

α -WISKUNDE/MATHEMATICS

Alpha Wiskunde Graad 10 / *Alpha Mathematics Grade 10*

Junie eksamen 2024 / *June examination 2024*

MEMORANDUM

Totaal / *Total*: 130 punte / *marks*

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Moderator: Lanice Liebenberg

**Hierdie memorandum bestaan uit 9 bladsye. /
*This memorandum consists of 9 pages.***

Vraag / Question 1

[10 punte / marks]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Vraag / Question 1

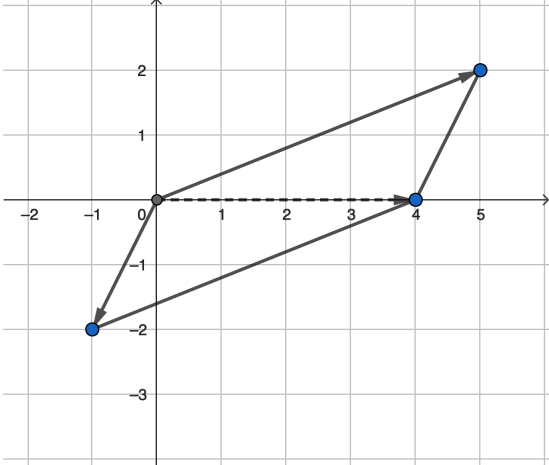
[10 punte / marks]

NR. NO.	ANTWOORD / ANSWER	PUNTE / MARKS
1.1	C $\sqrt{-9} \Rightarrow \sqrt{9}\sqrt{-1}$ $= 3i$	(2)
1.2	B $60^\circ \times \frac{\pi}{180^\circ} \Rightarrow \frac{\pi}{3} \Rightarrow 1,047$	(2)
1.3	B $2 + i$	(2)
1.4	C (A) Vals/false: Identiteitsmatriks is altyd 'n vierkantige matriks. <i>An identity matrix is always a square matrix.</i> (B) Vals/false: Die dimensies van 'n getransponeerde vierkantige matriks bly nie altyd dieselfde nie . <i>The dimensions of a transposed matrix and the original matrix not always remain the same.</i> (D) Vals/false: Die determinant van 'n matriks se antwoord is 'n skalaar . <i>The determinant of a matrix's answer is also a scalar.</i>	(2)

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1.5	<p>C</p> <p>(A) Kolomme matriks A nie gelyk aan rye van matriks B <i>Columns of matrix A is not equal to rows in matrix B.</i></p> <p>(B) Dimensies nie dieselfde nie. <i>Dimensions is not the same.</i></p> <p>(C) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \times [1 \quad 2] = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$</p> <p>(D) Dimensies nie dieselfde nie. <i>Dimensions is not the same.</i></p>	(2)
1.6	<p>A</p> $\det A = \begin{vmatrix} 2 & 3 \\ -1 & 2a \end{vmatrix} \Rightarrow 4a + 3$ $\det A_x = \begin{vmatrix} 8 & 3 \\ 3 & 2a \end{vmatrix} \Rightarrow 16a - 9$ $\frac{4a+3}{16a-9} = 1$ $4a + 3 = 16a - 9$ $12a = 12$ $a = 1$	(2)
1.7	<p>B</p> $a \cdot b = 1 \times x - 2 \times -3$ $-1 = x + 6$ $x = -7$	(2)
1.8	<p>A</p> $(1 + i)(2 - 3i)$ $= 2 - 3i + 2i - 3i^2$ $= 2 - i + 3$ $= 5 - i$ $\operatorname{Re}(5 - i) = 5$	(2)
1.9	<p>D</p> $1 \text{ radiaal} = 57,3^\circ$ $1 \text{ radian} = 57,3^\circ$	(2)
1.10	<p>D</p> $\frac{f(x)}{x^2(x-2)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$	(2)

Vraag / Question 2

[25 punte / marks]

