α-WISKUNDE/MATHEMATICS

Alpha Wiskunde Graad 10 / ***Alpha Mathematics Grade 10***

Finale eksamen 2024 / ***Final examination 2024***

MEMORANDUM

Totaal / ***Total*****: 150 punte /** ***marks***

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Hierdie memorandum bestaan uit 9 bladsye. /
This memorandum consists of 9 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

1.1	A	**B**	C	D
1.2	**A**	B	C	D
1.3	A	B	C	**D**
1.4	A	B	**C**	D
1.5	A	**B**	C	D
1.6	A	B	C	**D**
1.7	A	B	C	**D**
1.8	A	B	**C**	D
1.9	A	B	**C**	D
1.10	**A**	B	C	D

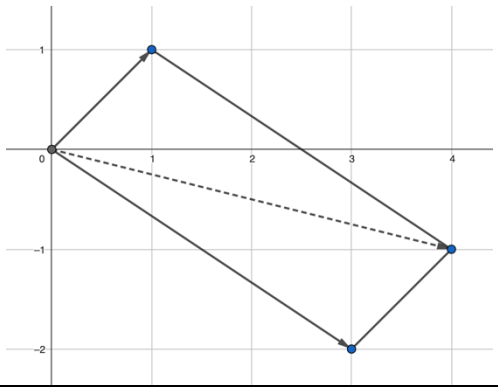
Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
1.1	B $x^2 = -4 \Rightarrow x = \sqrt{4}\sqrt{-1} = \pm 2i$	**(2)**
1.2	A $x - 1 = x + B \Rightarrow B = -1$	**(2)**
1.3	D $2x - 1 = 1 \Rightarrow x = 1$	**(2)**
1.4	C Booglengte is die radius × hoek van die sektor.	**(2)**
1.5	B $2(1) - (-1)(2) = 4$	**(2)**
1.6	D $(x) = \int 3x^2 dx = \frac{3x^3}{3} + k/C \Rightarrow (1)^3 + k = 0 \Rightarrow k = -1$	**(2)**
1.7	D $\frac{A}{x+1} + \frac{B}{x^2} + \frac{C}{X}$	**(2)**
1.8	C $\frac{1}{i^3} = \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-(i^2)} = \frac{i}{-(-1)} = i$	**(2)**
1.9	C $bgcos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \quad (30^o \; not \; in \; radians)$	**(2)**
1.10	A $\int_0^1 ax \, dx = 1$ $1 = \frac{ax^2}{2}\Big]_0^1 \quad \Rightarrow \quad 1 = \frac{a}{2} \quad \Rightarrow \quad a = 2$	**(2)**

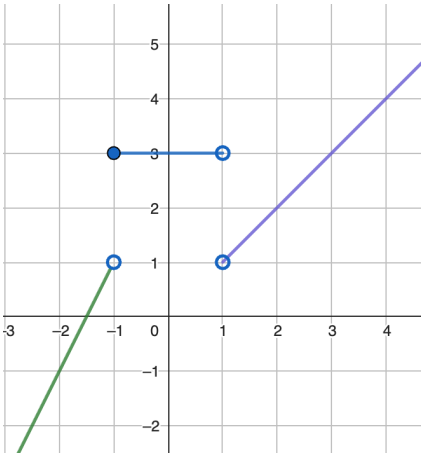
Vraag / *Question* 2 **[19 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1(a)	$\frac{2x^2-3x-1}{x(x-1)(x+1)}$ ✓ $\equiv \frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$ ✓ $2x^2-3x-1 \equiv A(x-1)(x+1)$ ✓ $+Bx(x+1)$ ✓ $+Cx(x-1)$ ✓ $x=0$ ✓: $-1=-A$ $A=1$ ✓ $x=1$ ✓: $2-3-1=2B$ $B=-2$ ✓ $x=-1$ ✓: $2+3-1=2C$ $C=2$ ✓ $\frac{2x^2-3x-1}{x(x-1)(x+1)} \equiv \frac{1}{x}+\frac{-2}{x-1}+\frac{2}{x+1}$ ✓	**(12)**
2.2	✓2 ✓−1 2 3 −1 | 4 4 −5 1 ✓ −(4 6 −2) −2 3 1 −(−2 3 1) 0 $\frac{4x^3+4x^2-5x+1}{2x^2+3x-1}=2x-1$ ✓	**(4)**
2.3(a)	$f(x)=2cosx-1$ $a=2$ ✓ $b=-1$ ✓	**(2)**
2.3(b)	$h(x)=2cosx+1$ ✓	**(1)**

Vraag / *Question* 3 **[18 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1(a)	$2i$ ✓$(\sqrt{2}-2i)$ $=2\sqrt{2}i-4i^2$ ✓ $=\ 2\sqrt{2}i+4$ ✓	**(3)**
3.1(b)	$\left(\frac{1}{2}-i\right)\left(1+\frac{i}{4}\right)=\frac{1}{2}-\frac{7i}{8}-\frac{i^2}{4}$ ✓ $=\frac{1}{2}-\frac{7i}{8}+\frac{1}{4}$ ✓ $=\frac{3}{4}$ ✓ $-\frac{7i}{8}$	**(3)**
3.1(c)	$\frac{2+i}{i}\times\frac{i}{i}$ ✓ $=\frac{2i+i^2}{i^2}$ ✓ $=\frac{2i-1}{-1}$ ✓ $=1-2i$	**(3)**
3.2(a)	$(1-i)^*=1+i$ ✓	**(1)**
3.2(b)	$Im(-2+i)=+1$ ✓	**(1)**
3.3(a)	$(3a-2i)-(1+ai)=3a-1-2i-ai$ Suiwer reëel/*pure real:* $-2i-ai=0$ ✓ $-2=a$ ✓	**(2)**
3.3(b)	$(3-2i)-(1+i)$ ✓ $(1;1)$ ✓ $(4;-1)$ ✓ $(3;-2)$ ✓ Vorm/*form* ✓ 	**(5)**

Vraag / *Question* 4 **[21 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1	$F(x) = \sqrt[3]{1-x} - (1-x)$ ✓ $f(x) = 1-x$ ✓ $g(x) = \sqrt[3]{x} - x$ ✓ OF/*OR* $F(x) = -\sqrt[3]{x-1} - (1-x)$ ✓ $f(x) = x-1$ ✓ $g(x) = -\sqrt[3]{x} + x$ ✓	**(3)**
4.2(a)	$(g \circ f)(x) = g\big(f(x)\big)$ ✓ $= \left(\sqrt{x+2}\right)^3$) ✓	**(2)**
4.2(b)	$(f \circ g)(x) = f\big(g(x)\big)$ ✓ $= \sqrt{x^3+2}$ ✓ $f\big(g(-1)\big) = \sqrt{(-1)^3+2}$ ✓ $= \sqrt{-1+2}$ $= 1$ ✓	**(4)**
4.3	 	**(5)** $(-1; 1)$ ✓**oop/*open*** $(-1; 3)$ ✓**toe/*closed*** $(1; 3)$ ✓**oop/*open*** $(1; 1)$ ✓**oop/*open*** **Vorm/*form*** ✓
4.4(a)	$f(3)$ ✓ $= 3^3 - 2(3)^2 - 5(3) + 6$ $= 27 - 18 - 15 + 6$ $= 0$ ✓	**(2)**
4.4(b)	$\begin{array}{r\|cccc} & 1 & -2 & -5 & 6 \text{ ✓} \\ 3 & \square & 3 & 3 & -6 \\ \hline & 1 & 1 & -2 & 0 \text{ ✓} \end{array}$ $x^2 + x - 2$ ✓ $= (x-2)(x+1)$ ✓ $f(x) = x^3 - 2x^2 - 5x + 6 = (x-2)(x+1)(x-3)$ ✓	**(5)**

Vraag / *Question* 5 **[21 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-3\\ 1\end{bmatrix}$ ✓ $detA = \begin{vmatrix}1 & 2\\ 3 & 1\end{vmatrix} = 1 - 6 = -5$ ✓ $detA_x = \begin{vmatrix}-3 & 2\\ 1 & 1\end{vmatrix}$ ✓$= -3 - 2 = -5$ ✓ $detA_y = \begin{vmatrix}1 & -3\\ 3 & 1\end{vmatrix}$ ✓$= 1 + 9 = 10$ ✓ $x = \frac{-5}{-5} = 1$ ✓ $y = \frac{10}{-5} = -2$ ✓	**(8)**
5.2	$\begin{vmatrix}2 & -5 & 3\\ 0 & 7 & -2\\ -1 & 4 & 1\end{vmatrix} = 2\begin{vmatrix}7 & -2\\ 4 & 1\end{vmatrix}$ ✓$+ 0\begin{vmatrix}-5 & 3\\ 4 & 1\end{vmatrix}$ ✓$- 1\begin{vmatrix}-5 & 3\\ 7 & -2\end{vmatrix}$ ✓ $= 2(7 + 8) - (10 - 21)$ $= 2(15$ ✓$) + 0 - (-11$ ✓$)$ $= 30 + 11$ $= 41$ ✓	**(6)**
5.3(a)	$B = \begin{bmatrix}-1 & 6\\ 3 & 1\end{bmatrix} \quad B^T = \begin{bmatrix}-1 & 3\\ 6 & 1\end{bmatrix}$ ✓	**(1)**
5.3(b)	$\begin{bmatrix}1 & 0\\ 2 & -1\\ 1 & 3\end{bmatrix} \times \begin{bmatrix}-1 & 6\\ 3 & 1\end{bmatrix} = \begin{bmatrix}-1+0 & 6+0\\ -2-3 & 12-1\\ -1+9 & 6+3\end{bmatrix}$ $= \begin{bmatrix}-1 & 6$ ✓$\\ -5 & 11$ ✓$\\ 8 & 9$ ✓$\end{bmatrix}$	**(3)**
5.3(c)	$2\begin{bmatrix}-1 & 6\\ 3 & 1\end{bmatrix} + \begin{bmatrix}1 & 3\\ a & 1\end{bmatrix} = \begin{bmatrix}-1 & 15\\ 3 & 3\end{bmatrix}$ $\begin{bmatrix}-2 & 12\\ 6 & 2\end{bmatrix}$ ✓$+ \begin{bmatrix}1 & 3\\ a & 1\end{bmatrix} = \begin{bmatrix}-1 & 15\\ 3 & 3\end{bmatrix}$ $6 + a = 3$ ✓ $a = -3$ ✓	**(3)**

Vraag / *Question* 6 **[19 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1(a)	$f(x) = 3x^{-\frac{1}{3}} + \pi x^{-2} + 5x^7$ $f'(x) = -x^{-\frac{4}{3}}\checkmark - 2\pi x^{-3}\checkmark + 35x^6\checkmark$	**(3)**
6.1(b)	$f(x) = 9(x-1)^{-1}$ $f'(x) = -9\checkmark(x-1)^{-2}\checkmark$	**(2)**
6.1(c)	$f(z) = (z^2 - z)^{\frac{1}{2}}$ $f'(z) = \frac{1}{2}\checkmark(z^2 - z)^{-\frac{1}{2}}\checkmark(2z-1)\checkmark$	**(3)**
6.2(a)	$\int (1 + 2x^2)\checkmark dx$ $= x\checkmark + \frac{2x^3}{3}\checkmark + k/c$ $-\mathbf{1}$ geen konstante/*no constant*	**(3)**
6.2(b)	$\int_0^1 (x^{\frac{1}{2}} + x^{-3})dx$ $= \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\checkmark + \frac{x^{-2}}{-2}\checkmark\Big]_0^2$ $= \left(\frac{2}{3}\right)1^{\frac{3}{2}} - \frac{(1)^{-2}}{2}\checkmark - \left(\left(\frac{2}{3}\right)0^{\frac{3}{2}} - \frac{(0)^{-2}}{2}\right)\checkmark$ $= \frac{2}{3} - \frac{1}{2}$ $= \frac{1}{6}\checkmark$	**(5)**
6.2(c)	$\int (1-2x)^{\frac{1}{2}}$ $-\mathbf{1}$ geen konstante indien eerste oortreding $= \frac{(1-2x)^{\frac{3}{2}}\checkmark}{\frac{3}{2}\checkmark(-2)\checkmark} + k/c$ -1 if no constant and first transgression	**(3)**

Vraag / *Question* 7 **[14 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1(a)	$x+2=x^2$ ✓ $x^2-x-2=0$ $(x-2)(x+1)=0$ ✓ $x=2$ ✓ $x=-1$ nvt/*na*	**(3)**
7.1(b)	$\int_0^2 (x^2-x-2)dx$ $=\frac{x^3}{3}$ ✓$-\frac{x^2}{2}$ ✓$-2x$ ✓$\Big]_0^2$ $=\frac{x^3}{3}-\frac{x^2}{2}-2x\Big]_0^2=\left(\frac{2^3}{3}-\frac{2^2}{2}-2(2)\right)-0$ vervang/*substitute* ✓ $=-\frac{2}{3}eenhede^2/unit^2$ ✓	**(5)**
7.2	$V=\pi\int_0^3\left(\sqrt{3x^2+a}\right)^2 dx$ ✓$=30\pi$ ✓ $\int_0^3 (3x^2+a)\,dx=30$ $\frac{3x^3}{3}$ ✓$+ax$ ✓$\Big]_0^3=30$ $3^3+3a=30$ ✓ $3a=3$ $a=1$ ✓	**(6)**

Vraag / *Question* 8 **[18 punte / *marks*]**

NR. NO.	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1(a)	$\|a\| = \sqrt{2^2 + 5^2} = \sqrt{29}$ ✓	**(1)**
8.1(b)	$\theta = bgtan\left(\frac{2}{-1}\right)$ ✓ θ =1,11 rad ✓	**(2)**
8.1(c)	Eenheidsvektor/*unit vector* $= (\frac{2}{\sqrt{29}}; \frac{5}{\sqrt{29}})$ ✓	**(1)**
8.1(d)	$\|b\| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$ ✓ $a \bullet b = -2 + 10 = 8$ ✓ $a \bullet b = \|a\|\|b\| \cos\theta$ $\theta = bgcos\left(\frac{8}{\sqrt{5}\sqrt{29}}\right)$ ✓ $\theta = 0{,}84$ ✓	**(4)**
8.2(a)	$AB = \sqrt{5^2 - 3^2} = 4$ ✓ $AD = BC - AC = 5 - 4 = 1$ ✓ Boog/*arc* $BD = r \times \alpha = 5 \times \frac{\pi}{6} = \frac{5\pi}{6}\ eenhede^2/units^2$ ✓	**(5)**
8.2(b)	Oppervlakte/Area $ABC = \frac{1}{2}(base)(height) = \frac{1}{2}(4)(3) = 6$ ✓ Oppervlakte/Area sector $A = \frac{1}{2}(r^2)\theta = \frac{1}{2}(5)^2\left(\frac{\pi}{6}\right)$ ✓ $= \frac{25\pi}{12}$ ✓ Oppervlakte/Area of $ABD = 25\pi - 6$ ✓ $= 0{,}54$ ✓ $eenhede^2/\text{units}^2$	**(5)**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -