

α -WISKUNDE

Alpha Wiskunde Graad 11 / *Alpha Mathematics Grade 11*
Finale Eksamen 2017 / *Final Examination 2017*

MEMORANDUM

Totaal / *Total*: 165 punte / *marks*

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Hierdie memorandum bestaan uit 9 bladsye. /
This memorandum consists of 9 pages.

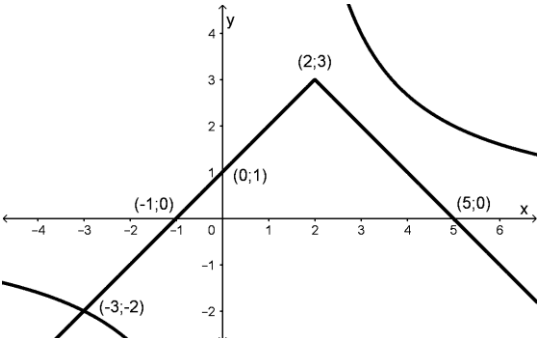
Vraag / Question 1

[20 punte / marks]

NR.	ANTWOORD ANSWER	BEREKENINGE (nie vir nasien doeleindes nie) CALCULATIONS (not for marking purpose)	PUNTE MARKS
1.1	D	$ x - 5 \leq -5$, dus geen oplossing / thus no solution	2
1.2	A	$x^3 + ax^2 + 4x + 20 = (x^2 + 4)(x + 5)$ $= x^3 + 5x^2 + 4x + 20$	2
1.3	B	Een meer term as n , dus 11 terme. / One more term than n , thus 11 terms.	2
1.4	A	Area = $\int_0^{\frac{\pi}{2}} \cos x \, dx - \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx = 2$	2
1.5	C	$ -4x^2 < 1 \Rightarrow x^2 < \frac{1}{4} \Rightarrow x < \frac{1}{2}$	2
1.6	A	$2n - 1 = 19$ of / or $2n + 1 = 21$, dus / thus $n = 10$ RK = $\frac{10}{2(10)+1} = 0,4762$	2
1.7	C	Neig na $x = 2$. Raaklyn net langs die draaipunt sal die x -afsnit net na $x = 2$ sny en dan sal die antwoord neig na $x = 2$. / Tends to $x = 2$. Tangent line next to turning point will intersect the x -intercept just after $x = 2$ and then the answer will tend to the answer $x = 2$.	2
1.8	B	Verwyderbare diskontinuiteit by / Removable discontinuity at $x = 2$	2
1.9	B	$\lim_{x \rightarrow -4} f(x) = -\infty$ WAAR / TRUE $\lim_{x \rightarrow 2} f(x) = 0$ WAAR / TRUE $\lim_{x \rightarrow 4} f(x)$ bestaan / exist VALS / FALSE	2
1.10	D	Reguitlyn met helling van -1 , dus $f'(8) = -1$ / Straight line with gradient of -1 , thus $f'(8) = -1$.	2

Vraag / Question 2

[26 punte / marks]

NR.	ANTWOORD / ANSWER	PUNTE MARKS
2.1 (a)	$f(0) = - -2 + 3 = 1 \checkmark\checkmark$	2
2.1 (b)	$- x - 2 + 3 = \frac{8}{x - 1}$ As $x < 2$: $(x - 2) + 3 = \frac{8}{x - 1} \checkmark\checkmark$ $x + 1 = \frac{8}{x - 1}$ $x^2 - 1 = 8$ $(x - 3)(x + 3) = 0 \checkmark$ $x = 3 \text{ (N.V.T / NA)} \checkmark \text{ of } x = -3 \checkmark$ Snypunt / Intersection is $(-3; -2) \checkmark$	6
2.1 (c)	x -afsnit (Stel $y = 0$): $- x - 2 + 3 = 0 \checkmark$ $ x - 2 = 3$ $x - 2 = 3$ of / or $x - 2 = -3$ $x = 5 \checkmark$ $x = -1 \checkmark$	3
2.1 (d)	 <ul style="list-style-type: none"> \checkmark knakpunt / salient point $(2; 3)$ \checkmark x-afsnitte / intercepts $(-1; 0), (5; 0)$ \checkmark y-afsnit / intercept $(0; 1)$ \checkmark vorm / shape \checkmark punt / point $(-3; -2)$ 	5
2.2 (a)	$x = 1 - \sqrt{3}$ is ook 'n wortel / is also a root \checkmark $(x - 1 - \sqrt{3})(x - 1 + \sqrt{3}) \checkmark = (x - 1)^2 - 3 \checkmark$ $= x^2 - 2x - 2$ is 'n faktor / is a factor OF / OR $x = 1 + \sqrt{3}$ $(x - 1)^2 = (\sqrt{3})^2 \checkmark\checkmark$ $x^2 - 2x + 1 = 3 \checkmark$, dus / thus $x^2 - 2x - 2$ is 'n faktor / is a factor.	3
2.2 (b)	Doen langdeling, dan is / Do long division, then: $(x^2 - 2x - 2)(x^2 + 2x + 2) \checkmark\checkmark\checkmark = 0$ $x^2 + 2x + 2 = 0$ is $x = \frac{-2 \pm \sqrt{2^2 - 4(2)}}{2} \checkmark = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i$ Dus / Thus $(x - 1 - \sqrt{3})(x - 1 + \sqrt{3})(x + 1 - i)(x - 1 + i) = 0$ $\therefore x = 1 \pm \sqrt{3} \checkmark$ of / or $x = -1 \pm i \checkmark\checkmark$	7

Vraag / Question 3

[22 punte / marks]

NR.	ANTWOORD / ANSWER	PUNTE MARKS
3.1	$\frac{4x^2 - x + 6}{x^3 + 2x} \equiv \frac{4x^2 - x + 6}{x(x^2 + 2)} \checkmark \equiv \frac{A}{x} + \frac{Bx + C}{x^2 + 2} \checkmark$ $4x^2 - x + 6 \equiv A(x^2 + 2) + x(Bx + C) \checkmark$ $\equiv Ax^2 + 2A + Bx^2 + Cx \checkmark$ $\equiv x^2(A + B) + x(C) + 2A \checkmark$ $\therefore 4 = A + B \quad C = -1 \checkmark \quad 2A = 6$ $B = 1 \checkmark \quad A = 3 \checkmark$ $\therefore \frac{4x^2 - x + 6}{x^3 + 2x} \equiv \frac{3}{x} + \frac{x - 1}{x^2 + 2} \checkmark$	9
3.2	<p>Stel / Let $n = 1$: LK = $\frac{1}{3}$ RK = $\frac{1}{3} \checkmark$</p> <p>LK = RK dus bewering is waar as / <i>the statement is true for $n = 1$.</i> \checkmark</p> <p>Aanvaar die bewering is waar as / Accept <i>the statement is true for $n = k$:</i> \checkmark</p> $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{(2k - 1)(2k + 1)} = \frac{k}{2k + 1} \checkmark$ <p>Beskou / Consider $n = k + 1$:</p> $\text{LK} = \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{(2k - 1)(2k + 1)} \checkmark + \frac{1}{(2k + 1)(2k + 3)} \checkmark$ $= \frac{k}{2k + 1} \checkmark + \frac{1}{(2k + 1)(2k + 3)}$ $= \frac{k(2k + 3) + 1}{(2k + 1)(2k + 3)} = \frac{2k^2 + 3k + 1}{(2k + 1)(2k + 3)} \checkmark$ $= \frac{(k + 1)(2k + 1)}{(2k + 1)(2k + 3)} = \frac{k + 1}{2k + 3} \checkmark \checkmark$ $\text{RK} = \frac{k + 1}{2k + 3} \checkmark \checkmark$ <p>Die bewering is waar as $n = 1$. As die bewering waar is vir $n = k$, is dit ook waar vir $n = k + 1$. Dus is die bewering waar vir alle $n \in \mathbb{N}$. /</p> <p><i>The statement is true for $n = 1$. If the statement is true for $n = k$, it is also true for $n = k + 1$. Thus, the statement is true for all $n \in \mathbb{N}$.</i> \checkmark</p>	13

Vraag / Question 4

[14 punte / marks]

NR.	ANTWOORD / ANSWER	PUNTE MARKS
4.1 (a)	$\binom{5}{r} (2x)^{5-r} \left(-\frac{3}{x}\right)^r = \binom{5}{r} 2^{5-r} \cdot x^{5-r} (-3)^r x^{-r} = \binom{5}{r} 2^{5-r} (-3)^r x^{5-2r} \checkmark\checkmark$ <p>x^3: dus / thus $5 - 2r = 3 \Rightarrow r = 1 \checkmark$</p> $\binom{5}{1} 2^4 (-3)^1 x^3 = -240x^3 \checkmark$ <p>x^1: dus / thus $5 - 2r = 1 \Rightarrow r = 2 \checkmark$</p> $\binom{5}{2} 2^3 (-3)^2 x = 720x \checkmark$	6
4.1 (b)	$\left(1 + \frac{2}{x^2}\right) \left(2x - \frac{3}{x}\right)^5 = 1 \left(2x - \frac{3}{x}\right)^5 + \frac{2}{x^2} \left(2x - \frac{3}{x}\right)^5$ $\dots + 1(720x) \checkmark + \frac{2}{x^2} (-240x^3) \checkmark + \dots = 240x \checkmark$	3
4.2	$(1 - 5x)^{-1} \checkmark = \dots + \frac{(-1)(-1-1)}{2!} (-5x)^2 \checkmark$ $+ \frac{(-1)(-1-1)(-1-2)}{3!} (-5x)^3 \checkmark + \dots$ <p>Dus 3^{de} en 4^{de} term is / Thus the 3rd and 4th term are</p> $= \dots + 25x^2 \checkmark + 125x^3 \checkmark + \dots$	5

Vraag / Question 5

[19 punte / marks]

NR.	ANTWOORD / ANSWER	PUNTE MARKS
5.1 (a)	Gelykbenige driehoek / <i>Isosceles triangle</i> , AM = MC ✓✓	2
5.1 (b)	$AC^2 = AM^2 + MC^2$ Pythagoras $= 15^2 + 15^2$ $AC = 15\sqrt{2}$ ✓✓ Area sektor / <i>sector</i> = $\frac{1}{2}r^2\theta = \frac{1}{2}(15\sqrt{2})^2 \left(\frac{\pi}{4}\right) = \frac{225}{4}\pi = 176,7145 \dots$ ✓✓ Area van / <i>of</i> $\Delta = \frac{1}{2} \times AM \times MC = \frac{1}{2}(15)^2 = 112,50$ ✓✓ Area van gearseerde deel / <i>Area of shaded area</i> is $= 176,7145 \dots - 112,50 = 64,21$ ✓	7
5.2 (a)	$\frac{3\pi}{4} = \arccos(-1) + p \Rightarrow p = -\frac{\pi}{4}$ ✓✓ OF / OR $\frac{\pi}{4} = \arccos(0) + p \Rightarrow p = -\frac{\pi}{4}$ ✓✓	2
5.2 (b)	$x = \arccos(-y) - \frac{\pi}{4}$ ✓ $x + \frac{\pi}{4} = \arccos(-y)$ ✓ $\cos\left(x + \frac{\pi}{4}\right) = -y$ ✓ $\therefore f^{-1}(x) = -\cos\left(x + \frac{\pi}{4}\right)$ ✓	4
5.2 (c)	$W_{f^{-1}}: y \in [-1 \text{ ✓}; 1 \text{ ✓}]$	2
5.2 (d)	Vertikale verskuiwing ✓ van f $\frac{\pi}{4}$ eenhede ✓ afwaarts. / <i>Vertical shift ✓ of f $\frac{\pi}{4}$ units ✓ downwards.</i>	2

Vraag / Question 6

[19 punte / marks]

NR.	ANTWOORD / ANSWER	PUNTE MARKS
6.1	(a) Waar / True ✓✓ (b) Vals / False ✓✓ (c) Waar / True ✓✓	6
6.2	$\lim_{x \rightarrow 3^-} h(x) = \lim_{x \rightarrow 3} (-x^2 + ax + 3) = 3a - 6$ ✓ $\lim_{x \rightarrow 3^+} h(x) = \lim_{x \rightarrow 3} (x + b) = 3 + b$ ✓ $h(3) = a - 5 + b$ ✓ $\therefore 3a - 6 = 3 + b = a - 5 + b$ ✓ $3 + b = a - 5 + b$ ✓ $3a - 6 = 3 + b$ ✓ $\therefore a = 8$ ✓ $3(8) - 6 = 3 + b \Rightarrow b = 15$ ✓	8
6.3	$f(a) = a^5 - 9a - 2$ $f'(a) = 5a^4 - 9$ ✓ $a_{n+1} = a_n - \frac{a_n^5 - 9a_n - 2}{5a_n^4 - 9}$ ✓✓ $a_0 = 1,8$ $a_1 = 1,7840$ $a_2 = 1,7836$ $a_3 = 1,7836$ $a \approx 1,784$ ✓✓	5

Vraag / Question 7

[20 punte / marks]

NR.	ANTWOORD / ANSWER	PUNTE MARKS
7.1	$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+5} - \sqrt{x+5}}{h} \checkmark \times \frac{\sqrt{x+h+5} + \sqrt{x+5}}{\sqrt{x+h+5} + \sqrt{x+5}} \checkmark$ $= \lim_{h \rightarrow 0} \frac{(x+h+5) - (x+5) \checkmark}{h(\sqrt{x+h+5} + \sqrt{x+5}) \checkmark}$ $= \lim_{h \rightarrow 0} \frac{h \checkmark}{h(\sqrt{x+h+5} + \sqrt{x+5})}$ $= \frac{1}{2\sqrt{x+5}} \checkmark$	6
7.2 (a)	$f(x) = x(x^2 + 1)^{\frac{1}{2}} \checkmark$ $f'(x) = (x^2 + 1)^{\frac{1}{2}} \checkmark + x \cdot \frac{1}{2} \checkmark (x^2 + 1)^{-\frac{1}{2}} \checkmark \cdot (2x) \checkmark$	5
7.2 (b)	<p style="text-align: center;">\checkmark oorspronklike funksies / original function</p> $\frac{dy}{dx} = \frac{-\sin x \checkmark \cdot x^2 - \cos x \cdot 2x \checkmark}{x^4 \checkmark}$	4
7.2 (c)	$3 \checkmark \tan^2(\text{bgsin } 2x) \checkmark \cdot \sec^2(\text{bgsin } 2x) \checkmark \cdot \frac{1}{\sqrt{1 - (2x)^2}} \checkmark \cdot 2 \checkmark$	5

Vraag / Question 8

[25 punte / marks]

NR.	ANTWOORD / ANSWER	PUNTE MARKS
8.1 (a)	$-5 \cdot \frac{1}{2} \checkmark \operatorname{cosec} 2x \checkmark + 10 \tan(5x + 1) \checkmark \cdot \frac{1}{5} \checkmark + k$	4
8.1 (b)	$\int 2(16 - 9x)^{-\frac{1}{2}} \checkmark dx$ $= \frac{2(16 - 9x)^{\frac{1}{2}} \checkmark}{\left(\frac{1}{2}\right) \checkmark (-9) \checkmark} + k$	4
8.1 (c)	$\int \frac{2}{\sqrt{16 \checkmark \left(1 - \frac{9}{16} x^2\right)}} dx$ $= \frac{2}{4} \int \frac{1}{\sqrt{1 - \left(\frac{3}{4} x\right)^2} \checkmark} dx$ $= \frac{2}{4} \checkmark \arcsin\left(\frac{3}{4} x\right) \checkmark \cdot \frac{4}{3} \checkmark + k$	5
8.2	$V = \pi \int_0^{\rho} \left[\sqrt{\frac{1}{x^2 - 6x + 10}} \right]^2 dx = \pi \int_0^{\rho} \frac{1}{x^2 - 6x + 10} dx \checkmark \checkmark$ <p>Vierkantsvoltooiing / Completing the squares:</p> $x^2 - 6x + 3^2 - 3^2 + 10 = (x - 3)^2 + 1 \checkmark \checkmark$ $= \pi \int_0^{\rho} \frac{1}{1 + (x - 3)^2} \checkmark dx$ $= \pi [\arctan(x - 3) \checkmark]_0^{\rho}$ $= \pi [\arctan(\rho - 3) - \arctan(-3)] \checkmark$	7
8.3	<p>Snypunt van / Intersection of $g(x)$ en / and $h(x)$ is</p> $\sqrt{7x} = \sqrt{36 - 9x} \Rightarrow 7x = 36 - 9x \Rightarrow x = \frac{9}{4} = 2,25 \checkmark$ $\text{Area} = \int_0^{2,25} g(x) - f(x) dx + \int_{2,25}^4 h(x) - f(x) dx$ $= \int_0^{2,25} \sqrt{7x} - (x^2 - 4x) \checkmark \checkmark dx + \int_{2,25}^4 \sqrt{36 - 9x} - (x^2 - 4x) \checkmark \checkmark dx$	5

- EINDE VAN DIE MEMORANDUM / END OF THE MEMORANDUM -