

# $\alpha$ -MATHEMATICS

## Alpha Mathematics Final Examination paper

**Reading time: 10 min**

**Writing time: 2 ½ hours**

**Total: 165 marks**

**Grade 11**

**Examination 2017**

### INSTRUCTIONS AND INFORMATION

Read the instructions carefully before answering the questions.

1. This question paper consist of 8 pages, a formula sheet of 1 page and an answering sheet of 2 pages.
2. Answer ALL eight questions.
3. Number the answers exactly the way the questions are numbered.
4. Non-programmable calculators may be used.
5. Unless otherwise indicated, all answers, where necessary, must be given correct to **two decimal figures**.
6. **All necessary calculations must be shown clearly, except in Question 1.** No marks will be awarded if only the answer is given.
7. The diagrams in the question paper are not necessarily drawn to scale.
8. All angles are given in radians. Answers must also be given in radians if necessary.
9. Write neatly and legible.

**Question 1****[20 marks]**

This question must be answered on the **answer sheet**, provided.

Each question has **ONLY** one correct answer and is worth two (2) points. Mark the correct answer with an **X** on the answer sheet.

1.1 Solve for  $x$  in  $-|x - 5| \geq 5$

- (A)  $x \in \mathbb{R}$
- (B)  $x \leq 0$  or  $x \geq 10$
- (C)  $0 \leq x \leq 10$
- (D) None of the above options.

1.2 Given  $p(x) = x^3 + ax^2 + 4x + 20$  and  $(x^2 + 4)$  is a factor of  $p(x)$ , then...

- (A)  $a = 5$
- (B)  $a = 4$
- (C)  $a = -4$
- (D)  $a = -5$

1.3 How many terms does the expansion of  $x(1 + 2x)^{10}$  have?

- (A) 10 terms
- (B) 11 terms
- (C) 12 terms
- (D) None of the above options.

1.4 The **area** enclosed under the graph of  $y = \cos x$  and the  $x$ -axis between the points  $x = 0$  and  $x = \pi$  is equal to...

- (A) 2
- (B) 1
- (C) 0
- (D)  $\pi$

1.5 For which values of  $x$  will the expansion of  $\frac{1}{\sqrt{1-4x^2}}$  converge:

- (A)  $|x| < \frac{1}{4}$
- (B)  $|x| > \frac{1}{2}$
- (C)  $|x| < \frac{1}{2}$
- (D) None of the above options.

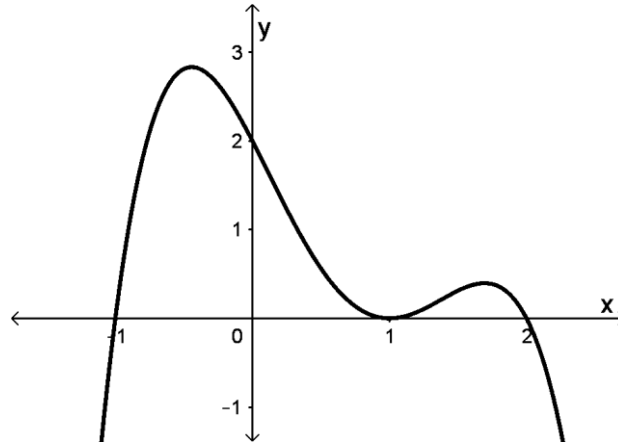
1.6 The following statement is proved by Mathematical Induction in **QUESTION 3.2**,

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Use this statement and calculate  $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{19 \times 21} =$

- (A) 0,4762
- (B) 0,0025
- (C) 0,4737
- (D) 0,4975

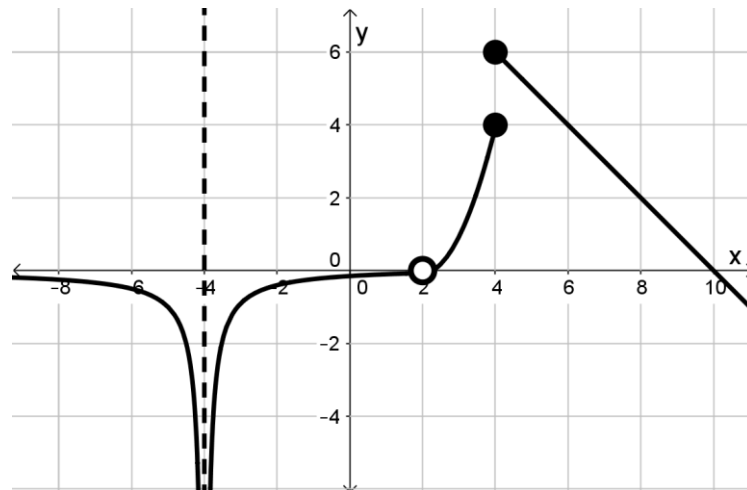
1.7 Given the following function of  $f(x)$ . The function has turning points at  $x = -\frac{1}{2}$ ,  $x = 1$  and  $x = 1,7$ .



If Newton's method was used to obtain an  $x$ -intercept of  $f(x)$  and we choose the initial value to be  $x_0 = -0.4$ , then Newton's method will tend to which  $x$ -intercept?

- (A)  $x = -1$
- (B)  $x = 1$
- (C)  $x = 2$
- (D) The chosen initial value will not tend to one of the  $x$ -intercepts.

Use the following sketch of  $f(x)$  for **QUESTION 1.8**, **QUESTION 1.9** and **QUESTION 1.10**:



1.8 Which one of the following statements are correct?

- (A) The function is continuous at  $x = 2$ .
- (B) The function has a removable discontinuity at  $x = 2$ .
- (C) The function has a jump discontinuity at  $x = 2$ .
- (D) None of the above assumptions are correct.

1.9 The following three assumptions are made with respect to  $f(x)$ :

$\lim_{x \rightarrow -4} f(x) = -\infty$	$\lim_{x \rightarrow 2} f(x) = 0$	$\lim_{x \rightarrow 4} f(x)$ exist
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- (A) Only one of the assumptions are correct.
- (B) Only two of the assumptions are correct.
- (C) All three assumptions are correct.
- (D) None of the assumptions are correct.

1.10 What is the value of  $f'(8) =$

- (A) 2
- (B) 1
- (C) -2
- (D) -1

**Question 2****[26 marks]**

2.1 Given the function  $f(x) = -|x - 2| + 3$  and  $g(x) = \frac{8}{x - 1}$ .

- (a) Determine the value of  $f(0)$ . (2)
- (b) The intersection of  $f$  and  $g$  exists only where  $x < 2$ . Calculate the coordinates of the intersection of  $f$  and  $g$ . (6)
- (c) Calculate the  $x$ -intercepts of  $f$ . (3)
- (d) The graph of  $g(x)$  is already sketched on **DIAGRAM SHEET 1**. Use this diagram sheet and sketch the graph of  $y = f(x)$ .  
Clearly show all intercepts with the axes and salient points on your sketch as well as your answer of QUESTION 2.1 (b). (5)

2.2 Given  $x^4 - 4x^2 - 8x - 4 = 0$  and  $x = 1 + \sqrt{3}$  is a root of the equation.

- (a) Determine a quadratic function for this equation. (3)
- (b) If  $x^2 - 2x - 2$  is a quadratic function, hence solve for  $x \in \mathbb{C}$  in the equation. (7)

**Question 3****[22 marks]**

3.1 Decompose  $\frac{4x^2 - x + 6}{x^3 + 2x}$  into partial fractions. (9)

3.2 Use Mathematical Induction and prove that the following statement applies to all  $n \in \mathbb{N}$ :

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \quad (13)$$

**Question 4****[14 marks]**

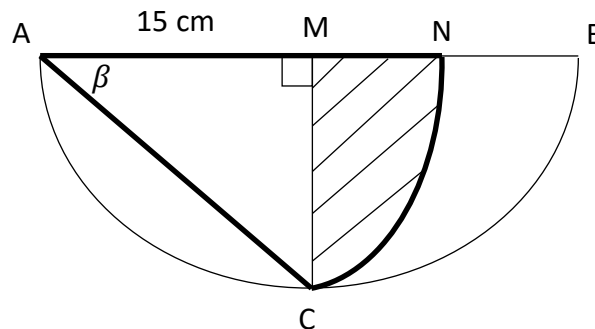
4.1 (a) Determine the coefficient of  $x^3$  and  $x$  in the expansion of  $\left(2x - \frac{3}{x}\right)^5$ . (6)

(b) **Hence**, determine the coefficient of  $x$  in  $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5$ . (3)

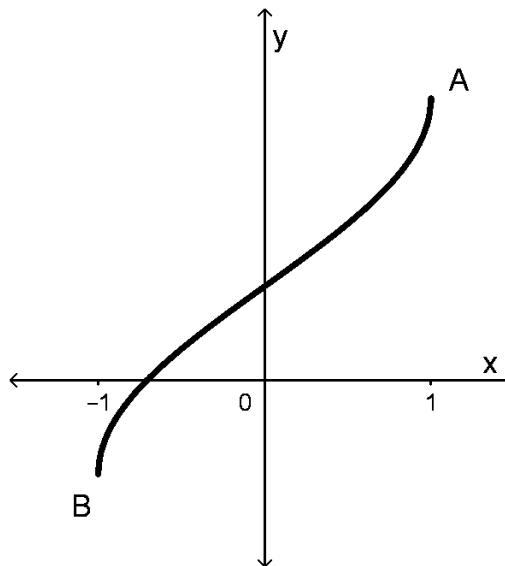
4.2 Determine and simplify only the **third** and **fourth** term in the expansion of  $\frac{1}{1-5x}$ . (5)

**Question 5****[19 marks]**

- 5.1 ABC is a semi-circle with centre M. ACN is a sector with centre A and corresponding arc NC.  $AM = 15$  cm,  $\widehat{AMC} = \frac{\pi}{2}$  radians and  $\widehat{MAC} = \beta$ .



- (a) Give a reason why  $\beta = \frac{\pi}{4}$  radians. (2)
- (b) Determine the area of the shaded region MNC. (7)
- 5.2 Given the function  $f(x) = \arccos(-x) + p$ . The function has end points at  $A(1; \frac{3\pi}{4})$  and B. The function has a y-intercept at  $(0; \frac{\pi}{4})$ .



- (a) Determine the value of  $p$ . (2)
- (b) Determine the inverse function of  $f$  and write it in the form  $f^{-1}(x) = \dots$  (4)
- (c) Give the range of the inverse function of  $f$ . (2)
- (d) Give the transformation from  $f(x)$  to the function  $y = \arcsin x$ . (2)

**Question 6****[19 marks]**

6.1 The following information are given with regards to a function  $f(x)$ :

$$\lim_{x \rightarrow 4} f(x) = 10 \quad f(4) = 0 \quad f(-2) = 5$$

Are the following assumptions/allegation below **TRUE** or **FALSE**:

(a)  $\lim_{x \rightarrow 4} f(x) = 10$  (2)

(b)  $f$  is continuous at  $x = 4$ . (2)

(c)  $\lim_{x \rightarrow -2} f(x) = f(-2)$  (2)

6.2 Given the function:

$$h(x) = \begin{cases} x^2 + ax + 3 & \text{if } x < 3 \\ a - 5 + b & \text{if } x = 3 \\ x + b & \text{if } x > 3 \end{cases}$$

Determine the values of  $a$  and  $b$  if the function  $h(x)$  is continuous at  $x = 3$ . (8)

6.3 Use Newton's method and calculate the value of  $a$  in  $a^5 - 9a - 2 = 0$  correct to 3 decimal places. Show all calculations. Use  $a_0 = 1.8$  as the initial value. (5)

**Question 7****[20 marks]**

7.1 Differentiate  $f(x) = \sqrt{x+5}$  by using first principles. (6)

7.2 Determine the following. It is not necessary to simplify.

(a)  $f'(x)$  if  $f(x) = x\sqrt{x^2+1}$  (5)

(b)  $\frac{dy}{dx}$  if  $y = \frac{\cos x}{x^2}$  (4)

(c)  $\frac{d}{dx} [\tan^3(\arcsin 2x)]$  (5)

**Question 8****[25 marks]**

8.1 Determine the following integrals:

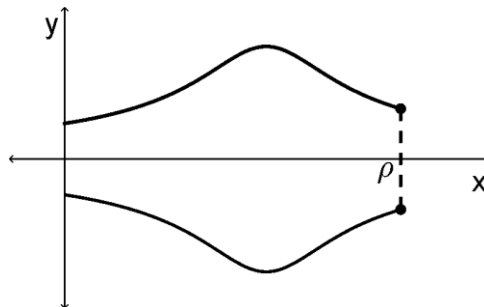
(a)  $\int 5 \operatorname{cosec} 2x \cot 2x + 10 \sec^2(5x+1) dx$  (4)

(b)  $\int \frac{2}{\sqrt{16-9x}} dx$  (4)

(c)  $\int \frac{2}{\sqrt{16-9x^2}} dx$  (5)

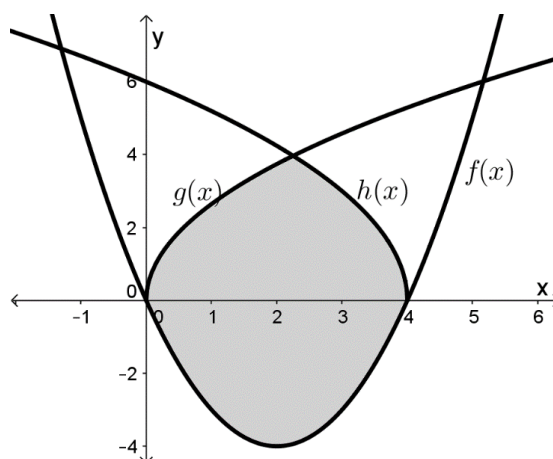
8.2 Given:  $f(x) = \sqrt{\frac{1}{x^2 - 6x + 10}}$

Determine the volume of revolution that exists if  $f$  rotates about the  $x$ -axis (as sketched below) between  $x = 0$  and  $x = \rho$ . Leave your answer in terms of  $\rho$  and an **inverse trigonometry function**.



(7)

8.3 The graph of  $f(x) = x^2 - 4x$ ,  $g(x) = \sqrt{7x}$  and  $h(x) = \sqrt{36 - 9x}$  are sketched. The region between these three graphs are shaded, as on the sketch.



**Firstly** determine the  $x$ -intersect of  $g$  and  $h$ , and then **write down** an equation that can be used to calculate the area of the shaded region.

**(Do not attempt to calculate the area!)**

(5)

**- END OF THE EXAMINATION PAPER -**

# Alpha Mathematics Formula Sheet

**ALGEBRA:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots ; \text{ if } |x| < 1$$

**CALCULUS:**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int_a^b x^n dx = \left[ \frac{x^{n+1}}{n+1} \right]_a^b$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

**TRIGONOMETRY:**

**In a sector:**  $s = r\theta$  en  $A = \frac{1}{2}r^2\theta$

**Identities:**  $\sin^2 x + \cos^2 x = 1$      $\tan^2 x + 1 = \sec^2 x$      $\cot^2 x + 1 = \operatorname{cosec}^2 x$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

In  $\Delta ABC$ :  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$      $a^2 = b^2 + c^2 - 2bc \cdot \cos A$      $\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$

**TABLE WITH DERIVATIVES:**

$F(x)$	$F'(x)$
$ax^n$	$nax^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \cdot \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$

$F(x)$	$F'(x)$
$\operatorname{bgsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arcsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\operatorname{bgcos} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\operatorname{arccos} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\operatorname{bgtan} x$	$\frac{1}{x^2+1}$
$\operatorname{arctan} x$	$\frac{1}{x^2+1}$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
$f[g(x)]$	$f'[g(x)] \cdot g'(x)$

**Alpha Mathematics Grade 11 - Final Examination 2017****ANSWER SHEET**

Name and Surname: \_\_\_\_\_

Question [Total]	1 [20]	2 [26]	3 [22]	4 [14]	5 [19]	6 [19]	7 [20]	8 [25]	TOTAL 165
Student mark									

**Question 1**

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

**DIAGRAM SHEET 1**

