

α -MATHEMATICS

Alpha Mathematics
Final exam 2019

Grade 11

Examinator: R Grobler

Moderator: A Muller

Time: 2 ½ ure

Total: 165 marks

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the question paper:

1. This question paper consists of 8 pages, a formula sheet of 1 page and a diagram sheet of 2 pages.
2. Answer ALL 9 questions.
3. Number the answers exactly the way the questions are numbered.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places
6. Clearly show all calculations, diagrams, graphs et cetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians if necessary.
10. Write neat and legible.

Question 1**[20 marks]**

This question must be answered **on the answer sheet**.

Each question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with a **X** on the answer sheet.

1.1 Solve for x in $-|x - 5| - 7 \leq 0$.

- (A) $x \in (2; 12)$
- (B) $x \leq 2$ or $x \geq 12$
- (C) $x \in \mathbb{R}$
- (D) No solution for x

1.2 The following transformations are applied to the graph of $y = \arctan x$:

- Enlarge with a factor of 2.
- Vertical transformation of $\frac{\pi}{2}$ units up, and

The new graph's range is:

- (A) $y \in (0; 2\pi)$
- (B) $y \in (-\frac{\pi}{2}; \frac{3\pi}{2})$
- (C) $y \in (-\frac{3\pi}{2}; \frac{\pi}{2})$
- (D) $y \in (-2\pi; 0)$

1.3 $f(x) = x^5 - 1$, then $f^{-1}(x) =$

- (A) $\frac{1}{\sqrt[5]{x+1}}$
- (B) $\frac{1}{\sqrt[5]{x-1}}$
- (C) $\sqrt[5]{x-1}$
- (D) $\sqrt[5]{x+1}$

1.4 Given: $f(x) = \begin{cases} 2x & \text{if } x < 0 \\ \arcsin x & \text{if } 0 < x < 1 \\ \arctan\left(x + \frac{\pi}{2}\right) + 0,5 & \text{if } x \geq 1 \end{cases}$

For what value(s) of x does $f(x)$ have a jump discontinuity?

- (A) $x = 1$
- (B) $x = 0$
- (C) $x = -1$
- (D) No value of x .

- 1.5 Which one of the following statements is TRUE:
- (A) A function that is differentiable for all values of x , will also be continuous for all values of x .
 - (B) A function that is continuous for all values of x , will also be differentiable for all values of x .
 - (C) If a function has a removable discontinuity, it is differentiable at this point.
 - (D) If a function is not differentiable at a point, it is never continuous at this point.
- 1.6 For what values of x will the expansion of $\frac{1}{1-2x}$ converge?
- (A) $|x| < 2$
 - (B) $|x| > \frac{1}{2}$
 - (C) $|x| < 2$
 - (D) $|x| < \frac{1}{2}$
- 1.7 $\int \frac{1}{1+4x^2} dx =$
- (A) $\arctan 2x + k$
 - (B) $\frac{1}{2} \arctan 2x + k$
 - (C) $\arctan\left(\frac{x}{2}\right) + k$
 - (D) $2\arctan\left(\frac{x}{2}\right) + k$
- 1.8 If $f(x) = \sqrt[3]{x}$, then $f'(-8) =$
- (A) -12
 - (B) 12
 - (C) $\frac{1}{12}$
 - (D) $-\frac{1}{12}$
- 1.9 The size of the vector $2i + 3j + ak$ is $\sqrt{38}$. Then the value of $a =$
- (A) 2
 - (B) 3
 - (C) -4
 - (D) -5
- 1.10 $\int_a^0 \sqrt{x^2 - 2x + 1} dx =$
- (A) $1 - a$
 - (B) $a - 1$
 - (C) $a - \frac{a^2}{2}$
 - (D) $\frac{a^2}{2} - a$

Question 2**[15 marks]**2.1 Solve for x :

(a) $|3 - 2x| < 7$ (4)

