α-WISKUNDE

Alpha Wiskunde Graad 12 / *Alpha Mathematics Grade 12*

Halfjaar eksamen 2019 / *Half year examination 2019*

MEMORANDUM

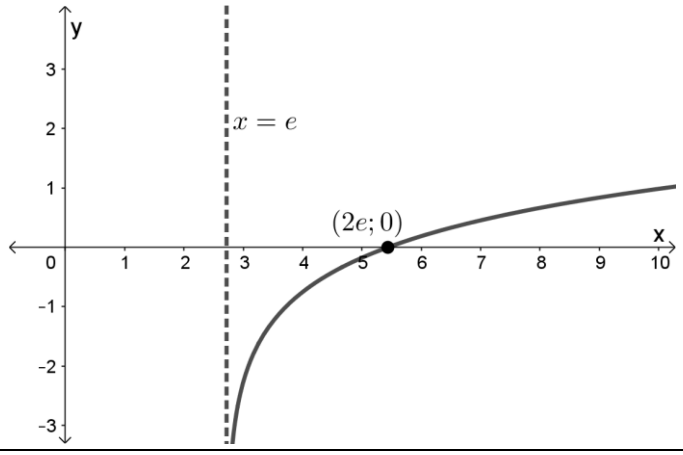
Totaal / *Total*: 200 punte / *marks*

Hierdie memorandum bestaan uit 11 bladsye. /
This memorandum consists of 11 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	A	$y = e^{2x}$ $\quad \frac{dy}{dx} = 2e^{2x}$ $m_{(0;1)} = 2$ $\quad y = 2x + 1$	**2**
1.2	C	$f'(x) > 0$ $2x + 2 > 0$ $x > -1$	
1.3	A	$f(x) = \log_2 e^x$ $f'(x) = \frac{e^x}{e^x . \ln 2} = \frac{1}{\ln 2}$	**2**
1.4	D	$y = 4e^{2x}$ $\frac{x}{4} = e^{2y}$ $y = \frac{1}{2}\ln\left(\frac{x}{4}\right)$	**2**
1.5	B	$f'(x) = 3x^2 + 2xa + b$ $f''(x) = 6x + 2a = 0$ $6(1) + 2a = 0$ $a = -3$ $6 = 1^3 - 3 + b + 1$ $b = 7$	**2**
1.6	A	$\begin{vmatrix} 2 & 3 \\ -1 & a \end{vmatrix} = 0$ $2a + 3 = 0$ $a = -\frac{3}{2}$	**2**
1.7	C	$y = \lim_{x \to 0} \frac{x^3\left(\frac{1}{x} + \frac{5}{x^2}\right)}{x^3\left(2 - \frac{1}{x} + \frac{4}{x^3}\right)}$ $y = 0$	
1.8	B	$(1 - i) = \sqrt{2}\, e^{-\frac{\pi}{4}i}$ $(1 - i)^4 = \left(\sqrt{2}\right)^4 e^{4 \times \frac{-\pi}{4}i} = 4e^{-\pi i}$	**2**
1.9	B	Lokale maksimum	**2**
1.10	D	$\ln f(x) = e^x \ln x$ $\frac{1}{f(x)} f'(x) = e^x \ln x + e^x \frac{1}{x}$ $\frac{1}{f(1)} f'(1) = e \ln 1 + e$ $f'(1) = e$ $\ln f(1) = e \ln(1)$ $f(1) = e^0 = 1$	**2**

Vraag / *Question* 2 **[20 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1 (a)	$e^{2-x} = 6$ $ln\, e^{2-x} = ln\, 6$ $(2-x)\, ln\, e = ln\, 6$ $x = 2 - ln\, 6$	✓ ln beide kante ✓ ln wet $(2-x)$ ✓ Antwoord **[3 punte / *marks*]**
2.1 (b)	$\frac{e^x}{e^x - 1} = 9$ $e^x = 9e^x - 9$ $8e^x = 9$ $e^x = \frac{9}{8}$ $x\, ln\, e = ln\frac{9}{8}$ $x = ln\frac{9}{8}$	✓ Vereenvoudig ✓ ln beide kante ✓ Antwoord **[3 punte / *marks*]**
2.1 (c)	$(ln\, x)^2 = ln\, e^2 x$ $(ln\, x)^2 = \ln e^2 + \ln x$ $(ln\, x)^2 - \ln x - 2 = 0$ $(ln\, x - 2)(ln\, x + 1) = 0$ $ln\, x = 2$ of $ln\, x = -1$ $x = e^2$ $\quad$ $x = e^{-1}$	✓ ln wet ✓ $ln\, e^2 = 2$ ✓✓ Faktoriseer ✓✓ Antwoorde **[6 punte / *marks*]**
2.2 (a)	$x = e$	✓✓ Antwoord **[2 punte / *marks*]**
2.2 (b)	x-afsnitte: $ln(x-e) - 1 = 0$ $ln(x-e) = 1$ $x - e = e^1$ $x = 2e = 5.44$ y-afsnit: $y = ln(0-e) - 1$ = geen oplossing 	✓ Asimptoot ✓✓✓ x-afsnit ✓✓ Vorm **[6 punte / *marks*]**

