

α -Mathematics

Grade 10

Examinator: Marco Botha

Moderator: Annelize Lippert

Time: 2 hours

Total: 130 marks

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 8 pages and an answer sheet of 2 pages.
2. Answer ALL 9 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, must be given correct to two decimal places, where necessary.
6. Clearly show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neatly and legibly.

Question 1

[20 marks]

This question must be answered **on the answer sheet**. Each question has **ONLY ONE** correct answer and each question counts **TWO** marks. Mark each correct answer with an **X** on the answer sheet.

1.1 If $x^2 + 9 = 0$, then:

- (A) $x = \pm 3$ (B) $x = \pm 3i$ (C) $x = \pm 9i$ (D) $x = \pm 9$

1.2 The radius of a sector is 5 cm and the arclength is 5π cm. Determine the angle of the sector.

- (A) π
(B) 2π
(C) $\frac{\pi}{2}$
(D) $\frac{3\pi}{2}$

1.3 Calculate $\arcsin\left(\frac{1}{2}\right)$.

- (A) Undefined
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$
(D) 30°

1.4 Given two vectors $a = (2; -3)$ and $b = (4; y)$ that are perpendicular. Determine the value of y .

- (A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{8}{3}$ (D) $\frac{8}{3}$

1.5 Determine $\begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix}$.

- (A) 0 (B) 4 (C) 3 (D) 5

1.6 Given $f(x) = x^3 + 2x$ and $f(x) = \int f'(x) dx$. Determine the equation of $f'(x)$.

- (A) $f(x) = 3x^2 + 2x$
(B) $f(x) = \frac{1}{3}x^2 + 2$
(C) $f(x) = \frac{1}{4}x^4 + x^2$
(D) $f(x) = 3x^2 + 2$

1.7 If $\frac{f(x)}{x^2-x}$ is decomposed into partial fractions, then $\frac{f(x)}{x^2-x} \equiv \dots$

(A) $\frac{A}{x} + \frac{B}{x^2-1}$

(B) $\frac{A}{x^2} + \frac{B}{x-1}$

(C) $\frac{A}{x} + \frac{B}{x-1}$

(D) $\frac{Ax+B}{x^2-x}$

1.8 Determine i^{63} .

(A) 1

(B) $-i$

(C) i

(D) -1

1.9 If $f(1) = f(-1) = 1$, $f(2) = f(-2) = 2$, $g(-1) = -2$, $g(1) = 2$, $g(-2) = -1$ en $g(2) = 2$, determine the value of $-f(g(1))$.

(A) -1

(B) 1

(C) 2

(D) -2

1.10 Determine the value of a if $\int_0^a x^2 dx = \frac{1}{3}$.

(A) $\frac{1}{6}$

(B) $\frac{1}{27}$

(C) -1

(D) 1

Question 2 – Complex Numbers

[18 marks]

- 2.1 Determine the conjugate of z , if $z = 2i - 3$. (1)
- 2.2 Determine the following:
- (a) $\sqrt{-4}(-3i + \sqrt{2})$ (3)
- (b) $(2 + 3i)(1 - 2i)$ (3)
- (c) $\frac{i^5}{1+2i}$ (4)
- 2.3 Consider a complex number, $z = x + yi$, with real component $Re(z) = 1$, where $x + y^2 = 5$. Determine the value(s) of x and y . (3)
- 2.4 Determine the value of $(3 - 2i) + (1 + i)$, graphically.
Use **DIAGRAM SHEET 1** on the answer sheet. (4)

Question 3 – Partial Fractions

[14 marks]

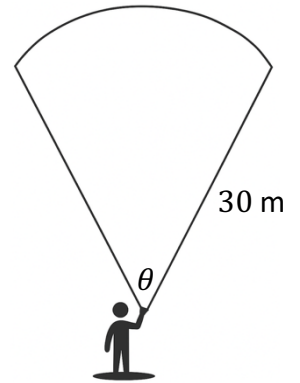
- 3.1 Decompose $\frac{3x^2 - 2x - 5}{x(x-1)(x+2)}$ into partial fractions. (11)
- 3.2 Determine the values of A, B and C if $\frac{-2x^2 + 5x + 3}{x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3}$. (3)

Question 4 – Trigonometry and Radian Measure

[12 marks]

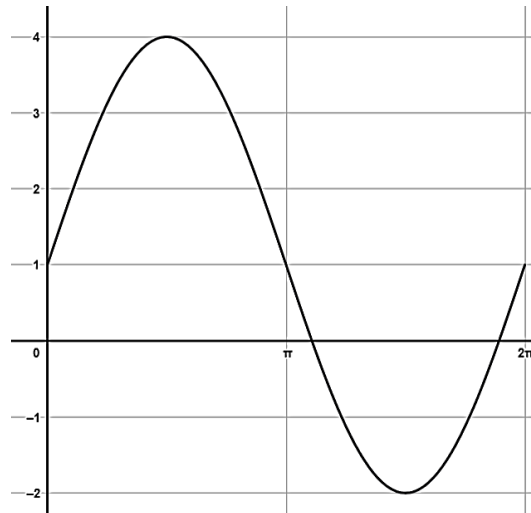
4.1 Righardt stands in the middle of a dark open field and is shining his flashlight.

The light disperses in the shape of a sector, as in the diagram to the right. He uses his protractor and measures the angle that the lightbeam makes as 60° . He also sees that the flashlight illuminates an area $d = 30\text{m}$ in front of him.



- (a) Convert 60° to radians. (1)
- (b) Determine the arclength of the illuminated area. (2)
- (c) Determine the area of the sector. (3)
- (d) If Righardt has another flashlight that shines twice as wide, but illuminates the area with the same arclength, determine the new distance d . (2)

4.2 Below is the graph of the function $f(x) = 3 \sin x + 1$.



- (a) Solve for x , if $f(x) = 4$. (3)
- (b) The graph of f is shifted 3 units down. Determine the y intercept of the new graph. (1)

Question 5 – Matrices and Cramer’s Rule

[20 marks]

5.1 The following system of equations is given.

$$x + y - z = -1$$

$$y + x = -2$$

$$4x - 2z = 2$$

Write the system of equations in matrix form and use Cramer’s rule to determine the value of y only. (7)

5.2 Given:

$$x + ay = 2$$

$$x - 2y = 5$$

Determine the value of a so the system of equation has no solutions. (3)

5.3 Given the matrices $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 5 \\ 7 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

5.3.1 Which type of matrix is I ? (1)

5.3.2 If possible, determine the following. If it is not possible, give a reason why.

(a) A^T (1)

(b) $B + C$ (2)

(c) C^2 (4)

(d) $A \times B$ (2)

Question 6 - Functions**[11 marks]**6.1 Sketch the following piecewise function on **DIAGRAM SHEET 2**.Show all intercepts with the x and y axes.

(6)

$$f(x) = \begin{cases} -2x - 1 & \text{if } x \leq 0 \\ x^2 - 2 & \text{if } 0 < x < 2 \\ 5 & \text{if } x > 3 \end{cases}$$

6.2 Given $f(x) = (x - 2)^3$ and $g(x) = \sqrt{x + 1}$. Determine $(g \circ f)(2)$. (2)6.3 The composite function $F(x) = (1 - x)^3 + 2(x - 1) + 4$ is given, where $F(x) = f(g(x))$. Determine $f(x)$ and $g(x)$. (3)**Question 7 – Polinomials and Vectors****[13 marks]**7.1 Given the vectors $\mathbf{a} = (-2; 4)$ and $\mathbf{b} = (3; 1)$.(a) Determine the size of \mathbf{a} and \mathbf{b} . (2)(b) Determine the unit vector of \mathbf{b} . (1)(c) Determine the angle between \mathbf{a} and \mathbf{b} . (3)7.2 Given $g(x) = -x^3 + 2x^2 + 5x - 6$.(a) Show that $x - 1$ is a factor of $g(x)$. (2)(b) Determine the zeros of $g(x)$. (5)

Question 8 - Calculus**[13 marks]**

8.1 Differentiate the functions with respect to the variable:

(a) $f(x) = x^4 - 2\sqrt[3]{x} + \frac{1}{2x^4} + \pi$ (3)

(b) $g(x) = \frac{2}{x^2+3x-1}$ (3)

8.2 Determine the following integrals:

(a) $\int \left(\frac{3}{x^5} + 1\right) dx$ (2)

(b) $\int \left(\frac{1}{2}x - 5\sqrt{x}\right) dx$ (2)

8.3 Determine the equation of $f(x)$, if $f'(x) = 2x - 3$ and $f(x)$ passes through $(0; 1)$. (3)**Question 9 - Area and Volume****[9 marks]**9.1 Given the functions $f(x) = \sqrt{6x+1}$ and $g(x) = ax^2$ where $a \in \mathbb{R}$.(a) Determine the area between $f(x)$ and the x -axis, between $x = 0$ and $x = 3$. (4)(b) It is given that $k(x) = f(g(x)) = \sqrt{6ax^2 + 1}$.The volume of the rotating body that is formed when $k(x)$ is rotated around the x axis, between $x = 0$ and $x = 3$ is 57π . Determine the value of a . (5)**- END OF PAPER -**

ALPHA MATHEMATICS FORMULA SHEET

MATRICES AND VECTORS:

$$\text{Cramer's Rule: } x_i = \frac{|A_i|}{|A|}$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y$$

CALCULUS:

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

TRIGONOMETRY:

$$\text{In a sector: } s = r\theta \text{ en } A = \frac{1}{2}r^2\theta$$

TABLE WITH DERIVATIVES:

$F(x)$	$F'(x)$
ax^n	nax^{n-1}
$f[g(x)]$	$f'[g(x)] \cdot g'(x)$

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ANSWER SHEET

Name and Surname: _____

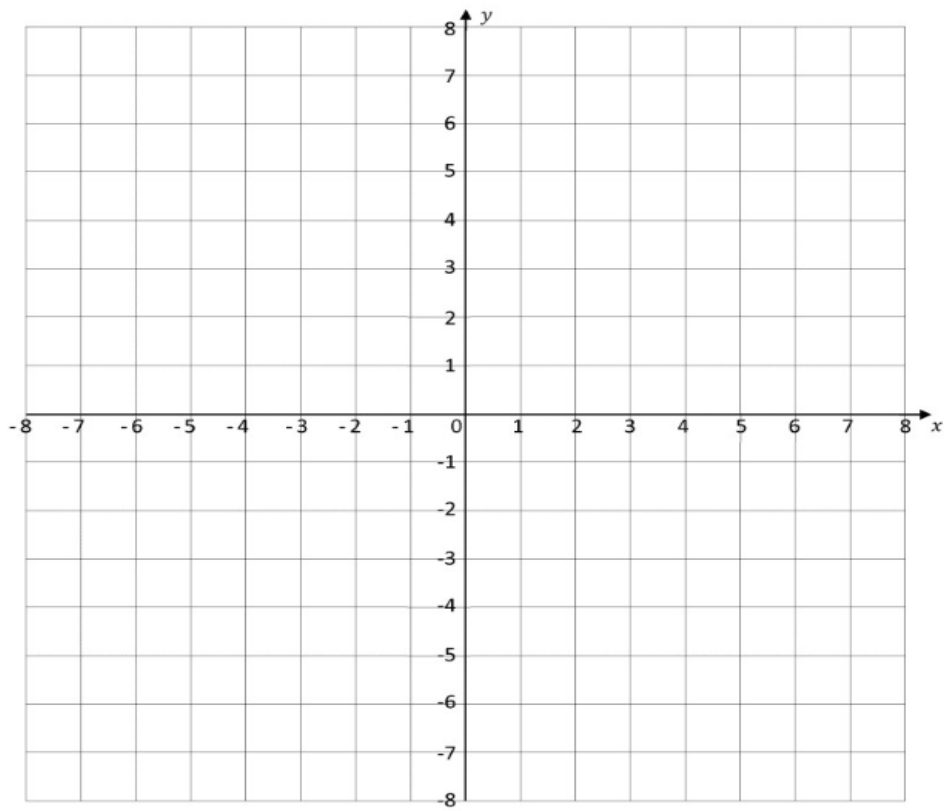
Question Total	1 [20]	2 [18]	3 [14]	4 [12]	5 [20]	6 [11]	7 [13]	8 [13]	9 [9]	TOTAL 130
Student Mark										

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAMSHEET 1

Question 2.4



Question 6.1