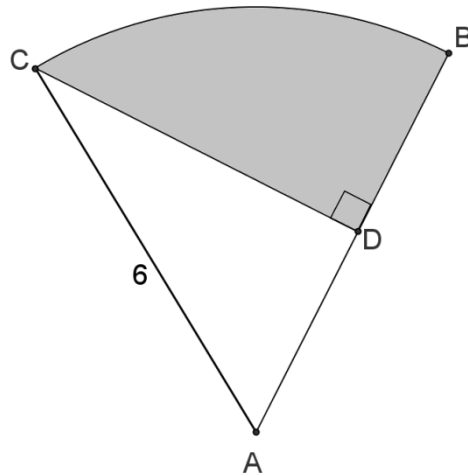
(b) $|x + 2| = -3x + 6$ (6)

2.2 Sketch the graph of $y = 1 - |x - 2|$. Show clearly all intercepts with the axes and the salient point on the sketch. Use **DIAGRAM SHEET 1** for the sketch. (5)**Question 3****[19 marks]**3.1 Factorise $x^4 + 4x^3 + 3x^2 + 4x + 2$ in $\mathbb{R}[x]$ if it is given that $-2 + \sqrt{2}$ is a zero. (8)3.2 Determine the 6'th term in the expansion of $\left(\frac{1}{2} - 4x\right)^8$. (6)3.3 Determine and simplify the first 3 terms of $(1 - 2x)^{-2}$. (5)**Question 4****[18 marks]**4.1 Decompose $\frac{5x^2 - 4x + 3}{(x-1)(x^2+1)}$ into partial fractions. (8)4.2 Use **mathematical induction** and prove that the following statement is true for $n \in \mathbb{N}$:

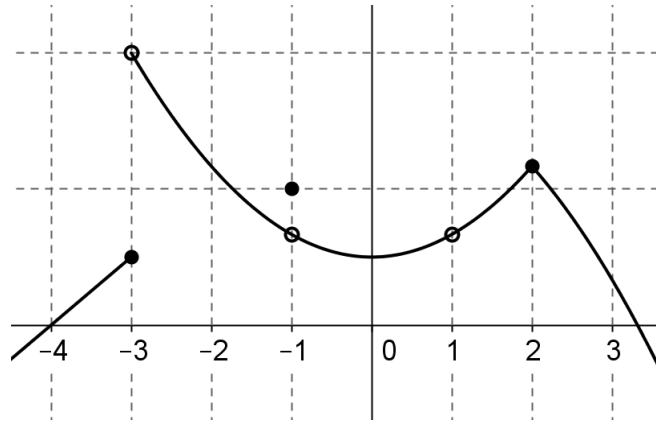
$$1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1) = \frac{1}{3}n(n+1)(n+2) \quad (10)$$

Question 5**[15 marks]**Given: $\mathbf{u} = 2i - j + 3k$, $\mathbf{v} = -2i + 5k$ and $\mathbf{w} = 5i - 2j + k$ 5.1 Determine the unit vector of \mathbf{w} . (3)5.2 Determine the size of the angle between \mathbf{u} and \mathbf{v} . (5)5.3 Determine the area of the parallelogram that is formed by vectors \mathbf{u} and \mathbf{w} . (7)**Question 6****[22 marks]**6.1 Determine the inverse function of $y = 2\cos(2x) - 1$.
Write it in the form $y = \dots$ (4)6.2 Make a neat sketchgraph of $f(x) = \arctan\left(\frac{x}{2}\right) + \frac{\pi}{4}$.
Clearly show all intercepts with the axes as well as all asymptotes.
If applicable, units must be given in terms of π .
Use **DIAGRAM SHEET 2** for the sketch. (6)

- 6.3 The sketch shows **sector** ABC with radius 6 cm and area 6π cm². D is a point on AB so that $CD \perp AB$.



- (a) Determine the size of \widehat{CAB} . (3)
- (b) Determine the arc length BC. Use $\widehat{CAB} = \frac{\pi}{3}$ radians. (2)
- (c) Calculate the length of AD. (3)
- (d) Hence calculate the area of the shaded region, BCD. (4)

Question 7**[20 marks]**7.1 The sketch shows the graph of $y = f(x)$.

- (a) The function is not everywhere continuous for $x \in [-4; 3]$. Give the points of discontinuity with the type. Motivate each one by using the **definition** of continuity. (6)
- (b) Motivate why the function is not differentiable in the following points:
- (i) $x = 1$ (1)
- (ii) $x = 2$ (1)

7.2 Differentiate as asked. It is not necessary to simplify the answers.

- (a) $D_x[\sin x^2 + 2x]$. (3)
- (b) $\frac{d}{dx}\left[\sqrt{x} \times \tan\left(\frac{2}{x}\right)\right]$. (5)
- (c) $f'(x)$ as $f(x) = [\arctan(x)^2]^4$. (4)

Question 8**[18 marks]**

8.1 Determine the following integrals:

- (a) $\int \operatorname{cosec}^2(5 - 4x) dx$. (3)
- (b) $\int \left(\frac{1}{\sqrt{4x-4x^2}}\right) dx$. (4)

8.2 (a) Given the identity $\cos 2x = \cos^2 x - \sin^2 x$.Use this and show that $\frac{d}{dx}(2x - \sin 2x) = 2 \sin^2 x$. (3)

- (b) Hence determine $\int \sin^2 4x dx$. (4)

8.3 Use Newton's method to determine the zero of $\tan x - (3x + 1) = 0$.

Give the answer correct to five decimal digits.

Use $-0,5$ as first approximation.

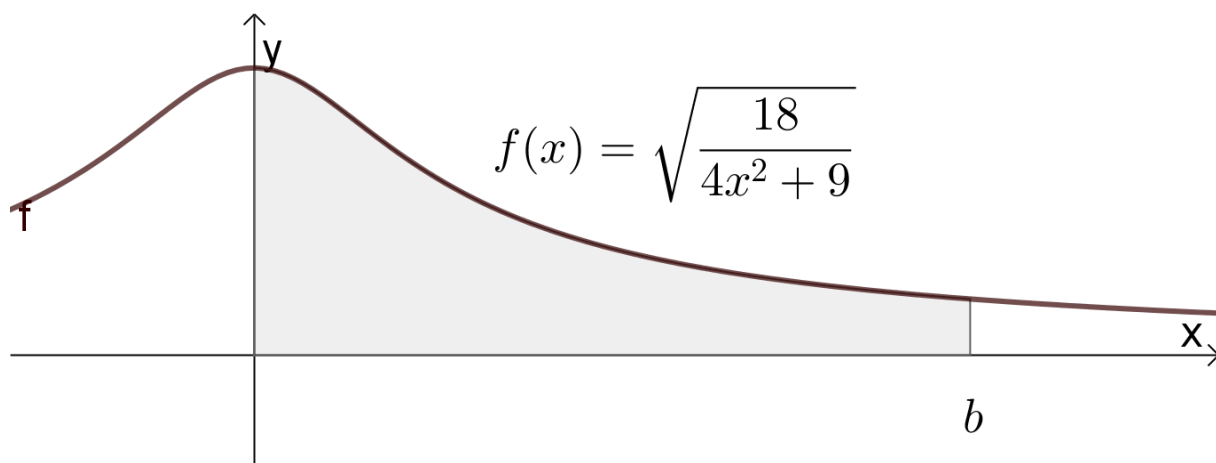
(4)

Question 9**[18 marks]**

9.1 The area between the graphs $f(x) = \sec^2 x$ and $g(x) = \sin 2x$ and between the lines $x = \frac{\pi}{4}$ en $x = \frac{\pi}{3}$ equals $\sqrt{p} + a$. Calculate the values of a and p which are rational numbers. Show all the steps of integration that you use.

(7)

9.2 The graph of $f(x) = \sqrt{\frac{18}{4x^2+9}}$ is shown. The region between the lines $x = 0$ and $x = b$ is shaded.



This area rotates around the x -as. The volume of the body of revolution that is formed is equal to $\frac{3\pi^2}{4}$. Calculate the value of b .

(11)

- END OF PAPER -

ALPHA MATHEMATICS FORMULA SHEET

ALGEBRA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & \text{as } x \geq 0 \\ -x & \text{as } x < 0 \end{cases}$$

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots ; \text{mits } |x| < 1$$

VECTORS:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

CALCULUS:

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

TRIGONOMETRY:

In a sector: $s = r\theta$ and $A = \frac{1}{2}r^2\theta$

Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $\cot^2 x + 1 = \operatorname{cosec}^2 x$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

In ΔABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

TABEL WITH DERIVETAVES:

$F(x)$	$F'(x)$
ax^n	nax^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \cdot \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$

$F(x)$	$F'(x)$
$\operatorname{bgsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arcsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\operatorname{bgcos} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\operatorname{arccos} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\operatorname{bgtan} x$	$\frac{1}{x^2+1}$
$\operatorname{arctan} x$	$\frac{1}{x^2+1}$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
$f[g(x)]$	$f'[g(x)] \cdot g'(x)$

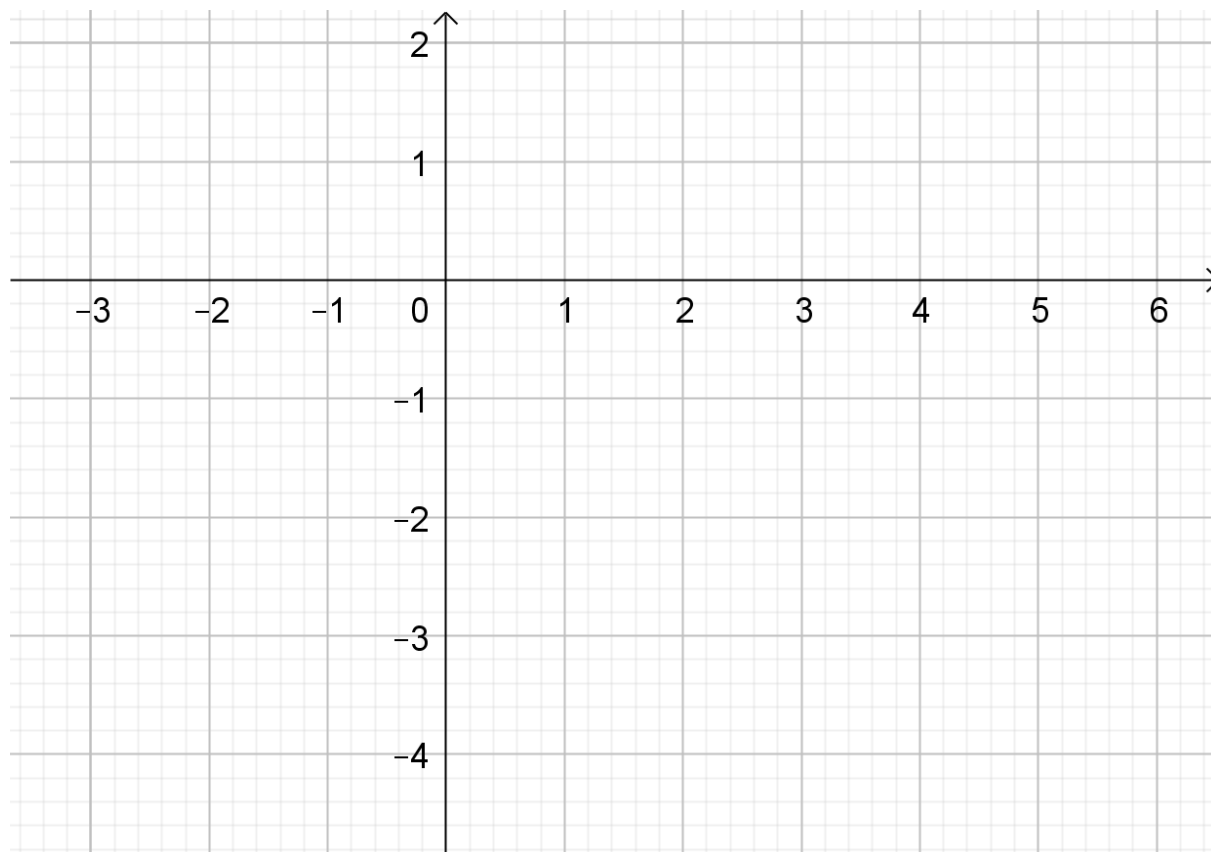
Alpha Mathematics Grade 11 - Finale exsam 2019**ANSWER SHEET**

Name and Surname: _____

Question Total	1 [20]	2 [15]	3 [19]	4 [18]	5 [15]	6 [22]	7 [20]	8 [18]	9 [18]	TOTAL 165
Learner mark										

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1 [Question 2.2]**DIAGRAM SHEET 2 [Question 6.2]**