Vraag / *Question* 3 **[24 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1 (a)	4000 kg	✓✓ Antwoord **[2 punte / *marks*]**
3.1 (b)	$\frac{dG}{dt} = \frac{0 + 8000(1.1^t \ln 1.1)}{(1.05 + 1.1^t)^2}$ Stygend as $\frac{dG}{dt} > 0$ $8000 \times 1.1^t \times \ln 1.1 > 0$ $1.1^t > 0$ $\Rightarrow 1.1^t > 0$ vir alle waardes van t. $\therefore G$ (gewig) neem voortdurend toe.	✓✓✓ $\frac{dG}{dt}$ (elke term) ✓ $8000 \times 1.1^t \times \ln 1.1 > 0$ ✓ $1.1^t > 0$ ✓ Stygend vir alle waardes van t **[6 punte / *marks*]**
3.1 (c)	$\left.\frac{dG}{dt}\right\|_{t=10} = 148.96$	✓✓ Antwoord **[2 punte / *marks*]**
3.1 (d)	$G = 4000 - \frac{8000}{1.05 + 1.1^t}$ $(G - 4000)(1.05 + 1.1^t) = -8000$ $1.1^t = \frac{8000}{4000 - G} - 1.05$ $1.1^t = \frac{8000 - 4200 + 1.05G}{4000 - G}$ $t = \log_{1.1}\left(\frac{3800 + 1.05G}{4000 - G}\right)$ Ouderdom van 3000 kg olifant: $\therefore t = 20.34$	✓ Vereenvoudig ✓ Vereenvoudig ✓ Alles op een KGV ✓ Kry t alleen ✓ Vervang 3000 in t ✓ Antwoord **[6 punte / *marks*]**
3.2 (a)	$\det A = \begin{vmatrix} b-1 & -3 \\ b+3 & -9 \end{vmatrix} = -9(b-1) + 3(b+3) = 18 - 6b$ $\det A_x = \begin{vmatrix} 3 & -3 \\ 11 & -9 \end{vmatrix} = -27 + 33 = 6$ $\therefore x = \frac{6}{18 - 6b}$	✓ $\det A$ ✓ Bepaal determinant ✓ $18 - 6b$ ✓ $\det A_x$ ✓ 6 ✓ Finale antwoord x **[6 punte / *marks*]**
3.2 (b)	$1 = \frac{6}{18 - 6b}$ $18 - 6b = 6$ $\therefore b = 2$	✓ Vervang x in ✓ Antwoord **[2 punte / *marks*]**

Vraag / *Question* 4 **[16 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1 (a)	$10\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\times 5\left(\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}\right)$ $=50\left(\cos\left(\frac{\pi}{3}+\frac{5\pi}{2}\right)+i\sin\left(\frac{\pi}{3}+\frac{5\pi}{2}\right)\right)$ $=50e^{\frac{17\pi}{6}i}$ **OF** $=10\,\text{cis}\frac{\pi}{3}\times 5\,\text{cis}\frac{5\pi}{2}$ $\quad=10e^{\frac{\pi}{3}i}\times 5e^{\frac{5\pi}{2}i}$ $=50\,\text{cis}\left(\frac{\pi}{3}+\frac{5\pi}{2}\right)$ $\quad=50e^{\left(\frac{\pi}{3}+\frac{5\pi}{2}\right)i}$ $=50e^{\frac{17\pi}{6}i}$ $\quad=50e^{\frac{17\pi}{6}i}$	✓ 50 ✓ $\frac{\pi}{3}+\frac{5\pi}{2}$ ✓ $e^{\frac{17\pi}{6}i}$ (maak seker die i is in die antwoord) **[3 punte / *marks*]**
4.1 (b)	$4\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)\div\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ $=4\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)\div\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ $=4\left(\cos\left(-\frac{\pi}{4}-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}-\frac{\pi}{4}\right)\right)$ $=4e^{-\frac{\pi}{2}i}$ **OF** $=4\,\text{cis}\left(-\frac{\pi}{4}\right)\div\text{cis}\left(\frac{\pi}{4}\right)$ $\quad=4e^{-\frac{\pi}{4}i}\div e^{\frac{\pi}{4}i}$ $=4\,\text{cis}\left(-\frac{\pi}{4}-\frac{\pi}{4}\right)$ $\quad=4e^{\left(-\frac{\pi}{4}-\frac{\pi}{4}\right)i}$ $=4e^{-\frac{\pi}{2}i}$ $\quad=4e^{-\frac{\pi}{2}i}$	✓ $-\frac{\pi}{4}$ ✓ 4 ✓ $-\frac{\pi}{4}-\frac{\pi}{4}$ ✓ $4e^{-\frac{\pi}{2}i}$ (maak seker die i is in die antwoord) **[4 punte / *marks*]**
4.2	$i=2\,\text{cis}\frac{\pi}{2}$ $-1-\sqrt{3}i=2\,\text{cis}\frac{4\pi}{3}$ $\left(\frac{2\,\text{cis}\frac{\pi}{2}}{2\,\text{cis}\frac{4\pi}{3}}\right)^{12}=\left[\text{cis}\left(\frac{\pi}{2}-\frac{4\pi}{3}\right)\right]^{12}$ $=\text{cis}\left(-\frac{5\pi}{6}\times 12\right)$ $=\text{cis}(-10\pi)$ $x=\cos(-10\pi)=1$ $y=\sin(-10\pi)=0$ Dus geen reële deel nie, want $y=0$.	✓ $2\,\text{cis}\frac{\pi}{2}$ ✓ 2 ✓ $\text{cis}\frac{4\pi}{3}$ ✓ $\text{cis}\left(\frac{\pi}{2}-\frac{4\pi}{3}\right)$ ✓ $\text{cis}\left(-\frac{5\pi}{6}\times 12\right)$ ✓ $\text{cis}(-10\pi)$ ✓ $x=1$ ✓ $y=0$ ✓ Antwoord **[9 punte / *marks*]**

Vraag / *Question* 5 **[24 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$(2x+y)^2 = 16x$ $2(2x+y)\left(2+1.\frac{dy}{dx}\right) = 16$ $\frac{dy}{dx} = \frac{16-4(2x+y)}{2(2x+y)}$ $= \frac{16-8x-4y}{4x+2y}$ **OF** $(2x+y)^2 = 16x$ $4x^2+4xy+y^2 = 16x$ $8x+4y+4x\frac{dy}{dx}+2y\frac{dy}{dx} = 16$ $\frac{dy}{dx} = \frac{16-8x-4y}{4x+2y}$	✓ 2 ✓ $(2x+y)$ ✓ 2 ✓ 1 ✓ $\frac{dy}{dx}$ ✓ 16 ✓ Kry $\frac{dy}{dx}$ alleen ✓ Punt vir elke term **[7 punte / *marks*]**
5.2	Horisontale raaklyn $\Rightarrow m = 0$ $\left.\frac{dy}{dx}\right\|_{(a;2)} = 0$ $\frac{16-8a-4(2)}{4a+2(2)} = 0$ $16-8a-8 = 0$ $-8a = -8$ $a = 1$	✓ Gradiënt $= 0$ ✓ Vervang in $\frac{dy}{dx}$ ✓ Antwoord **[3 punte / *marks*]**
5.3	$\frac{d^2y}{dx^2} = \frac{\left(0-8-4\frac{dy}{dx}\right)(4x+2y)-(16-8x-4y)\left(4+2\frac{dy}{dx}\right)}{(4x+2y)^2}$ $\left.\frac{dy}{dx}\right\|_{(1;;2)} = 0$ $\left.\frac{d^2y}{dx^2}\right\|_{(1;2)} = -1 < 0$ Lokale maksimum by $(1;2)$	✓ $0-8$ ✓ $-4\frac{dy}{dx}$ ✓ Kwosiënt reël ✓ 4 ✓ $2\frac{dy}{dx}$ ✓ Vervang $(1;2)$ in ✓ Antwoord -1 ✓ Lokale maskimum **[8 punte / *marks*]**
5.4	$(2(4)++y)^2 = 16(4)$ $(8+y)^2 = 64$ $8+y = \pm 8$ $y = 0$ of $y = -16$ $\therefore (4;0)$ $m_{\text{raaklyn}} = \left.\frac{dy}{dx}\right\|_{(4;0)} = -1$ $m_{\text{normaal}} = 1$ (loodreg op raaklyn) $\therefore y = x-4$	✓ Vervang koördinate in ✓ $y = 0$ ✓✓ $\frac{dy}{dx} = -1$ ✓ Gradiënt normaal $= 1$ ✓ $y = x-4$ **[6 punte / *marks*]**

Vraag / *Question* 6 **[22 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1	x-afsnitte: $(x-3)(x+2)^2=0$ $\therefore$ A $(-2;0)$ $\therefore$ D $(3;0)$ y-afsnit: $y=(0-3)(0+2)^2=-12$ $\therefore$ B $(0;-12)$ Stasionêre punte: $\frac{d}{dx}f'(x)=0$ $(x+2)^2+2(x-3)(x+2)=0$ $x^2+4x+4+2x^2-2x-12=0$ $3x^2+2x-8=0$ $(x+2)(3x-4)=0$ $x=-2$ of $x=\frac{4}{3}$ $y=\left(\frac{4}{3}-3\right)\left(\frac{4}{3}+2\right)^2=-\frac{500}{27}=-18\frac{14}{27}=-18.52$ $\therefore$ C $\left(\frac{4}{3};-18.52\right)$	✓ A ✓ D ✓ B ✓ Afgeleide $=0$ ✓✓ Differensieer ✓ Vereenvoudig ✓ $x=\frac{4}{3}$ ✓ $y=18.52$ **[9 punte / *marks*]**
6.2	$x=-2$: Buigpunt (f' draai op die x-as en $f'(x)<0$ links en regs van $x=-2$) $x=3$: Lokale minimum (f' verander van 'n negatiewe na positiewe teken)	✓ $x=-2$ ✓✓ Buigpunt ✓ $x=3$ ✓ Lokale minimum **[5 punte / *marks*]**
6.3	$x=-2, x=\frac{4}{3}$	✓✓ Antwoorde **[2 punte / *marks*]**
6.4	$\frac{d^2}{dx^2}[f'(x)]=6x+2$ $6x+2=0$ $x=-\frac{1}{3}$	✓ Kry 2de afgeleide van f' ✓ $6x+2$ ✓ Stel gelyk aan 0 ✓ Antwoord **[4 punte / *marks*]**
6.5	$x\in\mathbb{R};\ x\leq 3$	✓✓ Antwoord **[2 punte / *marks*]**

Vraag / *Question* 7 **[14 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	y-afsnit is $\frac{2}{9}$.	✓✓ Antwoord **[2 punte / *marks*]**
7.2	$x \in \mathbb{R}; x \neq 3; x \neq -1$	✓ $x \in \mathbb{R}$ ✓ $x \neq 3$ ✓ $x \neq -1$ **[3 punte / *marks*]**
7.3 (a)	Geen	✓ Antwoord **[1 punte / *marks*]**
7.3 (b)	$x = 3$	✓✓ Antwoord **[2 punte / *marks*]**
7.3 (c)	$y = -5x$	✓✓ Antwoord **[2 punte / *marks*]**
7.4 (a)	$f'(x) > 0$; $f''(x) < 0$	✓✓ Antwoorde **[2 punte / *marks*]**
7.4 (b)	$f'(x) < 0$; $f''(x) > 0$	✓✓ Antwoorde **[2 punte / *marks*]**

Vraag / *Question* 8 **[18 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1	Grafiek **C** $y=\frac{x^2-9}{x+3}=\frac{(x-3)(x+3)}{x+3}=x-3\,;x\neq -3$ • x-afsnit: $x=3$ • Verwyderbare diskontinuïteit by $(-3;-6)$	✓✓✓ Grafiek C **[3 punte / *marks*]**
8.2	Grafiek **B** $y=\frac{x^2+12x+27}{x-3}=\frac{(x+3)(x+9)}{x-3}$ • V/A: $x=3$ • x-afsnitte: $x=-3$ of $x=-9$ • Skuins asimptoot: $y=x+15$	✓✓✓ Grafiek B **[3 punte / *marks*]**
8.3	Grafiek **F** $y=\frac{2x^2-3x-9}{(x+3)^2}=\frac{(x-3)(2x+3)}{(x+3)^2}$ • V/A: $x=-3$ • H/A: $y=2$ • x-afsnitte: $x=3$ of $x=-\frac{3}{2}$	✓✓✓ Grafiek F **[3 punte / *marks*]**
8.4	Grafiek **G** $y=\frac{2x^3-50x}{x^2-4}=\frac{2x(x-5)(x+5)}{(x-2)(x+2)}$ • V/A: $x=\pm 2$ • x-afsnitte: $x=0; x=5; x=-5$ • Skuins asimptoot: $y=2x$	✓✓✓ Grafiek G **[3 punte / *marks*]**
8.5	Grafiek **E** $y=\frac{-2x^2}{x^2-4}=\frac{-2x^2}{(x-2)(x+2)}$ • V/A: $x=2;\ x=-2$ • H/A: $y=-2$ • x-afsnit: $x=0$	✓✓✓ Grafiek E **[3 punte / *marks*]**
8.6	Grafiek **A** $y=\frac{x-4}{x+3}$ • V/A: $x=-3$ • H/A: $y=1$ • x-afsnit: $x=4$ • y-afsnit: $y=-\frac{4}{3}$	✓✓✓ Grafiek A **[3 punte / *marks*]**

Vraag / *Question* 9 **[24 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
9.1 (a)	$f(x) = \int f'(x)\ dx$ $= \int 2^{4x+3} \ln 2\ dx$ $= \frac{2^{4x+3} \ln 2}{4.\ln 2} + k$ Vervang y-afsnit van 3 in: $3 = \frac{2^{4(0)+3}}{4} + k$ $k = 1$ $\therefore f(x) = \frac{2^{4x+3}}{2^2} + 1$ $= 2^{4x+1} + 1$	✓ Integraal ✓ 2^{4x+3} ✓ $4.\ln 2$ ✓ Vervang y-afsnit in ✓ $k = 1$ ✓ $f(x)$ ✓ Korrekte vorm **[7 punte / *marks*]**
9.1 (b)	$f''(x) = 0$ $2^{4x+3}.(\ln 2)^2.4 = 0$ $2^{4x+3} = 0$ Geen oplossing vir x. ∴ Geen buigpunt nie.	✓ $f''(x) = 0$ ✓ 2^{4x+3} ✓ $(\ln 2)^2.4$ ✓ Geen buigpunt **[4 punte / *marks*]**
9.2 (a)	$\frac{2^{5x-7}}{5.\ln 2} + k$	✓ 2^{5x-7} ✓ $\ln 2$ ✓ 5 **[3 punte / *marks*]**
9.2 (b)	$\frac{5}{2}\ln\lvert 2x\rvert + \frac{2}{5}.\frac{1}{2}x^2 + k$	✓ $\frac{5}{2}$ ✓ $\ln\lvert 2x\rvert$ ✓ $\frac{2}{5}x^2$ ✓ $\frac{1}{2}$ **[4 punte / *marks*]**
9.2 (c)	$2\log_5(x-3) + k$	✓ 5 ✓ log **[2 punte / *marks*]**
9.2 (d)	$\int e^{2x} - 2e^{-4x}\ dx$ $= \frac{e^{2x}}{2} - \frac{2e^{-4x}}{(-4)} + k$	✓✓ Vereenvoudig ✓ $\frac{e^{2x}}{2}$ ✓ $-\frac{2e^{-4x}}{(-4)}$ **[4 punte / *marks*]**

Vraag / *Question* 10 **[18 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
10.1 (a)	$m = \frac{e^{-1}-1}{1-0} = e^{-1}-1$ $y-1 = (e^{-1}-1)(x-0)$ $y = (e^{-1}-1)x+1$	✓✓ Gradiënt bereken ✓ Vervang in formule **[3 punte / *marks*]**
10.1 (b)	$\int_0^1 (e^{-1}-1)x+1-e^{-x}\ dx$ $= \left[\frac{(e^{-1}-1)x^2}{2}+x+e^{-x}\right]_0^1$ $= \left[\frac{(e^{-1}-1)}{2}+1+e^{-1}\right]-[e^0]$ $= \frac{1}{2}e^{-1}-\frac{1}{2}+1+e^{-1}-1$ $= \frac{3}{2}e^{-1}-\frac{1}{2}$	✓✓ Integraal ✓✓✓ Integreer elke term ✓ Vervang grense in ✓✓ Vereenvoudig ✓ Antwoord **[9 punte / *marks*]**
10.2	$V = \pi\int_2^3 \left(\frac{1}{4x-7}\right)\ dx$ $= \pi\left[\frac{1}{4}\ln\lvert 4x-7\rvert\right]_2^3$ $= \pi\left[\frac{1}{4}\ln 5-\frac{1}{4}\ln 1\right]$ $= \frac{\pi}{4}\ln 5$	✓ Vervang in formule ✓✓ Integreer ✓ Vervang grense in ✓ Vereenvoudig ✓ Antwoord **[6 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -