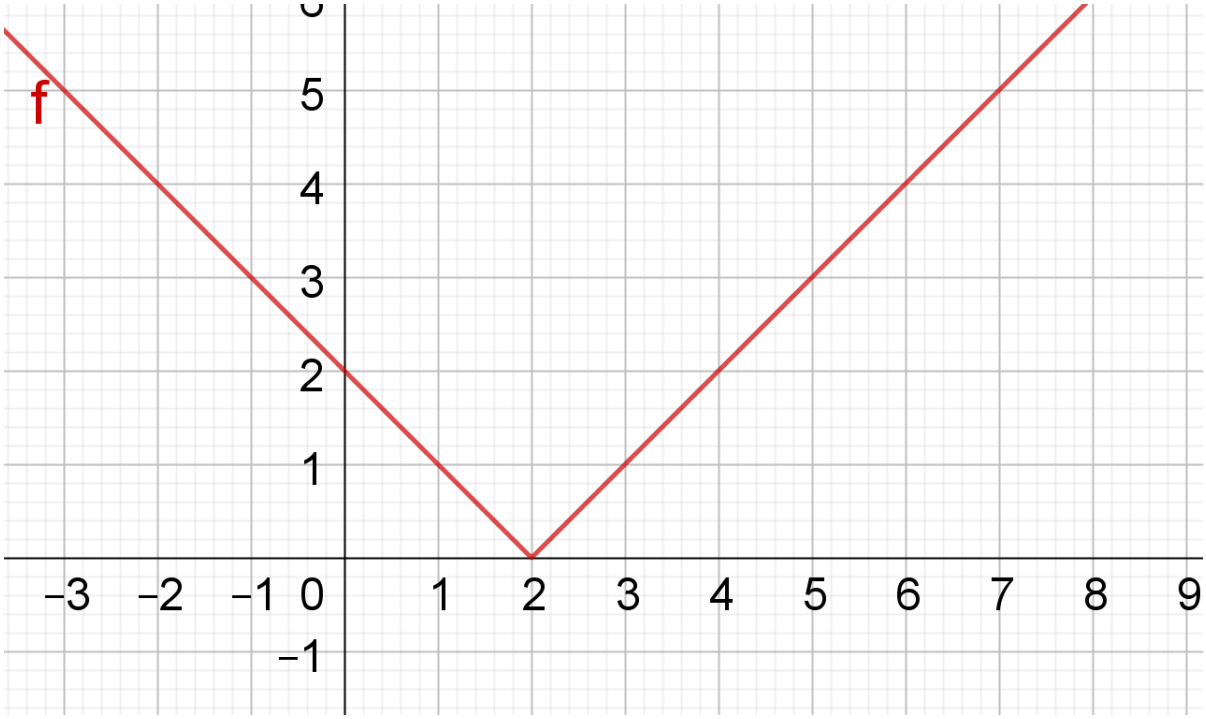
Answer sheet: VRAAG 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Marks

QUESTION	Possible mark	Mark
1	20	
2	20	
3	16	
4	21	
5	19	
6	20	
7	22	
8	14	
9	35	
10	13	
Total	200	

Question 4.1



Question 8.4