NR. NO.	ANTWOORD / ANSWER	PUNTE / MARKS
2.1(a)	$x^2 = -4 \checkmark$ $x = 2i \checkmark$ OF/OR $x = \pm 2i$	(2)
2.1(b)	$\frac{-2}{i} \times \frac{i}{i} \checkmark = \frac{-2i}{-1} \checkmark \Rightarrow 2i \checkmark$	(3)
2.2(a)	$(2i - 3)^* = -3 - 2i \checkmark$	(1)
2.2(b)	$i^{206}(2 - 3i)^2$ $206 \div 4 = 51 \text{ res } 2$ $= i^2 \checkmark (4 - 12i + 9i^2) \checkmark$ $= -1(4 - 12i - 9) \checkmark$ $= -1(-5 - 12i) \checkmark$ $= 5 + 12i \checkmark$	(5)
2.2(c)	$\frac{2+i}{2+2i} \times \frac{2-2i}{2-2i} \checkmark$ $= \frac{4 - 4i + 2i - 2i^2 \checkmark}{4 - 4i^2 \checkmark}$ $= \frac{6 - 2i \checkmark}{8 \checkmark}$ OF/OR $= \frac{3}{4} - \frac{1}{4}i$	(5)
2.3(a)	$(2a - i) - (2 - 3ai) = 2a - i - 2 + 3ai \checkmark$ Suiwer imaginêr/ <i>Purely imaginary: $2a - 2 = 0 \checkmark$</i> $a = 1 \checkmark$	(3)
2.3(b)	$-i + 3ai \checkmark$ $= -i + 3i$ $= 2i \checkmark$	(2)
2.3	 <p>Toets/test: $(-1 - 2i) + (5 + 2i) = 4 + 0i$</p>	(5; 2) ✓ (-1; -2) ✓ (4; 0) ✓ Vorm/shape ✓

Vraag / Question 3

[18 punte / marks]

NR. NO.	ANTWOORD / ANSWER	PUNTE / MARKS
3.1(a)	$\begin{pmatrix} 1 & 1 & -1 \\ -3 & -2 & -1 \\ 0 & -3 & 1 \end{pmatrix} \checkmark \begin{pmatrix} x \\ y \\ z \end{pmatrix} \checkmark = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix} \checkmark$	(3)
3.1(b)	$\det A = \begin{vmatrix} 1 & 1 & -1 \\ -3 & -2 & -1 \\ 0 & -3 & 1 \end{vmatrix} = 1 \begin{vmatrix} -2 & -1 \\ -3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix} \checkmark$ $= (-2 - 3) + 3(1 - 3) = -11 \checkmark$ <p>OF/OR</p> $\det A = \begin{vmatrix} 1 & 1 & -1 \\ -3 & -2 & -1 \\ 0 & -3 & 1 \end{vmatrix} = 1 \begin{vmatrix} -2 & -1 \\ -3 & 1 \end{vmatrix} - 1 \begin{vmatrix} -3 & -1 \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} -3 & -2 \\ 0 & -3 \end{vmatrix} \checkmark$ $= (-2 - 3) - 1(-3) - 1(9) = -11 \checkmark$ $\det A_z = \begin{vmatrix} 1 & 1 & -5 \\ -3 & -2 & 0 \\ 0 & -3 & 1 \end{vmatrix} \checkmark = 1 \begin{vmatrix} -2 & 0 \\ -3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & -5 \\ -3 & 1 \end{vmatrix} \checkmark$ $= -2 + 3(1 - 15) = -44 \checkmark$ <p>OF/OR</p> $\det A_z = \begin{vmatrix} 1 & 1 & -5 \\ -3 & -2 & 0 \\ 0 & -3 & 1 \end{vmatrix} \checkmark = 1 \begin{vmatrix} -2 & 0 \\ -3 & 1 \end{vmatrix} - 1 \begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 5 \begin{vmatrix} -3 & -2 \\ 0 & -3 \end{vmatrix} \checkmark$ $= -2 - 1(-3) - 5(9) = -44 \checkmark$ $z = \frac{\det A_z}{\det A} = \frac{-44}{-11} \checkmark = 4 \checkmark$	(7) <i>detA ✓✓</i> <i>detA_z ✓✓✓</i> <i>z ✓✓</i>
3.2(a)	$\begin{pmatrix} 1 & -1 \\ a & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} \checkmark$ $\det A = \begin{vmatrix} 1 & -1 \\ a & 1 \end{vmatrix} = 1 + a \checkmark$ $\det A_x = \begin{vmatrix} 5 & -1 \\ 7 & 1 \end{vmatrix} \checkmark = 5 + 7 = 12 \checkmark$ $x = \frac{12}{1 + a} \checkmark$	(5)
3.2(b)	$4 = \frac{12}{1 + a} \checkmark \quad 4 + 4a = 12 \checkmark$ $4a = 8 \quad a = 2 \checkmark$	(3)

NR. NO.	ANTWOORD / ANSWER	PUNTE / MARKS
4.1(a)	D $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \checkmark$	(1)
4.1(b)	B $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$	(1)
4.1(c)	E $\begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ $= \begin{pmatrix} -1+2 & 1+3 \end{pmatrix}$ $= \begin{pmatrix} 1 & 4 \end{pmatrix} \checkmark$	(1)
4.1(d)	F $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \checkmark$	(1)
4.1(e)	C $-1 \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ $= \begin{pmatrix} -1 & 1 \\ -2 & -3 \end{pmatrix} \checkmark$	(1)
4.2(a)	$\det \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \checkmark$ $= 3 + 2$ $= 5 \checkmark$	(2)
4.2(b)	$\begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix} - 2 \begin{pmatrix} 1 & \frac{1}{2} \\ -2 & 4 \end{pmatrix}$ $= \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ -4 & -8 \end{pmatrix} \checkmark$ OF/OR $= \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ 4 & 8 \end{pmatrix}$ $= \begin{pmatrix} -3 & 0 \\ 6 & 5 \end{pmatrix} \checkmark$	(2)
4.2(c)	$\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ -2 & 0 \end{pmatrix} \times \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$ $\checkmark \quad \checkmark$ $= \begin{pmatrix} -1+4 & 1-6 \\ -3-2 & 3+3 \\ 2 & -2 \end{pmatrix}$ $= \begin{pmatrix} 3 & -5 \\ -5 & 6 \\ 2 & -2 \end{pmatrix} \checkmark$	(3)

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4.3	$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 2 \\ 0 & 2 & -2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} \checkmark - 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} \checkmark$ $= (-2 - 4) \checkmark - (6) \checkmark$ $= -12 \checkmark$ <p>OF/OR</p> $\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 2 \\ 0 & 2 & -2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} \checkmark - 3 \begin{vmatrix} 0 & -1 \\ 2 & -2 \end{vmatrix} \checkmark$ $= (-2 - 4) \checkmark - 3(-2) \checkmark$ $= -12 \checkmark$	(5)

Vraag / Question 5

[16 punte / marks]

NR. NO.	ANTWOORD / ANSWER	PUNTE / MARKS
5.1	$\frac{5x^2 + 6x + 3}{x^3 + 2x^2 + x} = \frac{5x^2 + 6x + 3}{x(x+1)^2} \checkmark \equiv \frac{A}{x} + \frac{B}{x(x+1)} + \frac{C}{(x+1)^2} \checkmark$ $5x^2 + 6x + 3 \equiv A(x+1)^2 + Bx(x+1) + Cx \checkmark$ $5x^2 + 6x + 3 \equiv Ax^2 + 2Ax + A + Bx^2 + Bx + Cx \checkmark$ <p>$x = 0: \checkmark \quad 3 = A(1)^2 \Rightarrow A = 3 \checkmark$</p> <p>$x = -1: \checkmark \quad 5(-1)^2 + 6(-1) + 3 = C(-1)$ $2 = -C$ $C = -2 \checkmark$</p> <p>$x:$ OF/OR $x^2:$</p> $6 = 2A + B + C \qquad \qquad \qquad 5 = A + B$ $6 = 2(3) + B - 2 \qquad \qquad \qquad 5 = 3 + B$ $B = 2 \checkmark \qquad \qquad \qquad B = 2 \checkmark$ $\frac{3x^2 + 2x + 2}{(x+1)^2(x-2)} = \frac{1}{x+1} - \frac{1}{(x+1)^2} + \frac{2}{x-2} \checkmark$	(10)

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5.2	$1 - x + x^3 \equiv Ax^2(x - 1) + Bx(x - 1) + C(x - 1) + x^3$ $1 - x + x^3 \equiv Ax^3 - Ax^2 + Bx^2 - Bx + Cx - C + x^3 \checkmark$ $x = 0: \checkmark \quad 1 = -C \Rightarrow C = -1 \checkmark$ $x: -1 = -B + C \Rightarrow -1 = -B - 1 \Rightarrow B = 0 \checkmark$ $x^2: 0 = -A + B \Rightarrow 0 = -A \Rightarrow A = 0 \checkmark$ $\frac{(1 - x + x^3)}{x^4 - x^3} \equiv -\frac{1}{x^3} + \frac{1}{x - 1} \checkmark$	(6)

Vraag / Question 6

[14 punte / marks]

NR. NO.	ANTWOORD / ANSWER	PUNTE / MARKS
6.1(a)	$(f \circ g)(x) = (\sqrt{x+1})^2 - 2 \checkmark \Rightarrow x + 1 - 2 \Rightarrow x - 1 \checkmark$	(2)
6.1(b)	$(g \circ f)(x) = \sqrt{(x^2 - 2) + 1} \checkmark \Rightarrow \sqrt{x^2 - 1} \checkmark$	(2)
6.2	$f(x) = \sqrt[3]{x}$ $g(x) = 1 - x^2$	(2)
6.3	$x < 1: f(1) = (1)^2 - 2 \Rightarrow -1 \quad (1; -1) \checkmark$ $x = 1: f(1) = 2(1) \Rightarrow 2 \quad (1; 2) \checkmark$ $x > 1: f(1) = \frac{1}{1} \Rightarrow 1 \quad (1; 1) \checkmark$	(5)
		$x^2 - 2$ vorm/form \checkmark $\frac{1}{x}$ vorm/form \checkmark
6.4	$a = -1$ $b = 2$ $c = 1$	(3)

Vraag / Question 7

[20 punte / marks]

NR. NO.	ANTWOORD / ANSWER	PUNTE / MARKS
7.1(a)	$2(-1; 3) - (2; 4)$ $= (-2; 6) \checkmark - (2; 4)$ $= (-4; 2) \checkmark$	(2)
7.1(b)	$u = \sqrt{(-1)^2 + (-3)^2} \checkmark$ $= \sqrt{10} \checkmark$	(2)
7.1(c)	$\tan\theta = \frac{4}{2} \checkmark$ $\theta = 1,107148 \approx 1,11 \checkmark$	(2)
7.2(a)	$a = \left(-\frac{1}{\sqrt{5}} \checkmark; \frac{2}{\sqrt{5}} \checkmark\right)$	(2)
7.2(b)	$a \cdot b = -1 \times 3 \checkmark + 2 \times -2 \checkmark$ $= -3 - 4$ $= -7 \checkmark$	(3)
7.2(c)	$\cos\theta = \frac{a \cdot b}{ a b }$ $\cos\theta = \frac{-7}{5\sqrt{13}} \checkmark$ $\theta = 1,9696 \approx 1,97 \checkmark$	(2)
7.2(d)	$a \cdot c = 0 \checkmark$ $-1 \times 3 + 2y = 0 \checkmark$ $2y = 3$ $y = \frac{3}{2} \checkmark$	(3)
7.3(a)	$\frac{60^\circ}{180^\circ} \times \pi = \frac{\pi}{3} \checkmark$	(1)
7.3(b)	$D\hat{A}C = \pi - \frac{\pi}{3} \checkmark = \frac{2\pi}{3} \checkmark$ $s = \theta r = \frac{2\pi}{3} \times 10 = \frac{20\pi}{3} = 20,94 \text{ cm} \checkmark$	(3)

- EINDE VAN DIE MEMORANDUM / END OF THE MEMORANDUM -